Computer-Aided Design and Applications

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Emergent Crowd Behavior

Ching-Shoei Chiang[1], Christoph Hoffmann[2] and Sagar Mittal[3]

[1]Soochow University, Taipei, R.O.C., chiang@cis.scu.edu.tw
[2]Purdue University, West Lafayette, IN, USA, cmh@cs.purdue.edu
[3]Purdue University, West Lafayette, IN, USA, smittal@cs.purdue.edu

ABSTRACT

We investigate experimentally the behavior of *boids*, simple agents following three simple rules of conduct. In crowds these agents can exhibit a collective pattern of movement that imitates flocking animal behavior as well as other types of crowds.

Keywords: boids, flocks, collective behavior.
DOI: 10.3722/cadaps.2009.865-875

1. INTRODUCTION

A crowd is a large group of individuals collocated compactly. In a crowd, members may exhibit a behavior that differs from their ordinary, individual behavior. As eloquently argued by Reynolds in [12], crowd behavior occurs abundantly in nature, and very simple behavior of the members of a crowd can result in complex and pleasing collective behavior. Since Reynolds' paper on *boids*, a term short for birdoid describing artificial creatures that are members of flocks and crowds, there has been spotty progress characterizing how emergent crowd behavior comes about from simple, individual behavior. Part of the sparseness of firm results in the literature may be ascribed to the fact that a precise mathematical analysis of emergent crowd behavior must include the analysis of chaotic systems even in the case of just two or three boids [10]. Another factor to consider is that even the three rules of Reynolds can be interpreted in a number of ways, leading to related but different systems.

Reynolds used a few simple rules for the member behavior of which three are central to the emergent behavior of the crowd:

1. Two boids that are closer than a threshold will move apart so as to avoid collision.
2. A boid will seek to match the velocity (including direction) of other boids nearby.
3. A boid always seeks to be in the crowd, at the centroid of the surrounding boids.

The 1987 paper describes elements of implementing the rules and explains some results scripting boid behavior using these rules. Since, authors have adopted several different interpretations, such as how to avoid collision, how to determine the orientation of boids, whether boids have a margin of error in their determination of how to proceed, and so on; see, e.g., [2], [3].

There are a number of parameters that control the details of individual behavior, such as at which distance boids affect each other and in what strength, how fast a boid can turn, accelerate or decelerate, the field of vision of a boid, and so on. As noted by Reynolds, the valuation of these parameters affects the crowd behaviors, and, as made clear in [1], the effect can be considerable. Note, however, that, for many rule interpretations, convergence results such as the one derived in [5] remain robust when adding noise into the model.

In this paper, we explore empirically the crowd behavior as a function of parameter choices so as to better guide selecting models and parameter values. As noted in [13], getting a crowd to move in interesting ways may require extensive experimentation. Our larger motivation is to use emergent crowd behavior in computer games for nonplaying characters (NPC), and to interface them eventually with playing characters (PC) that would serve to give a global purpose to the crowd of NPCs by suitable directives. However, we do not restrict to this perspective alone. We note in particular that the literature on crowd behavior is highly diverse and extensive, including work in ethological biology, e.g., [4],[16]; traffic congestion, e.g., [8], and many other subjects. Boid flocking has even been proposed for simple problem solving tasks such as search and collect [15], a task not unlike ant foraging [7].

Emergent crowd behavior is based on rules such as the ones of Reynolds. One could argue that different rule sets should become activated based on circumstances. For example, the crowd in a soccer stadium will behave differently when a panic breaks out, rushing madly to the exits, a behavior in sharp contrast to the normal behavior when exiting at the end of a game. Thus a longer-term goal should be to explore how to steer such state changes seamlessly. Moreover, the leadership problem would seek solutions to imposing purpose on the emergent behavior that is absent when only executing the above rules. In later work, Reynolds develops such a framework with the Open Steer library [14] which is used in computer gaming.

2. PRIOR WORK AND GOALS

Reynolds' rules (see [13]) can amount to conflicting behavior, thus they have to be prioritized and blended. Reynolds proposes that the first rule, avoiding collision, take precedence. The rule is implemented by finding individuals within a specific range r_r and accelerating in the opposite direction. Next, observing the orientation and velocity of individuals within a range $r_o \geq r_r$ an acceleration is determined that is needed to match the average velocity and direction of motion. The average is weighted by a distance function where individuals further out contribute less to the average than individuals closer by. Reynolds proposes a weight proportional to the inverse distance squared. Finally, in rule 3, boids seek to move to the centroid of the flock. Seeking to avoid flock separation, Reynolds proposes to compute the centroid globally so flocks can join, but he also recognizes that inside a flock the perception range should be limited. Note that the paper does not discuss the mathematical model in detail.

Couzin [3],[4] uses a simplification of Reynolds' rules. Choosing specific parameters $r_r \leq r_o \leq r_a$ the three rules are implemented as follows:

1. Move away from the individuals within a circle of radius r_c choosing the direction opposite to the average direction of the nearby individuals.
2. With θ the average direction of all individuals in the annulus with radii r_c and r_o an individual tries to move in the direction θ.
3. Let c be the average direction from the individual to all others in the annulus between r_o and r_a. Then the individual tries to move in the direction of c.

While Reynolds implements the rules in a physical manner, computing acceleration and integrating velocity and position, Couzin focuses on the choice of direction and assumes constant velocity of boids, moving them purely kinematically. As in Reynolds' work, Couzin imposes a limit on how quickly a boid can turn. Larger turns will be truncated to that maximum. Moreover, there is a stochastic element in that a boid does its determination subject to an error that has a normal distribution characteristic to the individual. This error is meant to account for perceptual inaccuracies and difference in acuity of perception.

More precisely, let $r_{i,k}$ be the (unit length) direction vector from individual i to individual k. Compute the vector $d = \frac{1}{|S|}\sum_{k \in S} r_{i,k}$ and normalize it to unit length if it is not zero. For Rule 1 (repulsion), S is the set of individuals in the repulsion disk. If the vector d is not zero, the chosen direction is $-d$. For rule (3), S is the set of individuals in the annulus of attraction. If d is not zero, the chosen direction is d.

Both Reynolds and Couzin consider the field of perception an individual has. Using the direction of motion as reference, the individual's field of vision extends by the angle $\alpha/2$ in all directions. Thus, in a planar world, the individual sees the angular range α. Individuals that are situated in the blind cone at angle range 2π-α are not considered in the computations stipulated by the three rules.

Bajec et al [2] argue that the centroid-seeking rule is too mathematical. Basing their argument on visual perception, they posit that a flock member should seek to stay with the flock based on an evaluation of each flock mates relative position and speed, and then combine the resulting individual "urges" into motion based on fuzzy logic [12].

In this paper, we do not consider the question of leadership or purpose. This question has been investigated by Couzin [4] and by Han et al [9], considering both overt and covert leadership. We are also following Reynolds' and Couzin's lead and consider crowd members as particles with a direction. The techniques of Kang [11] to efficiently render large crowds of complex shaped characters therefore are not considered.

Our purpose is to investigate the three rules of Reynolds, focusing on the variant used by Couzin, and gain empirical insight into the emergent properties of a flock of boids, in 3D and in 2D, that can give some guidance to the choreographer of the crowd's behavior. As observed by many authors, devising interesting behavioral patterns may require a considerable amount of experimentation. Reynolds puts it as follows in [13]:

> *One of the charming aspects of the work reported here is not knowing how a simulation is going to proceed from the specified behavior and initial conditions; there are many unexpected, pleasant surprises. On the other hand, this charm starts to wear thin as deadlines approach and the unexpected annoyances pop up. This author has spent a lot of time recently trying to get uncooperative flocks to move as intended ("these darn boids seem to have a mind of their own!").*

Our own experiments attest to the complexity of choreographing boids. As we will see, there are strong influences resulting from the number of individuals, the initial formation, the initial density of the crowd, as well as whether the simulations are run in the plane or in 3-space.

3. BEHAVIOR PATTERNS AND ADOPTED MODEL

Consider a crowd of boids. We consider the crowd *coherent* if each member is close enough to at least one other member to be able to apply the centroid-seeking rule. The behaviors to be considered use the classification schema proposed by Couzin [3]. Assume there are *N* individuals at positions c_k and moving with velocity v_k. The *centroid* of the group is

$$c_{group} = \frac{1}{N}\sum_{k=1}^{N} c_k$$

The relative direction of member c_k is defined as $r_{kc}=c_k-c_{group}$. Define *group polarization* as the quantity

$$p_{group} = \frac{1}{N}\left|\sum_{k=1}^{N} v_k\right|$$

and the *group momentum* as the quantity

$$m_{group} = \frac{1}{N}\left|\sum_{k=1}^{N} \frac{r_{kc}}{\left|r_{kc}\right|} \times v_k\right|$$

Group polarization measures the degree to which the group members are aligned to a single direction, whereas the group momentum measures the degree of rotation around the center of the group. Note that both group polarization and momentum are time-dependent quantities.

Then the behavior will be called *swarming* if the group has no apparent motion and individuals emerge from the crowd's interior and re-enter into the interior of the group. In terms of the group polarization *p* and the group momentum *m*, the values observed after reaching steady-state are both low, less than 0.2. Swarm behavior in nature can be observed in gnats on a summer day.

We call the behavior *toroidal* if the members circle around a central point. Here, polarization *p* is not large, less than 0.4, but the momentum *m* is relatively large, 0.7 or larger. Toroidal behavior can be observed in nature with certain cave bats [1]. Couzin cites an example of fish behavior that is toroidal, such as tuna, barracuda and jack.

The behavior is *highly parallel* if the individuals are directionally aligned. Here, *p* is large, close to 1, and the momentum *m* is small, less than 0.2. Many fish schools are highly parallel while assuming a variety of group shapes. In this paper, we will analyze the behavior rules used by Couzin, here referred to as *Model C*:

Model C: This model uses the calculations of Couzin [3]. Three radii control the three rules of motion, named r_r, r_o, and r_a. With $r_r \leq r_o \leq r_a$ they define three zones, the *zone of repulsion*, *zor*, a sphere of radius r_r for the rule of collision avoidance; the *zone of orientation*, *zoo*, a spherical shell between the sphere of radius r_r and the radius r_o sphere; and the *zone of attraction*, *zoa*, a spherical shell between the sphere of radius r_o and the radius r_a sphere. A group member sees its surroundings except for a cone in the opposite direction of heading. The field of view is indicated by the angle α. Recall that the view angle extends by $\alpha/2$ in all directions from the heading, so a value of α=270° indicates that a boid sees everything except the boids in a cone with apex angle of 90° to the rear. See also the figure.

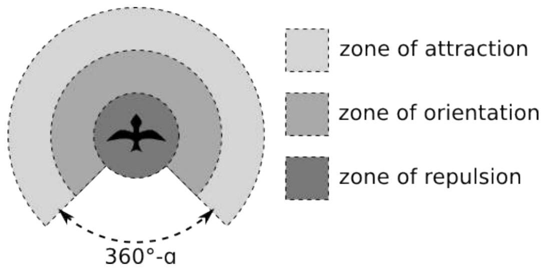

Fig. 1: A 2D cross-section of the shape of the three zones in *Model C*. The field of view does not affect the zone of repulsion, unlike the zones of orientation and attraction. The angle α is 270°.

Collision avoidance, the first rule of Reynolds, takes precedence. If there are other members in the zone of repulsion, the individual moves in the direction away from the average direction *d* of the others, as explained before. Otherwise, the motion of the member c_k is computed by blending the attraction and orientation directions. The orientation direction for c_k is computed as the average of the directions of all members in the zone of orientation, normalized to unit length. Likewise, the relative direction $r_{ik} = c_i - c_k$ of all members in the zone of attraction is computed, also normalized to unit length. If there are members in both zoa and zor, then the two directions are averaged and become the desired direction for moving. If one of the zones is empty or the desired direction is the zero vector, the direction computed from the members in the other zone is used, and if both zones are empty or the computed direction is zero, then the member continues to move as previously. If the desired direction requires turning, a member is restricted not to exceed a maximum turning angle θ, for each time step τ. A key aspect of model C is that flock members move at constant speed *s*. Thus, the computation is reduced to determining the direction of motion for the next time step. Model C assumes that the perception of flock members is subject to a random noise, thus the determined direction values are perturbed by a normally distributed error angle of deviation $\sigma_k<\sigma$. The magnitude σ_k is individual and chosen for each member in advance, also using a normal distribution with maximum σ. Perturbation is computed before the step that clips the turning angle to the maximum θ.

A key feature of Model C is that it focuses on the direction of motion by assuming that all boids have constant speed. This isolates the direction-seeking aspect of Reynolds framework, but it has the feature that a boid that has fallen behind the group has no good way of making up the lost distance and can rejoin the center of the flock only when the rest of the flock does some milling around.

4. COHESION EXPERIMENTS

The most basic emergent behavior we can inquire about is whether a flock with chosen parameter values will stay together or split up into several smaller flocks. We will use the incremental values of r_o and r_a denoting them $\Delta r_o = r_o - r_r$ and $\Delta r_a = r_a - r_o$. Couzin [3], Figure 3, identifies a region "e" in which there is a greater than 50% chance of the group breaking up. Visually, that region can be approximated by $\Delta r_o+\Delta r_a \leq 8$, for a flock of 100 members, and r_r=1, α=270°, θ=40°, s=3, τ=0.1, and σ=0.05 rad. Note that we require that $s\tau < r_r$.

4.1 Simulations in 3D

N	50, 100, 200, 400	s	3	R	10		
r_r	1	τ	0.1	ticks	5000		
Δr_o	0 to 15	σ	0, 0.05, 0.10 rad	trials	32		
Δr_a	0 to 15	θ	40^o	α	270^o		

Holding the parameters r_r, α, θ, s, τ and σ fixed, we investigate the property of cohesion as a function of Δr_o , Δr_a and N. Figure 2 shows the results for N=50, 100, 200 and 400 members. The values for Δr_o and Δr_a are incremented by 1 in the graph. Each data point is averaged from 30 trials. Initial position is random, with random direction, but such that each flock member sees at least one other member and participates in the same flock. As explained in detail later, the individuals are randomly placed into a sphere of radius R, in random orientation, but such that they form a flock.

The first batch of simulation experiments investigated the parameter valuations shown in the table. The cohesion graphs in Figure 2 show the percentage of trials in which the flock stays together for different values of N, with σ=0, as function of the attraction and orientation radii; the repulsion radius is fixed to 1 and the percentages are scaled to the range [0,1]:

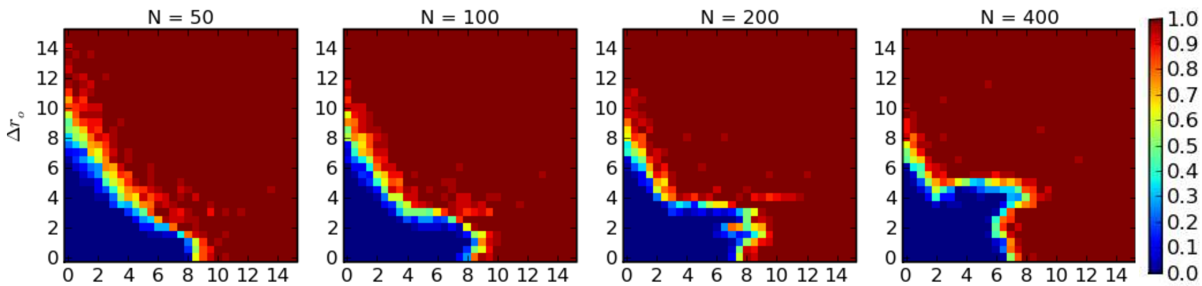

Fig. 2: Cohesion for flock sizes N=50, 100, 200 and 400; N=50 is leftmost, σ=0. Δr_a on the horizontal, Δr_o on the vertical axis.

The deep blue area, corresponding to zero values, is where the flock breaks up with certainty into more than one group. We note a dependence on the flock size as well as a change in the shape of the area in which the flock breaks up. The cohesion graphs remain essentially the same when simulating that individuals have a random error in their heading, when distributions with σ=0.05rad and 0.10rad are considered (angle values of 2.9 and 5.7 degrees respectively).

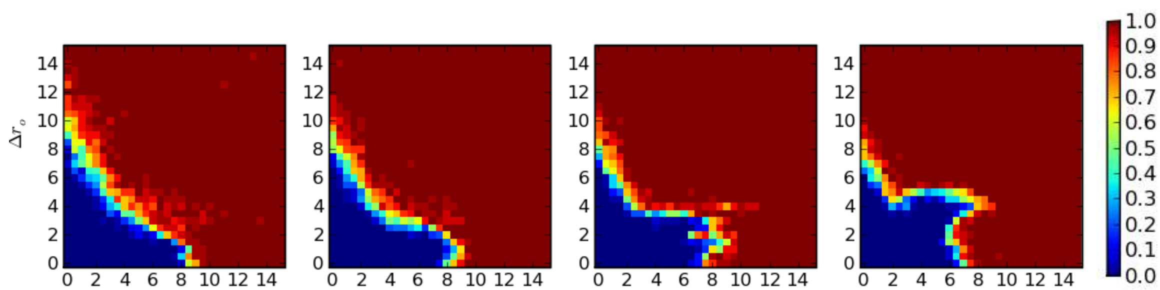

Fig. 3: Cohesion for flock sizes N=50, 100, 200, 400 and σ=0.05 rad.

Size dependence raises questions. It would seem that flock behavior in nature should be size-independent if it is to be based on individual cognition implementing the basic rules of flocking.

We investigated two possible hypotheses: (1) The simulations assume that once a flock breaks apart it does not recombine and so the simulation quits, perhaps wrongly. (2) As N increases but the initial volume of the flock does not, the individuals start out more crowded together and the initial spreading out to a comfortable distance between the individuals constitutes an outward pressure that impact cohesion. The hypotheses, if true, should be tested with large flock sizes so as to amplify the effects of crowding or of recombining.

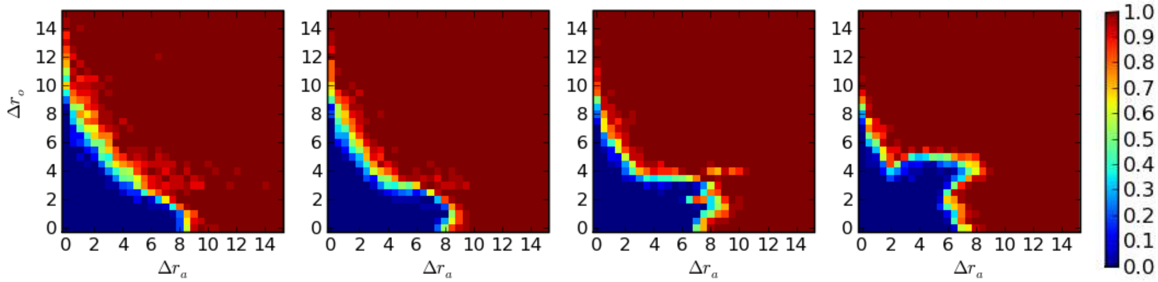

Fig. 4: Cohesion for flock sizes N=50, 100, 200, 400 and σ=0.10 rad. Δr_a on the horizontal, Δr_o on the vertical axis

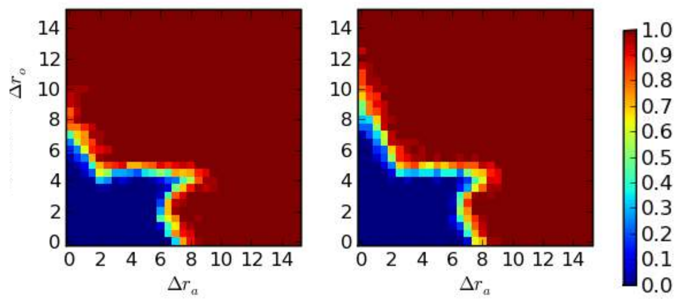

Fig. 5: Cohesion experiments for N=400, R=10, no early quit (left) and for N=400, R=15, early quit. In both cases σ=0.

We tested the two hypotheses with two simulation set-ups, with results shown in Figure 5. For hypothesis (1), we did not terminate the simulation on flock separation, but as seen in the plot on the left the outcome of the 32 trials is essentially the same. For hypothesis (2), we enlarged the initial volume from R=10 to R=15, effectively quadrupling the volume in which the flock starts out. Again, no appreciable difference is seen between this plot and the plot in Figure 2 to the right.

4.2 Shape and Size

We decompose the region of flock break-up in Figures 1-5 into two areas: first, a fundamental triangle approximately below the line $\Delta r_o + \Delta r_a = 8$. This region is relatively invariant to size. The initial configuration for the simulations is random position and random orientation in a sphere of the stated size.

The second area of break-up is a saw tooth shape bay whose upper border is approximately $\Delta r_o = v$ where v ranges from about 2 for N=50 to around 5 for N=400. Thus both the region size and its position are dependent on the number of individuals.

4.3 Cohesion in 2D

The behavior in 2D appears to be different than the 3D behavior, as shown in Figure 6. The region of cohesion is evidently smaller than in the 3D case.

The difference in cohesion is not the result of an initialization radius that would be too large. Running the simulations with R=10, a reduction by a factor of more than 2, does not yield materially different results. However, extending the range of Δr_a and Δr_o by a factor of 2 yields the following cohesion results (Fig. 7). These plots are in much better agreement to the cohesion plots of Figure 2, except that the saw-tooth notch of the 3D graphs is not evident. Note that the 2D runs have a higher variation in the highly-parallel region than the 3D counterparts.

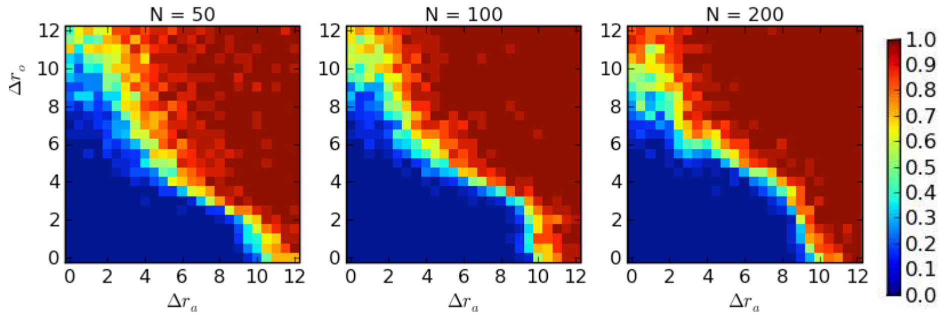

Fig. 6: 2D Cohesion for N=50, 100, 200, with R=15 and σ=0.

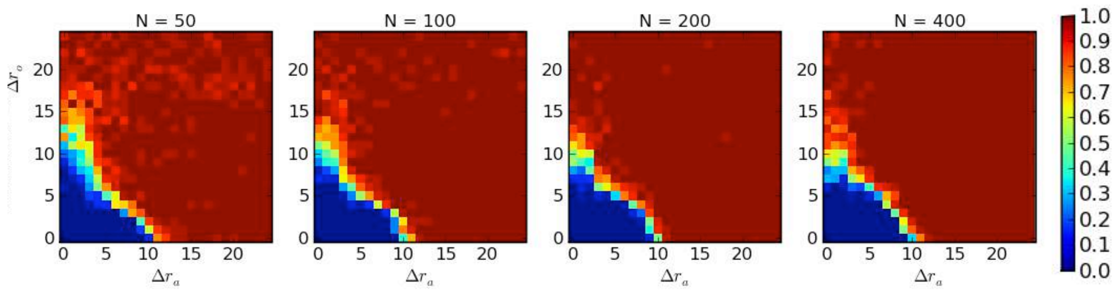

Fig. 7: 2D Cohesion for N=50, 100, 200, 400, and R=15, with double the range for Δr_a and Δr_o

4.4 Initial Configuration and Density

Couzin describes the initialization of a simulation as follows:

> *For each combination of parameters, individuals start with random orientations and at random positions within a sphere in which each can detect at least one individual. Group fragmentation is measured using an extension of the calculation of equivalence classes (...), where the criterion of interest is the presence of other individuals within the field of perception.*

We found that a substantial part of the incoherence region is dependent on this initialization. Our initialization computation is as follows: After fixing the radius of the sphere containing the crowd initially, boids are randomly instantiated inside the sphere, one by one. As they are added, they are accepted and aggregated into the group if it can see at least one boid previously instantiated. That boid should be at a distance no greater than r_a. If the boid cannot see any of the prior boids it is not accepted. This process repeats until the group has the required number of boids.

We also considered instantiating the crowd on a dense cubical grid, spaced apart at distance r_r and found that the region of cohesion becomes substantially different for regular formations, compared with the random position initializations. This is true even when running with 10,000 time steps instead of Couzin's 5,000. Likewise, a hexagonal grid alters the shape of the cohesion region.

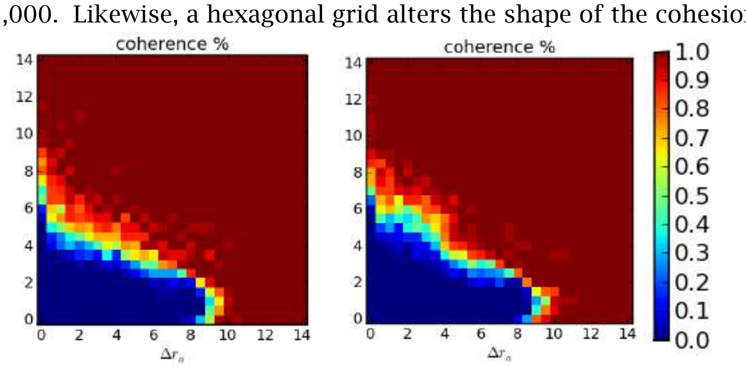

Fig. 8: N=100, σ=0.05, 30 trials per parameter combination, ticks = 10,000, 3D space. Initial position on a 5x5x4 cubic grid (left); and on a planar hexagon formation (right) with random orientation.

In 2D space the initial configuration does not seem to matter. That is, initial position in a formation with boids randomly oriented agrees with randomly positioned and oriented boids:

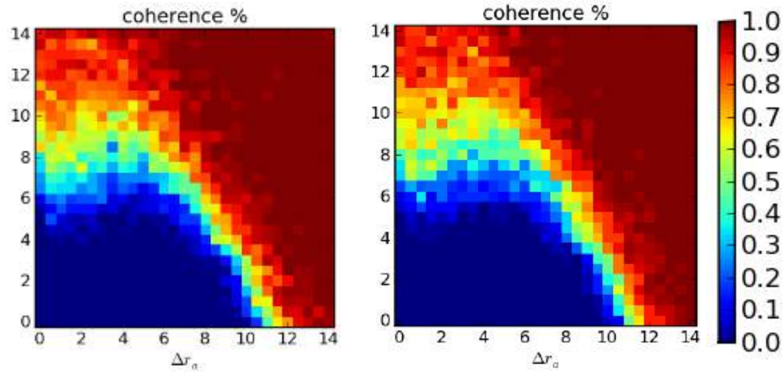

Fig. 9: N=100, σ=0.05, 48 trials per parameter combination, ticks = 5,000, 2D space. Initial position hexagonal formation with random orientation (left); Initial position random, R=5 (right).

5. SWARMING, ALIGNMENT AND MILLING

In [3], Couzin explores four emergent behaviors, *swarming*, *milling*, and *weak* and *strong* alignment. The behaviors are characterized in terms of the quantities *p* and *m* which we computed and graphed in an effort to confirm Couzin's 2002 findings. The experimental setup is the same as in the cohesion experiments. As in Couzin's case, we did not compute the quantities *p* and *m* when the flock broke up in more than 50% of the trials. The graphs and plots for N=100, σ=0.05rad are as follows (Fig. 10):

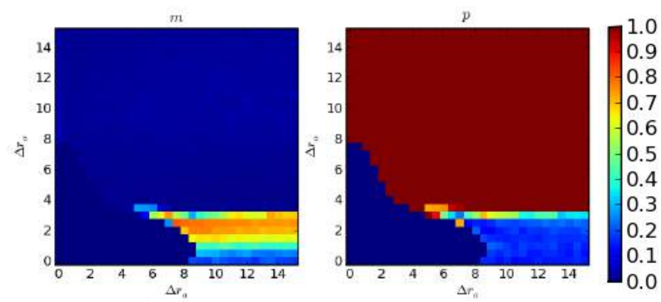

Fig. 10: Quantities *m* and *p* for N=100, σ=0.05rad.

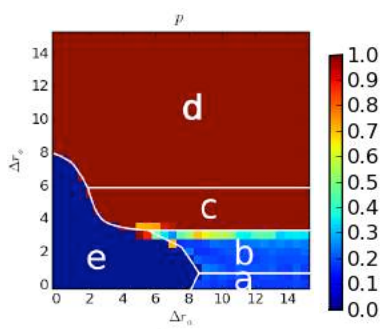

Fig. 11: Couzin (2002) regions of behavior type; see [3].

Couzin identifies several regions, sketched in Fig. 11: (a) swarm behavior, (b) torus movement, (c) dynamic parallel behavior, and (d) highly parallel behavior. Region (e) corresponds to parameter values in which it is highly likely that the swarm breaks up. The regional delineation is in good agreement with our experiments when using random initialization in 3D. However, we have been unable to identify the dynamic parallel region (c). As is evident from the Figure 10, there is no difference in our simulations between region (c) and region (d). Swarming behavior, region (a), is not sharply delineated against the toroidal behavior region (b). The following figures show *p* and *m* for larger values of N as well. In each case the initialization is random as described before.

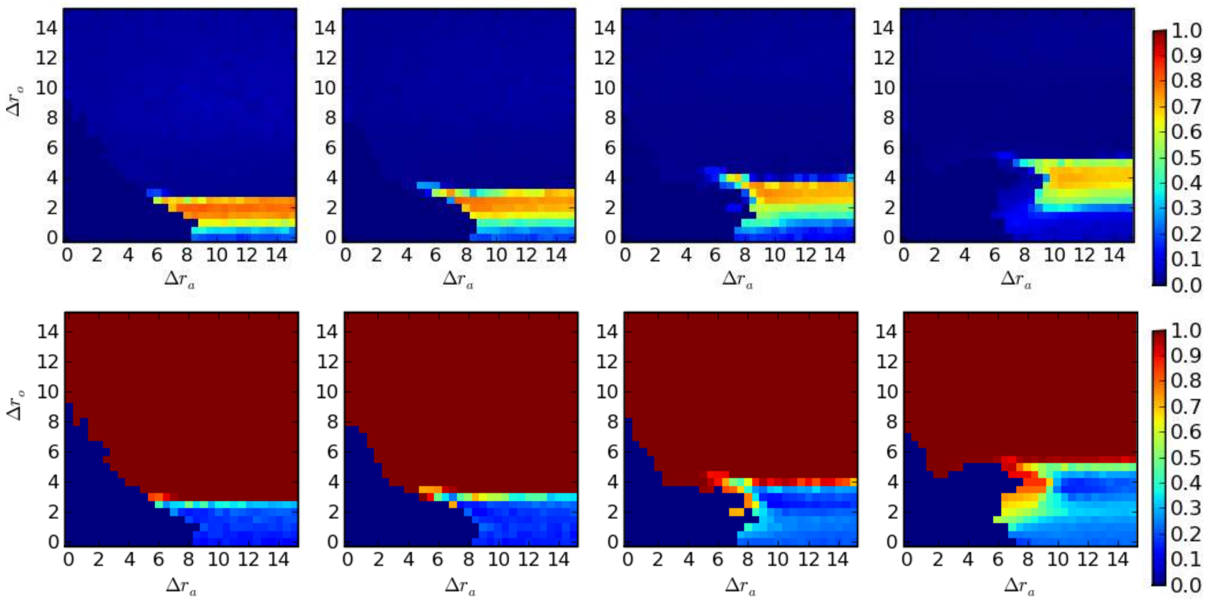

Fig. 12: Quantities *m* (top) and *p* (bottom) for N=50, 100, 200, and 400 (left to right) σ=0.05rad.

In our simulations with random position and orientation, the parameter region in which flocks tend to break up, region (e), is a triangle plus a saw tooth as described before. In the saw tooth areas we noticed that break-up often means that one or two boids break loose from the flock. An example is shown in Figure 13.

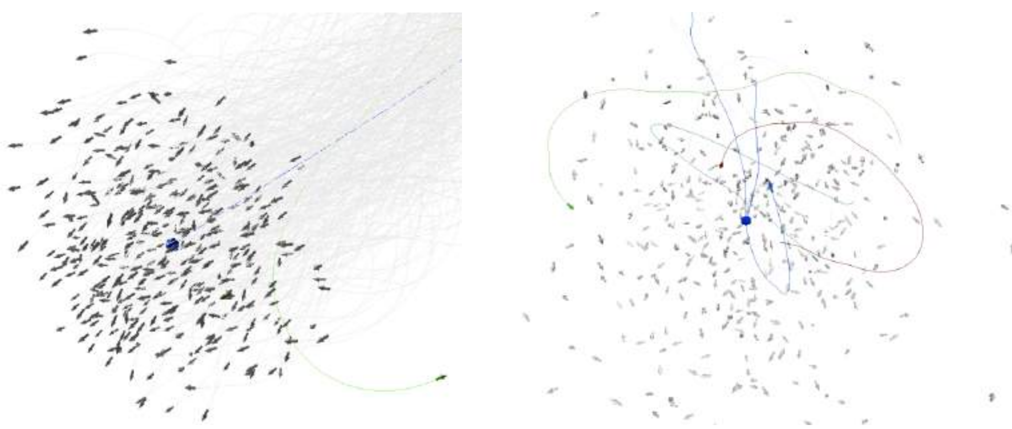

Fig. 13: Flock break-up by a boid breaking loose for r_o=5, r_a=10 (left); swarming for r_o=2, r_a=17 (right). Both figures have σ=0, α=270°, θ=40, *s*=3 and *N*=400. The blue line traces the centroid of the flock.

In highly parallel behavior the flock is guppy-shaped (Fig. 14 left). The extended tail section consists of individuals who took longer to align with the flock and so ended up at the rear of the group. In the toroidal case motion is not contained in a fixed torus (Fig. 14 right). Rather, boids move in partially elliptic tracks of considerable eccentricity, passing through the centroid at times, and occasionally even reversing their direction. Some of the tracks are shown in the picture. The track of the centroid of the flock exhibits a characteristic spiraling motion shown in blue.

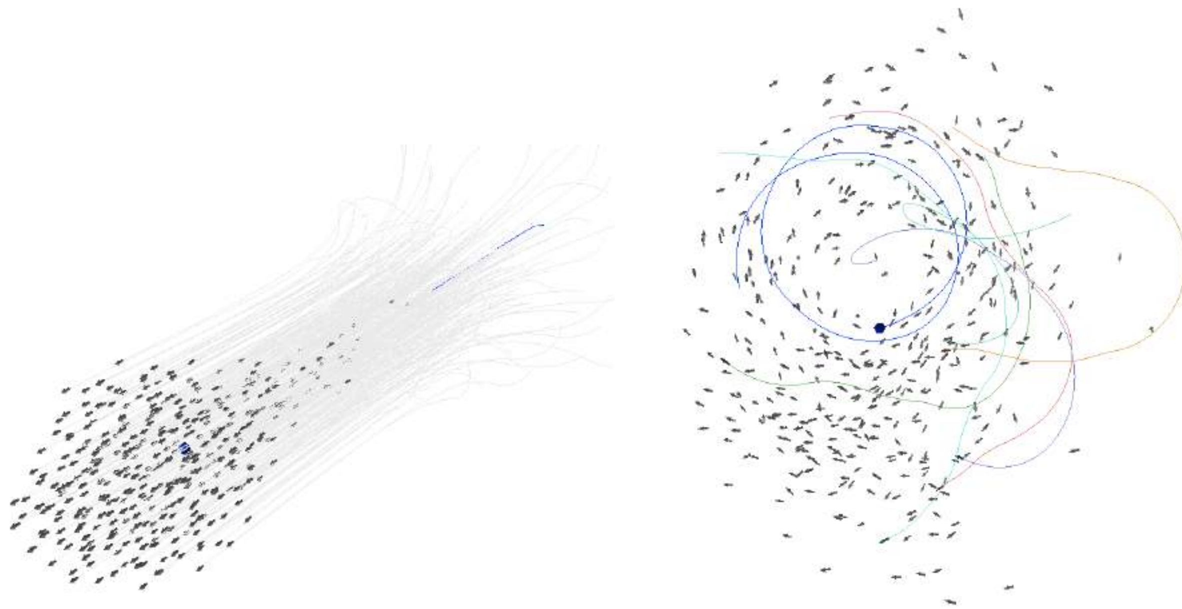

Fig. 14: Highly parallel behavior r_o=9, r_a=17 (left); toroidal behavior for r_o=5, r_a=17 (right). The blue curve traces the centroid position. Other parameters as in Figure 13.

6. SUMMARY AND CONCLUSIONS

Despite the simplicity of the rules for individuals, the emergent crowd behavior under Model C shows significant complexity, depending on the size of the crowd, its density, even its initial arrangement. The difference between 2D and 3D behavior is unexpected. If it is related to the greater freedom of motion in 3D there should be a better conformance when increasing the number of trials or the length of simulation. We observed neither. However, we did observe that the lack of cohesion in the fundamental triangle more often is due to the flock breaking up into many subgroups that develop, as exemplified in Figure 15. In the saw tooth area flocks break up more often because individuals wander off. In those regions the trial outcomes have a larger variation. The highly parallel behavior is the most stable region. All boids are aligned and motion is in a straight line.

Our experiments are in a preliminary stage. Regards Model C, the restriction to fixed speed, focusing behavior entirely on orientation, is a severe restriction that may be responsible for the high variability in the added break-up region.

In [3], Couzin discusses what he terms “collective memory”: He paces a flock through a sequence of changing values for r_o while holding r_a fixed. When increasing r_o from small to large, toroidal behavior transition is observed at some value. However, when decreasing it from large to small values no toroidal phase transition is observed. We believe that this phenomenon is connected with the flock’s behavior dependence on initial position.

7. ACKNOWLEDGEMENTS

Work partially supported by NSC Taiwan grant NSC 97-2212-E-031-002, by NSF grant DSC 03-25227, and by DOE award DE-FG52-06NA26290.

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Fig. 15: Flock breaking up into separate subgroups; r_o=3, r_a=5, N=100. Other parameters are as in Figure 13.