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The Evolutes of Log-Aesthetic Planar Curves and the Drawable Boundaries of the Curve Segments

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ABSTRACT

This paper investigates the two characteristics of log-aesthetic curves. We first show that the evolutes of log-aesthetic curves are also log-aesthetic curves. We provide a proof that the evolute of a log-aesthetic curve with the shape parameter α is a log-aesthetic curve with the shape parameter $-1/(\alpha-2)$. Then, we present a method for drawing the theoretical drawable boundaries of log-aesthetic curve segments with $\alpha<0$ or $\alpha>1$. We compare the theoretical drawable boundaries with the experimental drawable regions and show that they agree well except when α is close to 0.

Keywords: log-aesthetic planar curves, evolute, drawble region
DOI: 10.3722/cadaps.2012.721-731

1 INTRODUCTION

Various kinds of spirals appear in artificial and the natural objects. For example, for designing highways, the Clothoid curves whose curvature varies linearly according to the arc length are widely used. Logarithmic spirals, whose radius of curvature varies linearly according to the arc length appears in nature, for example, in certain growing forms such as nautilus shells and sunflower heads. Log-aesthetic planar curves [1,2,4,5] are curves which can be considered as a generalization of the Clothoid and the logarithmic spirals. Based on Haradaís analysis [1,2], it is known that the log-aesthetic curves appears in artificial and the natural objects, such as the characteristic lines of automobiles, butterflyís wings, and the Japanese swords.

This paper investigates the two characteristics about log-aesthetic planar curves. We first investigate the evolutes of log-aesthetic planar curves. The evolutes [3] are curves which are the locus of the curvature center of the generatrix. Then we present a method for drawing the drawable boundaries of log-aesthetic curve segments with $\alpha<0$ or $\alpha>1$. We compare the theoretical drawable boundaries with the experimental drawable regions [5] and show that they almost agree except when α is close to 0.

2 REVIEW OF LOG-AESTHETIC PLANER CURVES

Log-aesthetic planar curves are curves whose logarithmic curvature graphs[1,2,4,5,6] are represented by straight lines. Harada et al. have analyzed many aesthetic curves in artificial and the natural objects and found that the logarithmic curvature graphs(LCGs) can be approximated by straight lines. The linearity of the LCG constrains that the curvature is monotonically varying. Miura derived the general formula of log-aesthetic curves[4]. Then Yoshida and Saito have clarified the overall shapes of log-aesthetic curves and presented a method for interactively drawing curve segments.

Let s, ρ be the arc length and the radius of curvature, respectively. Since we assume that the curvature is monotonically varying, we assume that ρ increases as s increases. With this assumption, $\mathrm{d}s/\mathrm{d}\rho > 0$. The logarithmic curvature graph is shown in Fig. 1. Let α be the slope of a straight line in the LCG. We call α shape parameter. When $\alpha = -1, 0, 1$ and 2, the log-aesthetic curves becomes the Clothiod, Nielsenís spiral, the logarithmic spiral and the circle involute, respectively. Let the intercept of that straight line be c. The linearity of LCG can be represented by

$$\log\left(\rho\frac{\mathrm{d}s}{\mathrm{d}\rho}\right) = \alpha \log\rho + c\,. \tag{1}$$

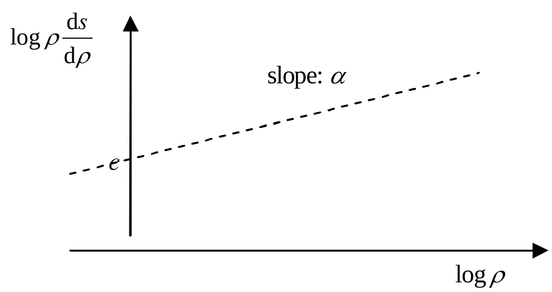


Fig. 1: Logarithmic curvature graph.

To derive the equations of log-aesthetic planar curves, we introduce the standard form[5]. In the standard form, we call any point on a log-aesthetic curve whose radius of curvature is not 0 or ∞ the reference point $\mathbf{P}_r$. $\mathbf{P}_r$ is determined by a parameter Λ, which is introduced shortly. In the standard form, we give the following constraints on $\mathbf{P}_r$. See Fig. 2.

(1) Translation: The reference point $\mathbf{P}_r$ is placed at the origin.

(2) Rotation: At the reference point $\mathbf{P}_r$, the tangent vector is directed toward the positive side of x-axis.

(3) Scaling: The radius of curvature at the reference point $\mathbf{P}_r$ is 1.

At the reference point, arc length s and tangential angle θ are both set to 0. See Fig. 2.

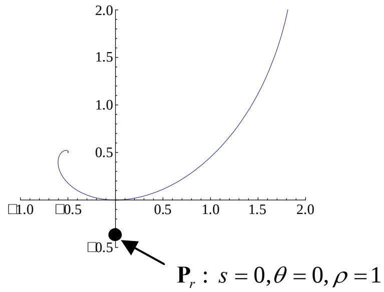


Fig. 2: A log-aesthetic curve ($\alpha = 1$) in the standard form.

Modifying Eq.(1), we get

$$\frac{\mathrm{d}s}{\mathrm{d}\rho} = \rho^{\alpha-1} e^{c} . \tag{2}$$

At the reference point $\mathbf{P}_r$, $\mathrm{d}s/\mathrm{d}\rho = e^c$ since $\rho = 1$. Let $\mathrm{d}s/\mathrm{d}\rho$ at the reference point $\mathbf{P}_r$ be $1/\Lambda$. Thus $\Lambda = e^{-c}$ and $0 < \Lambda < \infty$. Using Λ, Eq. (2) becomes

$$\frac{\mathrm{d}s}{\mathrm{d}\rho} = \frac{\rho^{\alpha-1}}{\Lambda} . \tag{3}$$

Integrating Eq.(3) with respect to ρ such that s becomes 0 when $\rho = 1$, we get

$$s = \int \frac{\mathrm{d}s}{\mathrm{d}\rho}\mathrm{d}\rho = \begin{cases} \frac{1}{\Lambda}\log\rho & \text{if } \alpha=0 \\ \frac{1}{\Lambda\alpha}\left(\rho^{\alpha}-1\right) & \text{otherwise} \end{cases} \tag{4}$$

Solving Eq.(4) with respect to ρ, we get

$$\rho = \begin{cases} e^{\Lambda s} & \text{if } \alpha = 1 \\ \left(\Lambda\alpha s + 1\right)^{\frac{1}{\alpha}} & \text{otherwise} \end{cases} . \tag{5}$$

Using Eq.(3) and $\mathrm{d}s = \rho\mathrm{d}\theta$,

$$\frac{\mathrm{d}\theta}{\mathrm{d}\rho} = \frac{1}{\rho}\frac{\mathrm{d}s}{\mathrm{d}\rho} = \frac{\rho^{\alpha-2}}{\Lambda} . \tag{6}$$

Integrating Eq.(6) with respect to ρ such that θ becomes 0 at $\rho = 1$, we get

$$\theta = \int \frac{\mathrm{d}\theta}{\mathrm{d}\rho}\mathrm{d}\rho = \begin{cases} \frac{1}{\Lambda}\log\rho & \text{if } \alpha=1 \\ \frac{\rho^{\alpha-1}-1}{\Lambda(\alpha-1)} & \text{otherwise} \end{cases} . \tag{7}$$

Since ρ changes from 0 to ∞, s and θ may have an upper or a lower bound depending on α. See Table 1. The lower (or upper) bound of s is derived by putting $\rho = 0$ (or $\rho = \infty$) into Eq. (4). Similarly, the lower (or upper) bound of θ is derived by putting $\rho = 0$ (or $\rho = \infty$) into Eq. (7).

	s			θ	
	lower bound ($\rho=0$)	upper bound ($\rho=\infty$)		lower bound ($\rho=0$)	upper bound ($\rho=\infty$)
$\alpha<0$	$-\infty$	$-1/(\Lambda\alpha)$	$\alpha<1$	$-\infty$	$1/(\Lambda(1-\alpha))$
$\alpha=0$	$-\infty$	∞	$\alpha=1$	$-\infty$	∞
$\alpha>0$	$-1/(\Lambda\alpha)$	∞	$\alpha>1$	$1/(\Lambda(1-\alpha))$	∞

Tab. 1: The bounds of s and θ

Solving Eq.(7) with respect to ρ,

$$\rho=\begin{cases} e^{\Lambda\theta} & \text{if } \alpha=1 \\ \left((\alpha-1)\Lambda\theta+1\right)^{\frac{1}{\alpha-1}} & \text{otherwise} \end{cases} \tag{8}$$

The equations of log-aesthetic curves $\mathbf{P}(\theta)=\left[\mathbf{P}_x(\theta) \quad \mathbf{P}_y(\theta)\right]$ in terms of tangential angle θ is

$$\begin{aligned} \mathbf{P}_x(\theta) &= \int_0^\theta \rho(\varphi)\cos\varphi \mathrm{d}\varphi \\ &= \begin{cases} \int_0^\theta e^{\Lambda\varphi}\cos\varphi \mathrm{d}\varphi & \text{if } \alpha=1 \\ \int_0^\theta \left((\alpha-1)\Lambda\varphi+1\right)^{\frac{1}{\alpha-1}}\cos\varphi \mathrm{d}\varphi & \text{otherwise} \end{cases} \end{aligned} \tag{9}$$

$$\begin{aligned} \mathbf{P}_y(\theta) &= \int_0^\theta \rho(\varphi)\sin\varphi \mathrm{d}\varphi \\ &= \begin{cases} \int_0^\theta e^{\Lambda\varphi}\sin\varphi \mathrm{d}\varphi & \text{if } \alpha=1 \\ \int_0^\theta \left((\alpha-1)\Lambda\varphi+1\right)^{\frac{1}{\alpha-1}}\sin\varphi \mathrm{d}\varphi & \text{otherwise} \end{cases} \end{aligned} \tag{10}$$

The equations of log-aesthetic curves in terms of arc length can be derived by representing θ in terms of s. Putting Eq.(5) into $\mathrm{d}\theta/\mathrm{ds}=1/\rho$ and integrating with respect to s such that θ becomes 0 when $s=0$, we get

$$\theta=\int\frac{\mathrm{d}\theta}{\mathrm{d}s}\mathrm{d}s=\begin{cases} 1-e^{-\Lambda s} & \text{if } \alpha=0 \\ \dfrac{\log(\Lambda s+1)}{\Lambda} & \text{if } \alpha=1 \\ \dfrac{(\Lambda\alpha s+1)^{\left(1-\frac{1}{\alpha}\right)}-1}{\Lambda(\alpha-1)} & \text{otherwise} \end{cases}. \tag{11}$$

Now the equations of log-aesthetic curves $\mathbf{Q}(s)=\left[\mathbf{Q}_x(s) \quad \mathbf{Q}_y(s)\right]$ in terms of arc length s is

$$\mathbf{Q}_x(s)=\int_0^s \cos(\theta(u))\mathrm{du}=\begin{cases}\int_0^s \cos\left(1-e^{-\Lambda u}\right)\mathrm{d}u & \text{if } \alpha=0\\ \int_0^s \cos\left(\frac{\log(\Lambda u+1)}{\Lambda}\right)\mathrm{d}u & \text{if } \alpha=1\\ \int_0^s \cos\left(\frac{(\Lambda\alpha u+1)^{(1-1/\alpha)}-1}{\Lambda(\alpha-1)}\right)\mathrm{d}u & \text{otherwise}\end{cases} \tag{12}$$

$$\mathbf{Q}_y(s)=\int_0^s \sin(\theta(u))\mathrm{du}=\begin{cases}\int_0^s \sin\left(1-e^{-\Lambda u}\right)\mathrm{d}u & \text{if } \alpha=0\\ \int_0^s \sin\left(\frac{\log(\Lambda u+1)}{\Lambda}\right)\mathrm{d}u & \text{if } \alpha=1\\ \int_0^s \sin\left(\frac{(\Lambda\alpha u+1)^{(1-1/\alpha)}-1}{\Lambda(\alpha-1)}\right)\mathrm{d}u & \text{otherwise}\end{cases}. \tag{13}$$

As written in [5], the Eq.(9) and (10) in terms of tangential angle is preferable when we compute the points of log-aesthetic curves near the inflection point. Eq. (12) and (13) are preferable when the value of Λ is close to 0, which means the curve is close to a circular arc. Detail characteristics of log-aesthetic curves are described in [5].

3 THE EVOLUTES OF LOG-AESTHETIC CURVES

The evolute[3] of a curve is the locus of the curvature center of the curve. Let $\mathbf{P}(t)$ represent a curve with parameter t. Let $\kappa(t),\mathbf{N}(t)$ be the curvature and the unit normal vector at parameter t. The evolute of the curve $\mathbf{P}(t)$ is

$$\mathbf{E}(t)=\mathbf{P}(t)+\frac{1}{\kappa(t)}\mathbf{N}(t). \tag{14}$$

Let $\breve{s},\breve{\rho},\breve{\theta},\breve{\alpha}$ be the arc length, the radius of curvature, the tangential angle and the slope of the LCG of the evolute of a log-aesthetic curve. From Eq. (6), if the evolute can be represented by

$$\frac{\mathrm{d}\breve{\theta}}{\mathrm{d}\breve{\rho}}=\breve{c}\breve{\rho}^{\breve{\alpha}-2} \tag{15}$$

with some constant $\breve{c}$, we can say that the evolute is also a log-aesthetic curve. We first consider the case when $\alpha\neq 1$, then we consider the case when $\alpha=1$.

3.1 The case when $\alpha\neq 1$

We first prove that the evolute of a log-aesthetic curve with $\alpha\neq 1$ is also a log-aesthetic curves. Taking the power of $(\alpha-2)$ of the both sides of Eq. (8), we get

$$\rho^{\alpha-2}=\left((\alpha-1)\Lambda\theta+1\right)^{\frac{\alpha-2}{\alpha-1}}. \tag{16}$$

From the relationship between an evolute and its generatrix, we have $\mathrm{d}\rho=\mathrm{d}\breve{s}$ and $\mathrm{d}\theta=\mathrm{d}\breve{\theta}$. Using these relationships and Eq.(6) and (16), we obtain

$$\breve{\rho}=\frac{\mathrm{d}\breve{s}}{\mathrm{d}\breve{\theta}}=\frac{\mathrm{d}\rho}{\mathrm{d}\theta}=\Lambda\left((\alpha-1)\Lambda\theta+1\right)^{-\frac{\alpha-2}{\alpha-1}}. \tag{17}$$

Solving Eq. (17) with respect to θ,

$$\theta = \frac{\left(\hat{\rho}/\Lambda\right)^{-\frac{(\alpha-1)}{(\alpha-2)}} - 1}{\Lambda(\alpha-1)}. \tag{18}$$

Since θ and $\hat{\theta}$ differ by $\pi/2$,

$$\hat{\theta} = \frac{\left(\hat{\rho}/\Lambda\right)^{-\frac{(\alpha-1)}{(\alpha-2)}} - 1}{\Lambda(\alpha-1)} + \frac{\pi}{2} \tag{19}$$

Differentiating Eq. (19) with respect to $\hat{\rho}$, we get

$$\begin{aligned}\frac{\mathrm{d}\hat{\theta}}{\mathrm{d}\hat{\rho}} &= \frac{-1}{\alpha-2}\frac{\left(\rho/\Lambda\right)^{-\frac{\alpha-1}{\alpha-2}-1}}{\Lambda^2} \\ &= \frac{-1}{\alpha-2}\Lambda^{\frac{\alpha-1}{\alpha-2}-1}\rho^{-\frac{\alpha-1}{\alpha-2}-1}.\end{aligned} \tag{20}$$

Comparing Eq. (15) and Eq. (20), we get

$$c = \frac{-1}{\alpha-2}\Lambda^{\frac{\alpha-1}{\alpha-2}-1}, \tag{21}$$

$$\hat{\alpha} - 2 = -\frac{\alpha-1}{\alpha-2} - 1. \tag{22}$$

Now we have proved that the evolutes of log-aesthetic curves with $\alpha \neq 1$ are also log-aesthetic curves. Solving Eq. (22) with respect to $\hat{\alpha}$, we get

$$\hat{\alpha} = -\frac{1}{\alpha-2}. \tag{22}$$

This means that the slope of the LCG of the evolute of a log-aesthetic curves with the slope of the LCG α is $-1/(\alpha-2)$.

3.2 The case when $\alpha = 1$

Now we consider log-aesthetic curves with $\alpha = 1$. Taking the power of $\alpha - 2$ of the both sides of Eq. (8), we get

$$\rho^{\alpha-2} = e^{\Lambda\theta(\alpha-2)}. \tag{23}$$

Using the relationships of $\mathrm{d}\rho = \mathrm{d}\hat{s}, \mathrm{d}\theta = \mathrm{d}\hat{\theta}$ and Eq. (6), (16), we get

$$\hat{\rho} = \frac{\mathrm{d}\hat{s}}{\mathrm{d}\hat{\theta}} = \frac{\mathrm{d}\rho}{\mathrm{d}\theta} = \Lambda e^{-\Lambda\theta(\alpha-2)}. \tag{24}$$

Solving Eq.(24) with respect to θ, we get

$$\theta = -\frac{\log\hat{\rho}}{\Lambda(\alpha-2)} \tag{25}$$

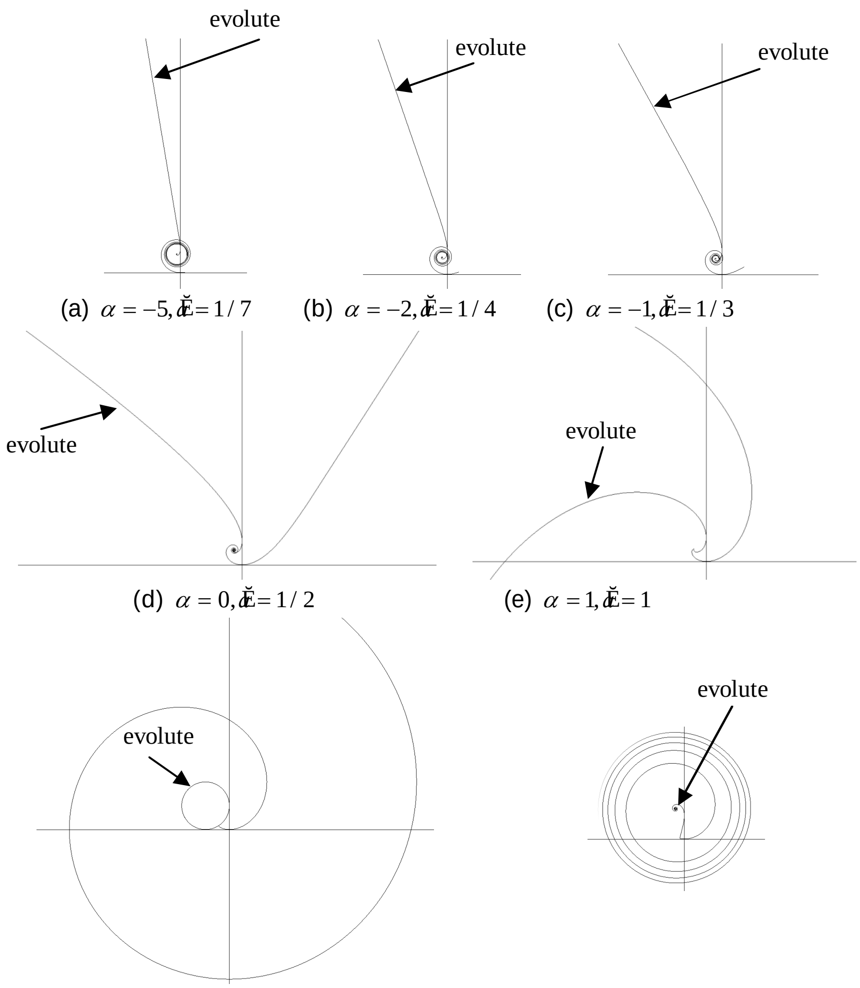
Since θ and $\hat{\theta}$ differ with $\pi/2$,

$$\hat{\theta} = -\frac{\log\hat{\rho}}{\Lambda(\alpha-2)} + \frac{\pi}{2}.$$

Differentiating $\hat{\theta}$ with respect to $\hat{\rho}$, we get

$$\frac{\mathrm{d}\hat{\theta}}{d\hat{\rho}} = \frac{1}{\Lambda(\alpha-2)}\hat{\rho}^{-1} \tag{26}$$

Comparing the right side of Eq. (15) with that of Eq. (26), we get



(a) $\alpha = -5,$ Ě$= 1/7$ (b) $\alpha = -2,$ Ě$= 1/4$ (c) $\alpha = -1,$ Ě$= 1/3$

(d) $\alpha = 0,$ Ě$= 1/2$ (e) $\alpha = 1,$ Ě$= 1$

(f) $\alpha = 2,$ Ě$= \pm\infty$ (g) $\alpha = 5,$ Ě$= -1/3$

Fig. 3: Log-aesthetic curves and their evolutes.

$$c = \frac{1}{\Lambda(\alpha - 2)}, \tag{27}$$

Ě$- 2 = -1$. (28)

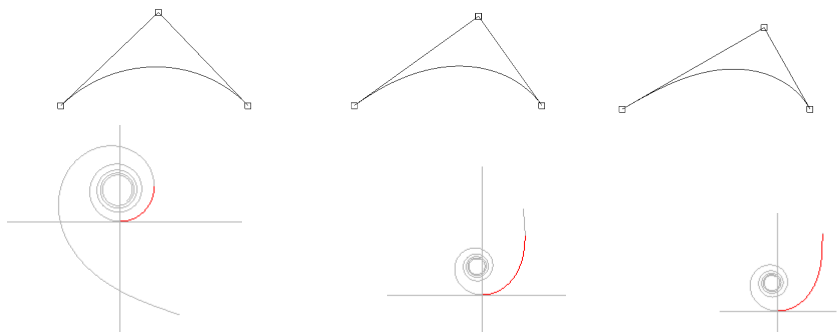
Thus the evolutes of log-aesthetic curves with $\alpha = 1$ are also log-aesthetic curves with Ě$= 1$. Therefore, Eq. (22) also holds when $\alpha = 1$.

3.3 Examples of evolutes of log-aesthetic curves

As proved above, the evolutes of log-aesthetic curves are also log-aesthetic curves and the shape parameters of the generatrix and its evolute are related by Eq. (22). Fig. 3 shows log-aesthetic curves with $\alpha = -5, -2, -1, 0, 1, 2, 5$ and their evolutes. The evolute of the logarithmic spiral ($\alpha = 1$) is also the logarithmic spiral. The evolute of a circle involute ($\alpha = 2$) is a circle ($\alpha = \pm\infty$). As the absolute value of α gets larger, the evolute gets closer to a Nielsenís spiral ($\alpha = 0$).

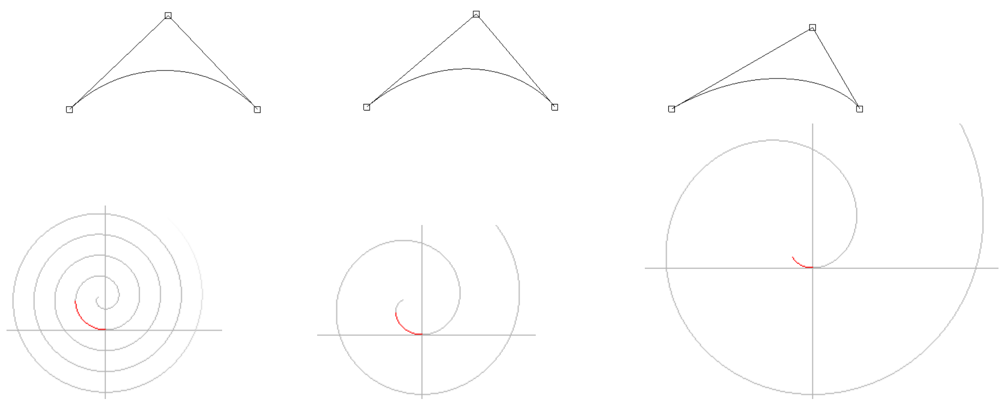
4 THE DRAWABLE BOUNDARIES OF LOG-AESTHETIEC CURVE SEGMENTS

In this section, we introduce theoretical drawable boundaries for log-aesthetic curve segments with $\alpha < 1$ or $\alpha > 1$ and compare the theoretical drawable regions with the experimental drawable regions. Log-aesthetic curves with $\alpha < 0$ or $\alpha > 1$ are curves with points at $\rho = \infty$ or points at $\rho = 0$, respectively. See [5] for the characteristics of overall shapes of log-aesthetic curves and the algorithm for drawing the curve segment.



(a) $\Lambda = 0.08$ (b) $\Lambda = 0.30$ (c) $\Lambda = 1/\pi$

Fig. 4: Log-aesthetic curve segments ($\alpha = -1$)



(a) $\Lambda = 0.08$ (b) $\Lambda = 0.30$ (c) $\Lambda = 2/\pi$

Fig. 5: Log-aesthetic curve segments ($\alpha = 2$).

We introduce the theoretical drawable regions based on the following observations. Fig. 4 and 5 shows log-aesthetic curve segments and the corresponding overall shapes with $\alpha = -1$ and $\alpha = 2$, respectively. Fig. 4 ($\alpha = -1$) is chosen as a representative case of log-aesthetic curves with inflection points. Similarly, Fig. 5 ($\alpha = 2$) is chosen as a representative case with points at $\rho = 0$.

In all of the curve segments in Fig. 4 and 5, the change of tangential angle are fixed to $\pi/2$. When $\alpha < 0$, Λ takes a value between 0 and $1/\left(\theta_d(1-\alpha)\right)$, where θ_d is the change of the tangential angle of the curve segment. As shown in Fig. 4, when Λ is close to 0, the curve segment is close to a circular curve segment. When Λ is 0, it is known that the curve segment is a circular arc[5]. As Λ gets larger to its bound $1/\left(\theta_d(1-\alpha)\right)$, the curve segment gets closer to the inflection point in the overall shape. At $\Lambda = 1/\left(\theta_d(1-\alpha)\right)$, the curve segment includes the inflection point. From this observation, we can find a point on the drawable boundary using $\Lambda = 1/(\theta_d(1-\alpha))$ when θ_d is fixed. Modifying θ_d, we can find a theoretical drawable boundary. We will introduce an algorithm for drawing theoretical drawable boundaries shortly.

When $\alpha > 1$, Λ takes a value between 0 and $1/(\theta_d(\alpha-1))$. As shown in Fig. 5, when Λ is close to 0, the curve segment is close to a circular arc. At $\Lambda = 1/(\theta_d(\alpha-1))$, the curve segment includes the point at $\rho = 0$.

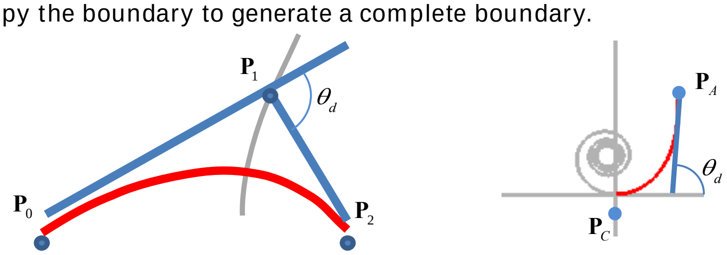
Now we introduce an algorithm for drawing a drawable boundary.

(1) Let the three òcontrol pointsó for drawing a curve segment be $\mathbf{P}_0, \mathbf{P}_1, \mathbf{P}_2$.

(2) Assume that θ_d is known. In the overall shape, we compute a curve segment using $\Lambda = 1/(\theta_d(1-\alpha))$ when $\alpha < 0$ or $\Lambda = 1/(\theta_d(\alpha-1))$ when $\alpha > 1$. As written in [5], when $\alpha < 0$, the curve segment whose tangential angle is between 0 and θ_d is used. When $\alpha > 1$ the curve segment whose tangential angle is between 0 and θ_d is used.

(3) Perform a similarity transformation such that the origin goes to $\mathbf{P}_2$ and the other endpoint of the curve segment goes to $\mathbf{P}_0$. Using the information of the curve segment, we draw tangent lines at $\mathbf{P}_0$ and $\mathbf{P}_2$ and then compute the intersection of the two tangent lines. The intersection point is the point of a drawable boundary corresponding to θ_d. In the example of Fig. 6, $\mathbf{P}_A$ and $\mathbf{P}_C$ are transformed to $\mathbf{P}_0$ and $\mathbf{P}_2$, respectively, to compute the point $\mathbf{P}_1$ on a drawable boundary.

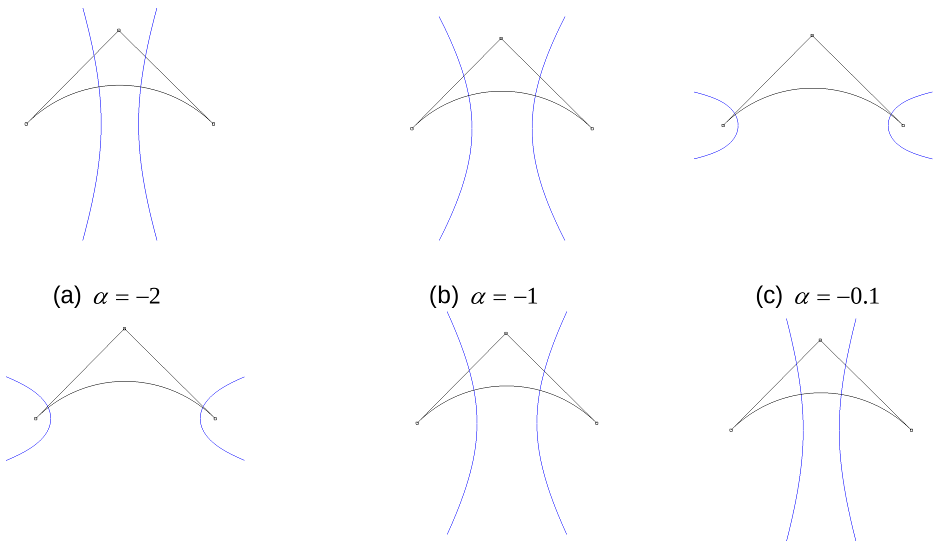
(4) By modifying θ_d from 0 to π, we can draw a drawable boundary as shown in Fig. 6(a). Using the symmetry, we copy the boundary to generate a complete boundary.



(a) A curve segment and a drawable boundary (b) Overallshape

Fig. 6: Drawing a drawable boundary.

Fig. 7 shows theoretical drawable boundaries drawn using the above algorithm. If the second òcontrol pointó, which corresponds to $\mathbf{P}_1$ in Fig. 6(a) is within the drawable boundaries, the curve segment is òtheoreticallyó drawable. By theoretically, we mean that there are cases where curve segments are not drawable even when the second òcontrol pointó is within the drawable boundary. Fig. 8 shows such a case. In our implementation, we found that when α is close to 0 the drawable boundary does not agree with the experimental drawable boundary due to inexact numerical computation caused by the use of floating-point numbers.



(a) $\alpha = -2$ (b) $\alpha = -1$ (c) $\alpha = -0.1$

(d) $\alpha = 1.1$ (e) $\alpha = 2$ (f) $\alpha = 3$

Fig. 7: Theoretical drawable boundary.

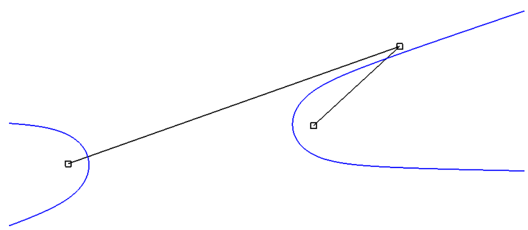
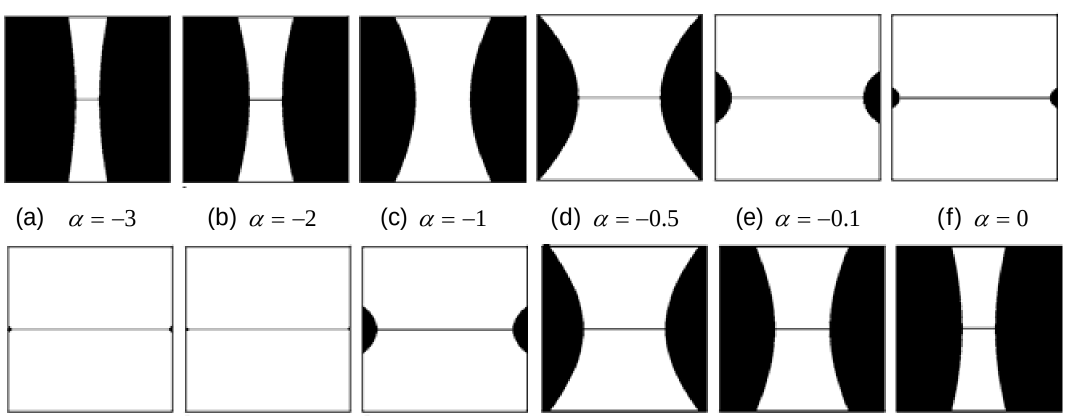


Fig. 8: The case where the drawable boundary does not agree with the experimental drawable region ($\alpha = -0.1$).



(a) $\alpha = -3$ (b) $\alpha = -2$ (c) $\alpha = -1$ (d) $\alpha = -0.5$ (e) $\alpha = -0.1$ (f) $\alpha = 0$

(g) $\alpha = 0.1$ (h) $\alpha = 0.2$ to 0.8 (i) $\alpha = 1.1$ (j) $\alpha = 1.5$ (j) $\alpha = 2$ (k) $\alpha = 3$

Fig. 9: Experimental drawable regions.

Fig. 9 shows the experimental drawable regions of log-aesthetic curves segments with various α. In each rectangle, the coordinates of the bottom left corner is (-1,-1) and the coordinates of the top right corner is (1,1). In the figure, the first “control point” is placed at (-1,0) and the third “control point” is placed at (1,0). The second control point is moved within the rectangles. If a curve segment is drawable, the point (pixel) of the second control point is drawn with white. If not drawble, the point is drawn with black. In comparison with the theoretical boundaries, we confirmed that the drawable boundaries agree well except when α is close to 0. The experimental drawable region in Fig. 9(e) is larger than the theoretical drawable boundary in Fig. 7(c), especially when $\alpha = -0.1$. When α is between 0 and 1, we see black regions (not drawable region) in the experimental drawble regions. This is due to numerical inaccuracies caused by floating point computations, but we don’t have a proof that the curve segment is always drawable when α is between 0 and 1.

5 CONCLUSIONS

We have investigated the two characteristics of log-aesthetic curves. We first provided a proof that the evolute of a log-aesthetic curve with shape parameter α is also a log-aesthetic curve with shape parameter $-1/(\alpha-2)$. Then we presented a method for drawing the theoretical drawable boundaries of log-aesthetic curve segments when $\alpha < 0$ or $\alpha > 1$. We compared the drawable boundaries with the experimental drawable regions and confirmed that they agree well except when α is close to 0 or 1. When α is between 0 and 1, we cannot draw a theoretical drawable boundary. This may indicate that the curve segment is (at least theoretically) drawable. The question that the curve segment is always theoretically drawable when α is between 0 and 1 remains open.

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