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Medial Axis Transform for Tolerance Verification

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ABSTRACT

Medial Axis Transformation (MAT) is proposed as an intermediate representation for tolerance verification. It is assumed that the manufactured part has been scanned and that points on the boundary are available as a dense cloud of points with respect to a single reference frame (Inspection Reference Frame, IRF) that is typical of most scanners in use today. Tolerance verification involves first locating the IRF with respect to the CAD reference frame (CRF) followed by verification as laid out according to appropriate standards. The former involves segmenting the point cloud and fitting surfaces to the segmented set of points. The use of MAT eliminates the need for both segmentation and surface fitting while enabling the verification of tolerances specified with respect to the datum. MAT also avoids the problem of rejection of parts within tolerances that arises in current practice due to the over estimation of the tolerance zone when using the least squares method or other optimization techniques in conventional verification with point data. As only characteristic points in the MAT of the point cloud are processed, the computational effort and the combinatorial complexity are also significantly reduced. A simple case study is presented to illustrate the power of MAT as an intermediate representation in tolerance verification. Current limitations and remaining work to develop the idea further are identified.

Keywords: CAD model, medial axis transformation (MAT), tolerance verification.
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1 INTRODUCTION

With the advent and increasing penetration of scanning devices (both contact and optical), it is now possible to obtain a large number of points on the surface of a part very rapidly. Inspection of a manufactured part is therefore increasingly be a set of points that has to be processed further for verifying if the part is within the prescribed tolerances.

For tolerance verification, measured point data or the geometric object reconstructed from the point data is compared with the nominal CAD model. The verification process varies a little depending on whether the tolerances in the part have been prescribed with respect to one or more datum or not. Typically, the measured point set is obtained with respect to the Inspection Reference Frame (IRF) that is different from the reference frame with respect to which the nominal CAD model is defined (CRF). An important step in the verification process, therefore, is registration (also referred to as localization) of the IRF with respect to the CRF or vice versa. The goal of localization is to find a transformation that optimally positions the measured point set in the CRF. The main step involved in this is establishing a correspondence between the CAD model and the measured point set.

When there are no datum planes involved in the tolerances specified, a correspondence between the CAD model and the point cloud data is established during the localization process itself. In the case of datum being available in the specification, generally, the correspondence is established between the CAD model and the point cloud data before the localization step. For establishing the correspondence between the datums for localization, computation is needed for surface fitting to unstructured measured point data and feature detection. Feature detection in point data is done by the segmentation process [3]. After the segmentation process the surface is fitted to the cloud points using various optimization techniques. Both segmenting and surface fitting are error-prone processes. Thus, there is a need to establish a correspondence between the CAD model and the measured point data without fitting and segmenting point cloud data.

The Iterative closest point (ICP) algorithm is best known and widely used for localization in both the cases. In the case of a specified datum plane, a correspondence is established by fixing the datum plane in the measured data and the ICP algorithm is applied to complete the localization. For models without datum planes, localization is done in two steps- initial pose estimation (approximate solution) and solution refinement. In "approximate solution", the distance is minimized between the CAD model and the point data by using the least mean square distance considering the entire CAD model and point cloud. In "solution refinement", the distance is further reduced by applying the ICP algorithm for points fixed by establishing a correspondence based on geometry.

The ICP shows good convergence only if the design reference frame (DRF) and inspection reference frames (IRF) are kept close to each other to find the general correspondence or otherwise which requires a proper estimation of initial rigid transformation. Also, the ICP algorithm has a basic complexity of $O(N_c N_m)$ where N_c and N_m represent the number of points in the CAD model and point cloud data and requires heavy computations. Several algorithms are proposed to speed up the algorithm; some of them are reduction of the number of iteration, reduction of the number of data points and acceleration of the closest points search [13].

After localization, the verification of tolerances is done by determining the closest points between the measured point data and the surface of the CAD model by using the shortest distance criterion. The verification process does not consider corresponding points between the CAD model and measured point data. This could result in the rejection of good parts or the acceptance of parts out of tolerance. A Singularity problem could also arise at the vertices of polyhedral shapes or other sharp corners of the shape as discussed by Pasupathy et al. [17]. These points are called singular points or points with multiple tolerance values. It is difficult to both specify tolerances and verify the deviation of the manufactured part in the singular regions.

In this context, Medial axis transform (MAT), a conceptually elegant abstraction of shape models can be used for the verification of tolerances. There is a one-to-one correspondence between the geometric model and its medial axis transform. The MAT of the nominal CAD model and the MAT of the cloud points are compared to verify tolerances that have been specified independently or with respect to datum planes. MAT exhibits dimensional reduction and hence reduces verification time as compared with the time required for processing huge amounts of point cloud data. Also, this method has an edge over other methods as it verifies the tolerance of corresponding points as compared to the closest point using the shortest distance method or criterion. It eliminates the segmentation of unstructured point data along with surface fitting for detected features and also eliminates the singularity problem encountered at the corners of the polyhedral shapes. In this paper, it is assumed that the measured data available in a single frame albeit different from that of the CAD frame.

This paper illustrates the power of MAT as an intermediate representation in tolerance verification. The remainder of the paper is organized as follows. Section 2 deals with a brief review of related work. Section 3 describes the procedure for verification of dimensional tolerance of a part using medial axis transform. Section 4 presents some preliminary results followed by a discussion of some issues regarding the tolerance verification process. The paper ends with some concluding remarks.

2 LITERATURE REVIEW

Tolerance specification is a definition of classes of objects that are interchangeable in assembly operations and are functionally equivalent. These classes are variational classes and are sets of solids which, in turn, are sets of points in E^3.Tolerance specifications are representations of variational classes. [19]

These variations in the classes of objects are specified and controlled by the symbolic language of geometric dimensioning and tolerancing (GD&T). This language is used to specify features on a part and is regulated by ASME standard Dimensioning and Tolerancing ASME Y14.5M–2009 principles [1].

These specified tolerances are required to be verified on the manufactured part. The verification process carried out with mechanical gauges, jigs and fixtures is termed *hard-metrology*, whereas verification using measured point data is termed *soft-metrology*. Unfortunately, however, no standard for the verification of tolerance specifications exists. The verification method can be defined as a method which uses point data sampled from an actual part to determine if the part complies with the given dimensional and tolerance specifications [7].

Tolerances are specified with a datum or without depending on the type of tolerance. Localization is done between the CAD model and point cloud data by using specified datum planes or points with geometrical characteristics. As the accuracy of the verification process mainly depends on accurate localization, different methods are discussed in the literature for the localization of polyhedral objects and free form surfaces with and without specified datum planes.

A popular method for aligning 3-D shapes including freeform surfaces based on an ICP algorithm was proposed by Besl and McKay [4]. The algorithm assigns a correspondence to each data point from another data point (model) with the least distance as the criterion. The transformation is applied on the correspondence points in order to minimize the mean square error between them. This process is iterated until some convergence criteria are reached. Rusinkiewicz and Levoy [20] classified variants of the ICP algorithm into six categories. Various researchers used these variants to increase accuracy and accelerate the localization process based on the ICP algorithm. Some drawbacks of the method have already been mentioned in the previous section. The main drawback of the ICP method is that of monotonic convergence to local minima instead of global optimal alignment. Some drawbacks can be overcome by prior knowledge of point correspondence or when an initial estimate of the relative pose is known.

Qualified datums [16] are datum surfaces or planes that can vary inside tolerance zones. Localization based on these datum planes does not provide optimal alignment and this result in an inaccurate comparison of surfaces. To overcome this, the concept of a datum direction frame was proposed by Li and Gu [16] to solve the localization of point cloud data to the design system with datum planes and without. For the proposed method, the datum for the measured data is defined using the least square principle. Also, it does not use point to point correspondence between the design and manufactured data for localization and this result in inaccurate results for subsequent tolerance verification.

Various point cloud analysis algorithms have been developed to segment point cloud data into different zones corresponding to basic CAD modeling surfaces [3]. Based on this, Campana and Germani [6] described a method to identify datum geometries on a point cloud corresponding to an axis and planes using the outer-point fitting approach (OPF).They demonstrated that the least square method (LSM) approach overestimates distances as compared with the OPF approach.

In all of the methods used for localization, the sole purpose is to minimize the distances between the various features and not between the corresponding points. For the localization of the datum plane, at least some corresponding points representing the datum plane of the manufactured part should coincide with the design datum plane to get accurate results for tolerance verification. The closest point with the least distance criterion is suitable for the matching of surfaces but not for the tolerance verification process when datum planes are prescribed.

3 PRESENT WORK

In this section, it will be shown how the medial axis can be used to verify dimensional tolerance from the manufactured model.

The medial axis transform was first introduced by Blum [5] to describe biological shapes efficiently. Several researchers have pointed out the use of Medial axis transform (MAT) in various fields of applications including geometric modelling. Recently Bahlen *et al.* [2] introduced a method to extract and visualize dimension from a geometric model, especially thickness and angles and certain properties like symmetry using MAT.

The definition of the medial axis transform reported in [18] is as follows. *The Medial Axis (MA), or skeleton of the set D, denoted M (D), is defined as the locus of points which lie at the centers of all closed balls (or disks in 2-D) which are maximal with respect to D, together with the limit points of this locus. A closed ball (or disk) is said to be maximal in a subset D of the 3D (or 2D) space if it is contained in D but is not a proper subset of any other ball (or disk) contained in D. The radius function of the MA of a connected set D is a continuous, real-valued function defined on M (D) whose value at each point on the MA is equal to the radius of the associated maximal ball or disk. The Medial Axis Transform (MAT) of D is the MA together with its associated radius function.*

An important characteristic of MAT is that it can be used to simplify the original object and still retain the original information about the objects. MAT has several useful properties such as

- *Uniqueness: There is a unique MAT for a given object.*
- *Dimensional reduction:* The dimensionality of the MAT is lower than its object.
- *Homotopical equivalence:* The MAT is topologically equivalent to its object.
- *Invertibility:* With the axis and the radius function one can reconstruct the MAT.
- *Bijective mapping:* For every point on the object boundary there is a unique point on the MAT.

These properties enable the use of MAT for the verification of tolerance for polyhedral objects as well as freeform surfaces with and without datum planes. The scope of this paper is limited to verification of the dimensional tolerance of 3D objects with specified datum. It is also assumed that MATs are available for the CAD model.MAT of the nominal CAD model (henceforth referred to as *nominal-MAT)* and the MAT of the measured model (henceforth referred to as *measured-MAT)*. Techniques such as those in [18] and [9] are used for obtaining the MAT. Procedure Verify-Tol takes the MAT of the CAD model (nominal-MAT) and the MAT of the measured points (measured-MAT) as input and verifies if the manufactured part is within the specified tolerances.

```
Procedure Verify-Tol (nominal-MAT, measured-MAT) {
        define-reference-template (nominal-MAT, nominal-template)
        determine-corresponding-template(nominal-template, measured-MAT, meas-template)
           primary–localization (nominal-template, meas-template)
        determine–soft-datum (datum-plane, soft-datum-plane, nominal-MAT, measured-MAT)
           secondary – localization (datum plane, soft-datum-plane)
           verify-tolerance-along reference system (nominal mat, measured-MAT, soft-datum-plane)
        }
```

3.1 Localization of MAT with Datum

Similar to the conventional digital tolerance verification process, localization is an essential and important step in the tolerance verification process using MAT. It requires establishing a correspondence between the nominal-MAT and the measured-MAT. This can be achieved by localizing the vertices of the datum plane in the nominal MAT with respect to the corresponding vertices of a measured-MAT. The datum plane in a nominal-MAT represents the *theoretical datum plane* whereas the corresponding plane in a measured-MAT represents the *soft datum* plane. In hard-metrology, the datum plane is considered a reference to verify the remaining metrology. Practically speaking, it is not possible to manufacture an ideal geometry and a non-ideal datum can lead to inaccurate results at the end of the verification process [16]. In soft metrology, the CAD model is used as reference; geometrical tolerances can be applied to the soft datum plane and can be easily verified, too. Thus if the soft datum plane satisfies the geometrical tolerances, then the remaining corresponding vertices of an object are verified as per the specifications of dimensions and tolerances.

3.1.1Characteristic Segments in a Nominal- MAT

The various elements that generally compromise the 3D MAT have been defined and generated by Ramanathan and Gurumoorthy [18]. Only the elements and terminologies used in the paper are defined. If both end points of a MAT segment are shared with other MAT segments, then the MAT segment is termed a *loop segment* (segment A,B,C,D in fig.1) and the remaining are termed *free segments* (1,2,...8 in fig.1). The MAT segment may have its starting point from the corner of an object or from the junction point. Thus, the free segments have only a start point or end point in common with the loop segments. The free end of the MAT segments represents the sharp convex corner or fillet of an object. End points in a loop segment are termed *junction points* (J1, J2, J3 and J4 in fig.1).

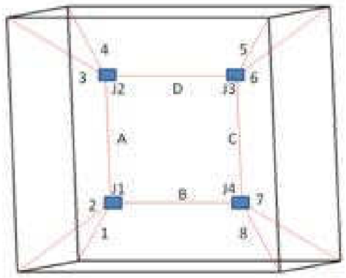

Fig. 1: Characteristic points of MAT.

3.1.2 Establishing Reference Template for Nominal-MAT:

The total number of MAT segments with their end point coordinates representing the nominal CAD model is calculated. The nominal MAT is searched for identifying the number of loops. The centroid of junction points is determined and considered the *first reference point.* The distance from the centroid to the junction points of the loops is determined, and the junction point with the maximum distance is termed the *second reference point.* The *third reference point* is searched for from the remaining junction points of the loop in such a way that the included angle formed by the reference points does not exceed 120 degrees. The triangle formed from the reference point acts as a template to find a correspondence with the measured-MAT. Fig.2 (a) shows the nominal-MAT and measured-MAT for rectangular cuboids in different reference systems. Fig.2. (b) shows the MATs along with the respective triangle formed by the three reference points, R1, R2 and R3 respectively.

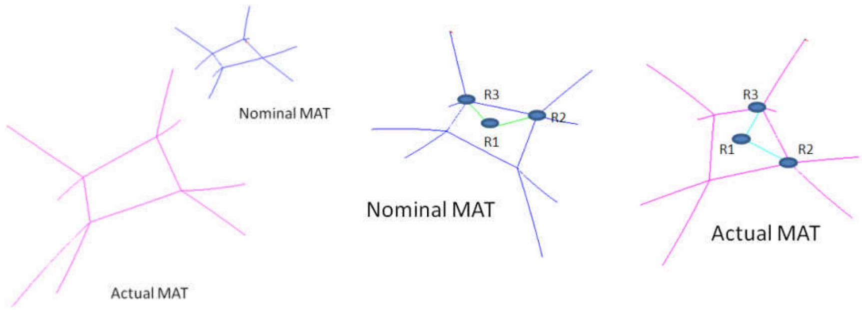

Fig. 2: (a) Nominal and measured MAT (b) Nominal and measured MAT with reference points.

The procedure find-datum- template constructs the template from the input mat. The template constructed can then be used to establish correspondence.

Procedure define-reference-template (nominal-MAT, nominal-template) {
ref-point1 ← centroid of junction points in mat
ref-point2 ← junction point farthest from the centroid
vec1← ref-point1 – ref-point2

```
            for remaining junction points {
                    vec2 ← ref-point2 – junction-pt_j
                    vec3 ← ref-point1 – junction-pt_j
                    angle1_j ← included-angle(vec1 ,vec3)
                    angle2_j ← included-angle(vec1 ,vec2)
                    angle3_j ← included-angle(vec2 ,vec3)
                    if (angle1_j, angle2_j and angle3_j are all < 120°)
                            ref-point3 ← junction-pt_j
        }
    }
```

3.1.3 Determining Template for Measured-MAT:

A similar procedure is adapted as for a nominal-MAT for calculating the number of MAT segments, loop segments, free segments, junction points, number of loops and the centroid for the measured-MAT. The centroid acts as a *first reference point* for the measured-MAT. The second and third reference points are searched for based on the template from the junction points with the distance as a criterion. If the distance criteria are satisfied, then angle criteria are applied for angle verification. After the verification of distance and angle criteria, junction points are fixed as reference points and are used to establish a correspondence between the two mats. The procedure establish-correspondence-datum-reference illustrates the procedure for the establishment of correspondence between the nominal and measured MAT using the template formed in the procedure define-reference-template.

```
Procedure determine-corresponding-template (nominal-template, measured-MAT, meas-template) {
        meas-template. ref-point1 ← centroid of junction points in measured-MAT
        for junction points jpt_j  in measured-MAT{
        if (dist(centroid, jpt_j) - dist(nominal-template.ref-point1,nominal-template.ref-point2) < TOL) {
                    ref-point2 ←  jpt_j
                    for junction points jpt_k in measured-MAT{
                            if (jpt_k != ref-point2){
                                    if (dist (centroid, jpt_k) -  dist (nominal-template.ref-point1,
                                            nominal- template.ref-point3) < TOL){
                                            ref-point3 ←  jpt_k
                                            break;
                                    }
                            } /* if not equal */
                    } /* for */
                    nom-vec1 ←  nominal-template.ref-point1 – nominal-template.ref-point2
                    nom-vec2 ←  nominal-template.ref-point1 – nominal-template.ref-point3
                    meas-vec1 ← meas-template.ref-point1 – ref-point2
                    meas-vec2 ← meas-template.ref-point1 – ref-point3
                    if ( included-angle(meas-vec1 , meas-vec2) -  included-angle(nom-vec1, nom-
                    vec2) < TOL{
                            meas-template.ref-point2 ←  ref-point2
                            meas-template.ref-point3 ←  ref-point3
                    }
            }
        }
```

3.1.4 Primary Localization:

The nominal-MAT is constructed in a design coordinate system (DCS) whereas the measured-MAT is constructed in a measurement coordinate system (MCS). For the verification of tolerances, it is essential to bring them into a common coordinate system. This process is called localization. The localization of nominal and actual MATs is done in two stages, primary localization and secondary localization. In the first stage, localization is done with the correspondence developed with the reference triangle, whereas in the second stage, it is done with correspondence with specified datum planes.

The centroid of the junction points of the nominal model is set to be the origin. The first step in primary localization is to localize the centroid by translating the centroid of the measured-MAT to the centroid of the nominal-MAT. After 3D translation, the triangles formed by the reference points in the two MATs are aligned by a rotational transformation [12]. Details are given in the procedure Primary-localization.

```
Procedure Primary–localization (nominal-template, meas-template, measured-MAT) {
        origin-nominal-template ← nominal-template. ref-point1
        origin-meas-template ← meas-template. ref-point1
        trans ←origin-nominal-template  – origin-meas-template
        Translate (measured-MAT, trans)
        nom-vec1 ←  nominal-template.ref-point1 – nominal-template.ref-point2
        meas-vec1 ← meas-template.ref-point1 – meas-template.ref-point2
        axis ← vect-prod (nom-vec1, meas - vec1)
        angle ← angle (nom-vec1, meas-vec1)
        Rotate (measured-MAT, axis, angle)
        nom-vec2 ←  nominal-template.ref-point1 – nominal-template.ref-point3
        meas-vec2 ← meas-template.ref-point1 – meas-template.ref-point3
        axis ← nom-vec1
        angle ← angle (nom-vec2, meas-vec2)
        Rotate (measured-MAT, axis, angle)
}
```

The function Translate takes the set of points in the first argument and translates them by the vector that is defined by the second argument. The function Rotate takes the set of points in the first argument and rotates them about the axis represented by the second argument through the angle that forms the third argument. By transforming the measured-MAT such that the reference templates defined for the respective MATs are aligned, Primary localization ensures a correspondence between all the MAT segments of nominal and measured data. Fig.3 shows the nominal and measured MAT for rectangular cuboids after the primary localization, where reference points R1, R2 and R3 defined for both MATs coincide with each other.

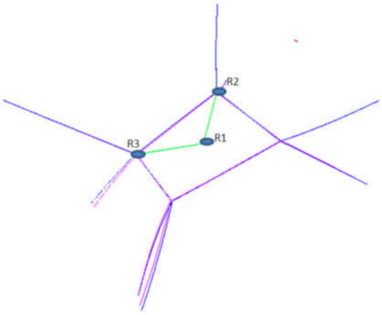

Fig. 3: Nominal and measured MATs after primary localization.

3.1.5. Secondary localization:

In actual hard-metrology, the face defined as the datum is placed on a surface plate for further dimensional and geometrical tolerance verification. If there are other datum planes, the corresponding faces are located on calibrated surfaces as well. This is captured in soft-metrology by aligning faces in the measured part with the respective datum surfaces in the nominal part. Secondary localization is used to achieve this. Here we do not have the faces in the measured part. Therefore the MAT points corresponding to the datum surfaces in the measured point set are aligned with those of the nominal model. The alignment is done starting with the primary datum followed by secondary and tertiary datum where applicable.

The first step in secondary localization is the determination of soft-datum-plane(s) corresponding to the prescribed datum plane(s). Since the faces in the actual part are not available, the Mat points corresponding to the corner points of the prescribed datum plane are used to define the soft-datum plane. Procedure determine–soft-datum takes the prescribed datum-plane and the two Mats and obtains the soft-datum-plane.

```
Procedure determine–soft-datum (datum-plane, soft-datum-plane, nominal-MAT, measured-MAT) {
        for each corner/vertex in the datum-plane {
          nominal-free-end  ← nominal-MAT.free-end that is closest to the corner
          soft-datum-plane.corner ← measured-MAT.free-end  corresponding to nominal-free-end
        }
}
```

The soft-datum-plane obtained is then registered with respect to the prescribed datum and then used for dimensional verification. The free end point closest to the datum plane in the nominal model is taken as the origin. The corresponding free end point in the measured-MAT is assigned as the origin and the measured-MAT is translated to align the two corresponding free end points. Rotational transformation to align two other free end points associated with the datum junction points, with the corresponding free end points in the nominal-MAT, are identified [12]. The transformed measured-MAT then will be localized with the nominal-MAT with respect to the datum planes. The procedure secondary localization will take the two MATs and the datum planes defined for the two MATS for secondary localization. The procedure Secondary-localization implements the registration process described above.

```
Procedure Secondary–localization (datum-plane, soft-datum-plane) {
        Determine the pair of closest corner points in the datum-plane and soft-datum-plane
        Translate the datum-plane and soft-datum-plane such that the respective corner point
           identified above is at the global origin
        for the remaining corner points in the two datum-planes
                Find the next closest pair of points between the corners in datum-plane and soft-
        datum-plane
        nom-vec ←  origin – datum-plane.corner-point
        meas-vec ← origin – soft-datum-plane.corner-point
        axis← vect-prod (nom-vec, meas-vec)
        angle← angle (nom-vec, meas- vec)
        Rotate (soft-datum-plane, axis, angle)
        for the remaining corner points in the two datum-planes
                Find the next closest pair of points between the corners in datum-plane and soft-
        datum-plane
        axis← nom-vec
        nom-vec ←  origin – datum-plane.corner-point
        meas-vec ← origin – soft-datum-plane.corner-point
        angle← angle (nom-vec, meas-vec)
        Rotate(soft-datum-plane, axis, angle)
}
```

3.1.6 Verifying Flatness of Datum Plane:

The distance between corresponding free ends representing the corner points of a theoretical and soft datum plane are calculated. Generally, for a datum plane, flatness tolerance is assigned. The difference between the maximum and minimum distance from the theoretical datum plane gives the variation in the flatness of the soft datum plane. Here the Chebychev min-max principle is used, as the least square methods used for the localization gives a tight tolerance zone and may result in the rejection of acceptable objects [22, 23].

3.2 Verification of the Dimensional Tolerance

A dimension is specified between faces in the nominal model along with the tolerance on the dimension. For each of the two faces related by the dimension, the free end points in both the nominal-MAT and measured-MAT are determined. The distance between the corresponding points in the two MATs is computed for each face. The component of the computed distance along the normal to the faces is used for verification. Procedure verify-tolerance-along reference system performs this computation given the two mats. The worst case combination of the components for the two faces is then checked against the prescribed tolerance. It is proposed that the dimensional tolerances be verified using the free end points in the MAT (corresponding to corners in the object) and the geometrical tolerances be verified using the interior points in the MAT.

4 RESULTS AND DISCUSSION

This section illustrates the use of MAT as an intermediate representation in tolerance verification with two examples. Some related issues are discussed at the end of the section.

Figs 4 (a) and (b) shows the nominal trapezium model with MAT and the actual trapezium model respectively. The actual model is obtained by randomly perturbing some of the vertices of the nominal model. The nominal-MAT and the measured-MAT that are used for verification are also shown in the respective frames.

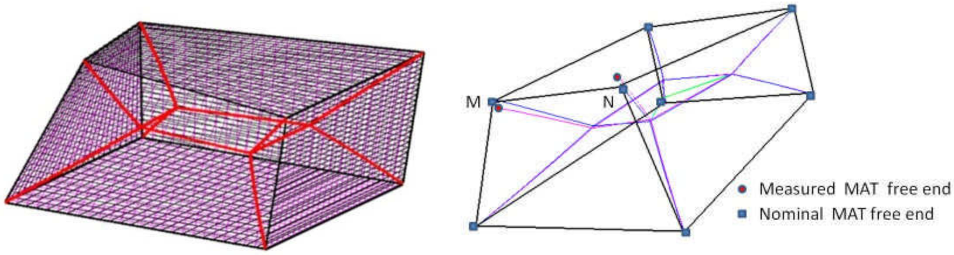

Fig. 4: (a) Nominal trapezium model with MAT (b) Measured Trapezium model with nominal and measured MAT.

Table1 shows the coordinates of the vertices of the nominal and measured model with theoretical and measured distances calculated with the algorithms prescribed.

Corner Number	Co-ordinates Nominal model(X,Y,Z)	Co-ordinates Measured model (X,Y,Z)	Distance (Actual)	Distance (Algorithm)	Distance(Algorithm) component along dimension		
					X	Y	Z
M	-8 -10 12	-8 -11 12	1	1	0	1	0
N	8 -10 14	8 -9 14	1	1	0	1	0

Tab.1: Results from comparing corresponding points of nominal and measured MAT for trapezium.

Degeneracy /Singularity case:

Fig 5 (a) shows a 'T' shape part along with the MAT. It consists of a curved edge and surface, concave corners and a concave edge. The concave corners are denoted by CC1 and CC2 while the concave edge is shown by CE1. Face A-B-C-D represents the primary datum, face A-B-b-a represents the secondary datum and face A-a-c-C represents the tertiary datum. The corner 'P' of the object is perturbed in the direction of the X and Y axes as shown in fig 5 (b). Thus, the perturbed corner is at a unit distance from corner 'P' in the 'X' and 'Y' directions respectively, as shown in fig.6. The free ends of the remaining MAT segments coincide with each other. This is evident from the result as zero variation is obtained in location for all vertices of the part except corner 'P'.

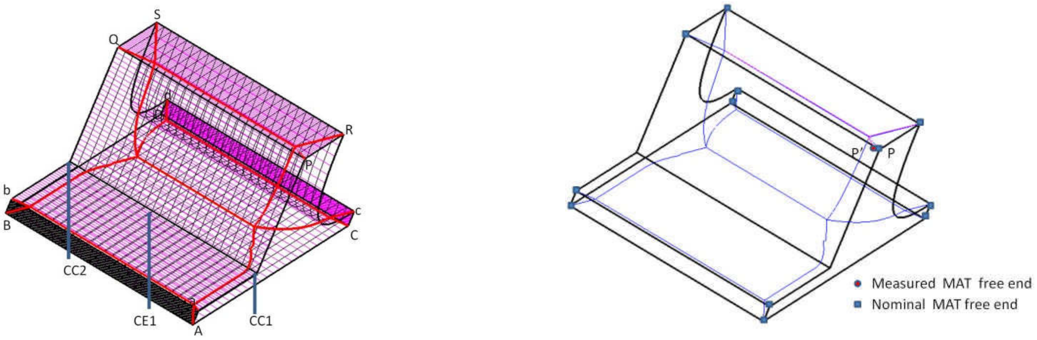

Fig. 5: (a) Nominal 'T' shape model with MAT (b) Measured perturbed 'T' shape model with nominal and measured MAT.

Assume a unit dimensional tolerance is applied to face P-Q-R-S with respect to the primary datum (parallel to the X- axis) and the secondary datum (parallel to the Y-axis). The current state of the art uses the shortest distance criterion from the point to the surface for tolerance verification. Thus the corner point 'P' 'will be shown at a distance of 1.43 unit from the corner 'P', and the part will be rejected in this case. But in the actual case, the perturbed point is in the variational limit and should be accepted.

Table 2 shows the coordinates of the vertices of the nominal and measured model with the theoretical and actual distances calculated for point P with the algorithms prescribed.

Corner Number	Co-ordinates Nominal model (X,Y,Z)	Co-ordinates Measured model (X,Y,Z)	Distance (Actual)	Distance (Algorithm)	Distance (Algorithm) component along dimension		
					X	Y	Z
P	-5 20 20	-6 21 20	1.43	1.43	1	1	0

Tab. 2: Results from comparing corresponding points of nominal and measured MAT for 'T' shape model.

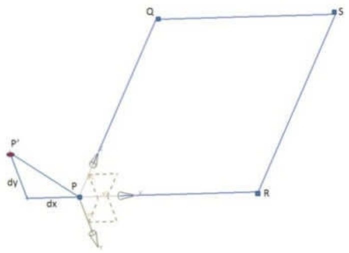

Fig. 6: Deviation of perturbed corner P.

In the prescribed procedure, however, the component of the deviations along the normal to the faces related by the dimension is used for verification. The projection gives the actual distance of the actual corner from the corner 'P' and ensures correct verification. MAT handles all such degenerate cases including the case where different tolerances are applied for the dimension associated with the faces associated with a given point [17]. The problem stated above actually represents the singularity condition [17] occurring at the corner points of polyhedral objects and can be resolved by using the MAT for verification purposes.

It has been shown that, it is possible to use MAT as an intermediate representation to achieve tolerance verification without having to do surface fitting or segmenting. While these two steps are computationally expensive, it must be mentioned that the effort in the computation of MAT has to be accounted for. It may be noted that several efforts are underway for developing robust and reliable constructions of MAT from both smooth models and discrete point data [8, 11, 15]. However, a comparison of the efforts made remains to be done. However, MAT, as a representation of the object, has other applications as well and therefore the computational effort could well be amortized over more than one application. The use of MAT for representing tolerances has been suggested [17]. This might be interesting to pursue as the verification step could then become simple as both the specification and verification would be in terms of the same representation.

In this paper, the verification of tolerance is limited to dimensional tolerance with a specified datum. In future work, algorithms for the verification of tolerances will be modified to accommodate all types of geometrical tolerances applied to all types of surfaces. The accuracy of the algorithms will be further increased with the inclusion of the interior points of the MAT in the alignment of soft datum planes with nominal datum planes.

5 CONCLUSIONS

This paper proposed the algorithms to verify dimensional tolerance for polyhedral objects with specified datums. The algorithms use input in the form of a MAT for nominal model and measured point data respectively. Localization and correspondence between the two models are established without the need for either surface fitting procedures or segmentation procedures. The prescribed algorithms use the distance between the corresponding points and not the closest distance between the point and the surface as adopted in general verification methods. As only characteristic points in the MAT of the point cloud are processed, the computational effort and the combinatorial complexity are also significantly reduced. Though it is too early to claim that the entire verification of tolerances can be undertaken by the procedures prescribed in this paper, this is a first step towards an effective and efficient tolerance verification system using MAT as an intermediate representation.

REFERENCES

[1] ASME Y14.5M-2009, Dimensioning and tolerancing, The American Society of Mechanical Engineers, New York.

[2] Bahlen, T.; Bronsvoort, W.; Spence, A.: Extraction and Visualization of Dimensions from a Geometric Model, Computer-Aided Design & Applications, 7(4), 2010, 579-589.

[3] Benko, P.; Varady, T.: Segmentation methods for smooth point regions of conventional engineering objects, Computer-Aided Design 36, 2004, 511–523. doi:10.1016/S0010-4485(03)00159-3

[4] Besl, P.; McKay, N.: A Method for Registration of 3-D Shapes, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 14, No. 2, February 1992, 239-256. doi:10.1109/34.121791

[5] Blum, H.: A transformation for extracting new descriptors of shape, Models for the Perception of Speech and Visual Form, 1967, 362–381.

[6] Campana F.; Germani, M: Datum Identification for Tolerances Control on Dense Clouds of Points, Computer-Aided Design & Applications, 5(1-4), 2008, 209-219.

[7] Carr, K.; Ferreirat, P.: Verification of form tolerances Part I: Basic issues, flatness and straightness, Precision Engineering, 17, 1995, 131-143. doi:10.1016/0141-6359(94)00017-T

[8] Chazal, F.; Lieutier, A.: The λ - Medial Axis, Graphical Models 67, 2005, 304–331. doi:10.1016/j.gmod.2005.01.002

[9] Dey, T.; Sun, J.: Defining and Computing Curve-skeletons with Medial Geodesic Function, Euro graphics Symposium on Geometry Processing, Cagliari, Italy. (SGP 2006), 143-152.

[10] Giesen, J.; Miklos, B.; Pauly, M.: Medial Axis Approximation of Planar Shapes from Union of Balls: A Simpler and more Robust Algorithm, CCCG 2007, Ottawa, Ontario, August 20–22, 2007.

[11] Giesen, J.; Miklos B.; Pauly M.; Wormser, C.: The Scale Axis Transform, Aarhus, Denmark. SCG'09, June 8–10, 2009.

[12] Ghosal, A.: Robotics fundamental concepts and analysis, Oxford University Press, New Delhi, 2006.

[13] Jost, T.; Hugli, H.: A Multi-Resolution Scheme ICP Algorithm for Fast Shape Registration, Proceedings of the First International Symposium on 3D Data Processing Visualization and Transmission (3DPVT.02), IEEE 2002,540-543.

[14] Juttler, B.; Poteaux, A.; Song, X.: Medial Axis Computation using a Hierarchical Spline Approximation of the Signed Distance Function, FSP Report No.105,July 2010.

[15] Katz, R. A.; Pizer, S. M.: Untangling the Blum medial axis transform, International Journal of Computer Vision, 55(2/3), 2003, 139-153. doi:10.1023/A:1026183017197

[16] Li, Y.; Gu, P.: Sculptured surface tolerance verification with design datums, International Journal of Production Research, Vol. 43, No. 7, 1 April 2005, 1465–1482. doi:10.1080/00207540412331299675

[17] Pasupathy, T.M.K.; Morse, E.P.; Wilhelm, R.G.: A survey of Mathematical Methods for the construction of Geometric Tolerance Zones, Journal of Computing and Information Science in Engineering, Vol. 3, March 2003, 64-75. doi:10.1115/1.1572519

[18] Ramanathan, M.; Gurumoorthy B.: Interior Axis Transform computation of 3D objects bound by free-form surfaces, Computer-Aided Design 42, 2010, 1217–1231. doi:10.1016/j.cad.2010.08.006

[19] Requicha, A.: Representation of Tolerances in solid modelling: Issues and alternative Approaches, Tech Memo.No.41, Production Automation Project, University of Rochester, August 1983, 1-26.

[20] Rusinkiewicz, S.; Levoy, M.: Efficient Variants of the ICP Algorithm, Third International Conference on 3D Digital Imaging and Modelling, 3DIM, Québec City, Canada, Proceedings: IEEE Computer Society Press, May28-June1, 2001,145-152.

[21] Sherbrooke, E.; Patrikalakis, N.; Brisson, E.: An Algorithm for the Medial Axis Transform of 3D Polyhedral Solids, IEEE Transactions on Visualization and Computer Graphics, Vol.2, No.1, March 1996, 44-61. doi:10.1109/2945.489386

[22] Traband, M.; Joshi, S.; Wysk, R.; Cavalier, T.: Evaluation of straightness and flatness tolerances using the minimum zone, Manufacturing Rev. 2, 1989, 2, 189-195.

[23] Weber, T.; Motavalli, S.; Falahi, B.; Cheraghi, S. H.: A unified approach to form error evaluation, Precision Engineering, 26, 2002, 269-278. doi:10.1016/S0141-6359(02)00105-8