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Fuzzy Tolerancing Based on Available Manufacturing Resources

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ABSTRACT

Tolerancing is an essential part of the manufacturing process. Previous research on tolerancing is based on process capability and robust design. Few have considered the fuzzy factors in shop floor manufacturing. This paper presents a fuzzy tolerancing method which can ensure the robustness of the design and the fuzziness in manufacturing. First, the fuzziness of the manufacturing is analyzed and a new mathematical model is developed. Then, Simulation Annealing (SA) algorithm is applied to solve the model. An example is included to demonstrate the effectiveness of the new method.

Keywords: dimensioning and tolerancing, fuzzy system, simulation annealing.
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1. INTRODUCTION

In engineering design, tolerancing is always an important issue [1]. It has a huge impact on the product quality, manufacturing process, assembly process and cost. As a result, much research has been carried out, especially in recent decades because of the advancement of the computer technology.

Most current tolerancing methods focus on tolerance analysis and synthesis, in which tolerances are assigned to the dimensions of a part according to assembly and cost requirements [2]. The manufacturing process capability has also been widely considered. For example, Lee and Wei [3] proposed a nonlinear programming model for tolerancing. They standardized the tolerances in conjunction with the process capability to minimize the total manufacturing loss that occurs due to non-conforming parts. They also introduced a fuzzy method for tolerancing to maximize the use of the process capability [4]. Huang and Zhong [5] established a sequential linear optimization model for tolerancing based on the process capabilities. Their method was designed to balance the tolerances, manufacturing costs, and acceptance rate of the parts. Gao and Huang [6] presented an optimization approach for tolerancing based on process capabilities. In their approach, the estimated standard deviations of the dimensions, constraints of the tolerancing chains, manufacturing process capability, and economics of machine tools are considered as constraints, while the objective function is the total cost functions. Fang and Wu [7] proposed an optimization model based on the concurrent engineering concept where the objective function was the total cost and the constraints include the assembly requirements, machine tools, and stock removal tolerance. Hu and Peng [8] presented a tolerance modeling and robust design approach to support concurrent engineering. Their method allowed the designer to specify geometric dimensions and tolerances based on assembly requirements and manufacturing costs. Willhelm and Lu [9] developed a framework which can provide automated tolerance synthesis. However, it is found that most existing methods do not consider the fuzziness in the manufacturing process. Shah et al. [10] also proposed a geometric tolerance analysis method.

Cutting tools may have different states of wear affecting the actual dimensions of the part. As a result, it is difficult to know the true performance of a manufacturing process and thus, find suitable machines that can satisfy the designed tolerance [11, 12]. This may cause a decrease in product quality and an increase in manufacturing cost.

In this paper, a fuzzy tolerancing method is introduced. The rest of the paper is organized as follows. In Section 2, the tolerance design problem is formulated as an optimization problem and the mathematical model of fuzzy concurrent tolerance design is developed. In Section 3, the Simulation Annealing (SA) method is adopted to solve the optimization problem. In Section 4, a demonstration example is given via computer simulation. Finally, Section 5 contains conclusions.

2. THE FUZZY TOLERANCING MODEL

It is known that the conventional tolerancing model can usually be described by the optimization problem below [13]:

$$\text{Min } C(T) \tag{1}$$

$$\text{subject to } g_i(T) \le G_i \tag{2}$$

where, $T = (T_1, T_2, \ldots, T_n)^T$, T_i is the designed tolerance for component i, $C(T)$ is the cost function, $g_i(T)$ is the design function and G_i is the functional requirement or the technological requirement of component i.

Such a model is in fact oversimplified for many complex design problems. These problems involve many fuzz factors, which may come from the following sources:

- The linearization of non-linear functions;
- The conditions in manufacturing process, such as the accuracy of the fixture, use of coolant, use of different tools, tool conditions, etc;
- Transitions from qualitative to quantitative specifications, such as the fitting, and the lubrication;
- The qualitative description of the functional requirement of the part.

As a result, the aforementioned optimization model gives only idealized situation.

The proposed idea is to minimize the total cost while satisfying the functional requirement. In general, the total cost consists of the manufacturing cost and quality loss cost. A tight tolerance may easily satisfy the functional requirement (hence, result in no quality loss) but would cause higher manufacturing cost. A number of functions have been used to define the relationship between the tolerance and the cost. These functions may be divided into two types [16-20], (1) Elementary functions based cost model such as the Sutherland function, reciprocal function, reciprocal square function and exponential function. (2) Neural network based functions such as BP neural network and fuzzy neural network. In this research, we adopt the Taguchi's definition: the quality is the loss of customer or society created by the quality fluctuation [14, 15].

2.1 The Design Parameters

We consider the manufacturing may be influenced by seven factors: (1) the machine, (2) the process, (3) the number of processes, (4) the operation time, (5) the tool, (6) the measurement, and (7) the operator. To accommodate the fuzziness of these factors, a set of coefficients, or the weights, is introduced as follows:

$$A = (a_1, a_2, a_3, a_4, a, a_6, a_7) \tag{3}$$

where a_1, a_2, a_3, a_4, a_5, a_6; and a_7 are the weights of the seven factors mentioned above.

Following the fuzzy theory, each factor is divided into several grades and the influence coefficients of each grade can be found through statistical analysis. For example, in machining, the process can be divided into nine grades, including broaching, grinding, cutting, reaming, planning, etc. Hence, the grade can be expressed as follows:

$$B = (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9) \tag{4}$$

where, b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, and b_9 are the influence coefficients. Accordingly the fuzzy cost coefficient, C, is defined as follow:

$$C = \sum_{i=1}^{7} a_i b_{ij} \tag{5}$$

where, b_{ij} is the influence coefficient of the j^{th} grade of the i^{th} factor.

2.2 Objective Function

Due to the fuzziness in the manufacturing, the variation of the product is inevitable. According to Taguchi's view of quality [14, 15], the quality of a product is the loss incurred due to the deviations of the products' characteristics from their target values. The quality loss is defined.

$$C_Q = K\left[s^2 + \left(m - m_0\right)^2\right] \tag{6}$$

where, K is the quality loss coefficient, s the standard deviation of the product, m the mean of the product, and m_0 the target product mean. Assuming the loss created by an unqualified product is C_{MA} and the tolerance specification is T. The quality loss coefficient, K, can be expressed as follows:

$$K = \frac{C_{MA}}{\left(T/2\right)^2} \tag{7}$$

Moreover, the process capability, C_p, and the deviation coefficient, k, can be calculated as follows [5]:

$$C_p = \frac{T}{6s} \tag{8}$$

$$k = \frac{\left|m - m_0\right|}{T/2} \tag{9}$$

Substitute Equations (7), (8) and (9) into Equation (6), it follows that:

$$C_Q = C_{MA}\left(\frac{1}{9C_p^2} + k^2\right) \tag{10}$$

To reach a balance between the quality loss and the manufacturing cost, the objective function of the proposed tolerance design model is therefore the total cost, C_T , defined below:

$$C_T = \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{r=1}^{o_{ij}} x_{ijr}\left(\left(C_M\right)_{ijr} + \left(C_Q\right)_{ijr}\right) = \sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{r=1}^{o_{ij}} x_{ijr}\left[\left(C_M\right)_{ijr} + \left(C_{MA}\right)_{ijr}\left(\frac{1}{9\left(C_p\right)_{ijr}^2} + k_{ijr}^2\right)\right] \tag{11}$$

where, n is the number of components in the dimension chain, m_i the production mean of Component i, Q_{ij} the quality of using i^{th} machine to make j^{th} component, $(C_M)_{ijr}$, $(C_{MA})_{ijr}$, and $(C_Q)_{ijr}$ are the manufacturing cost, rejection cost, and quality loss cost of using i_{th} machine to make j^{th} component *with* r^{th} processing method respectively, Finally, x_{ijr} is the selection coefficient, $x_{ijr} = 1$ implies the selection is made, and $x_{ijr} = 0$ otherwise.

2.3 The Constraints

There are three types of constraints are considered.

(1) Process capacity constraint. Each process has its limits, thus:

$$\left(C_p\right)_{ijr}^{L} \le \left(C_p\right)_{ijr} \le \left(C_p\right)_{ijr}^{U} \tag{12}$$

where $\left(C_p\right)_{ijr}^{L}$ and $\left(C_p\right)_{ijr}^{U}$ are the upper bound and lower bound of the process capability when using i^{th} machine to make j^{th} component with r^{th} processing method.

(2) Constraints on processing methods selection. Only one processing method can be selected for each component, thus:

$$\sum_{r=1}^{o_{ij}} x_{ijr} = 1 \tag{13}$$

(3) Assembly requirements. The assembly requirements represent the quality characteristics required by the product. Suppose s_{ij} is the standard deviation when using i^{th} machine to make j^{th} component, it should satisfy the following constraints:

$$s_{ij} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m_i}\sum_{r=1}^{o_{ij}}\left(\frac{f}{T_{ij}} x_{ijr} s_{ijr}\right)^2} \le s_{\lim} \tag{14}$$

where, $s_{\lim}$ is the upper limit of s_{ij}.

(4) The minimum processing allowances for operations. This constraint represents the minimum requirement of the machines.

$$D_{ij} = \sum_{r=1}^{o_{ij}} 6x_{ijr} \left(C_p\right)_{ijr} s_{ijr} \le \left(D_{\lim}\right)_{ij} \tag{15}$$

where, D_{ij} is the allowances when using i^{th} machine to make j^{th} component, and $(D_{\lim})_{ij}$ is the upper limit of D_{ij}.

In the optimization model, the design parameters includes the selection index, x_{ijr}, and the required process capacity $(C_p)_{ijr}$. Note that the former is a discrete variable while the latter is a continuous variable. Moreover, the process factors are fuzzy variables, and hence, it is a fuzzy optimization model.

3. OPTIMIZATION ALGORITHM

To solve the aforementioned fuzzy optimization problem, the first step is to transform the fuzzy optimization problem to a non-fuzzy optimization problem. This can be done using the membership function defined as shown in Figure 1. More details can be seen in [13].

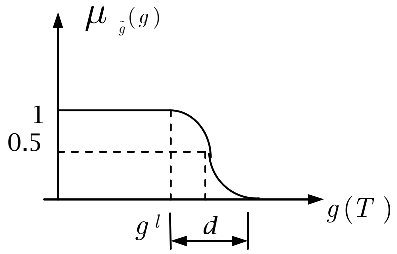


Fig. 1: Illustration of the fuzzy membership function.

The Simulated Annealing (SA) algorithm [21, 22] is used to solve the optimization problem. By using this method, the optimization problem is compared with thermal equilibrium problem of statistical mechanics. It is a heuristic random search algorithm based on Monte Carlo Simulation. Figure 2 shows the flow chart of tolerance design based on SA algorithm.

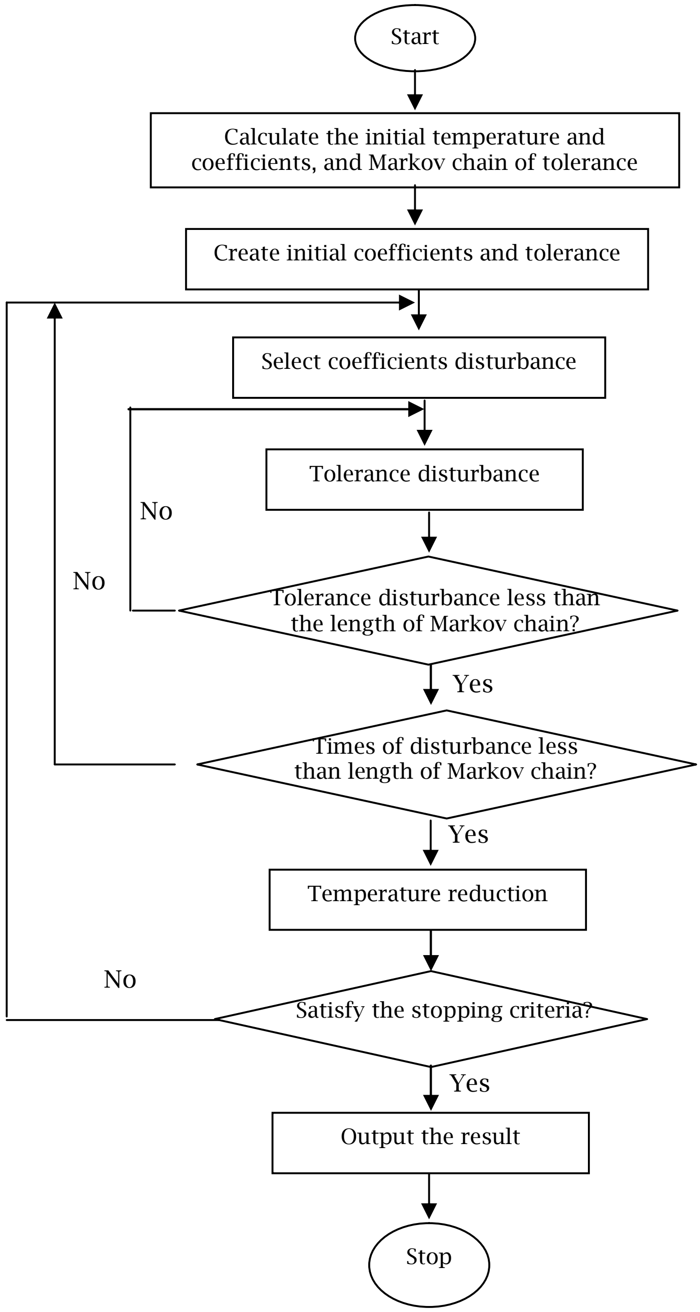
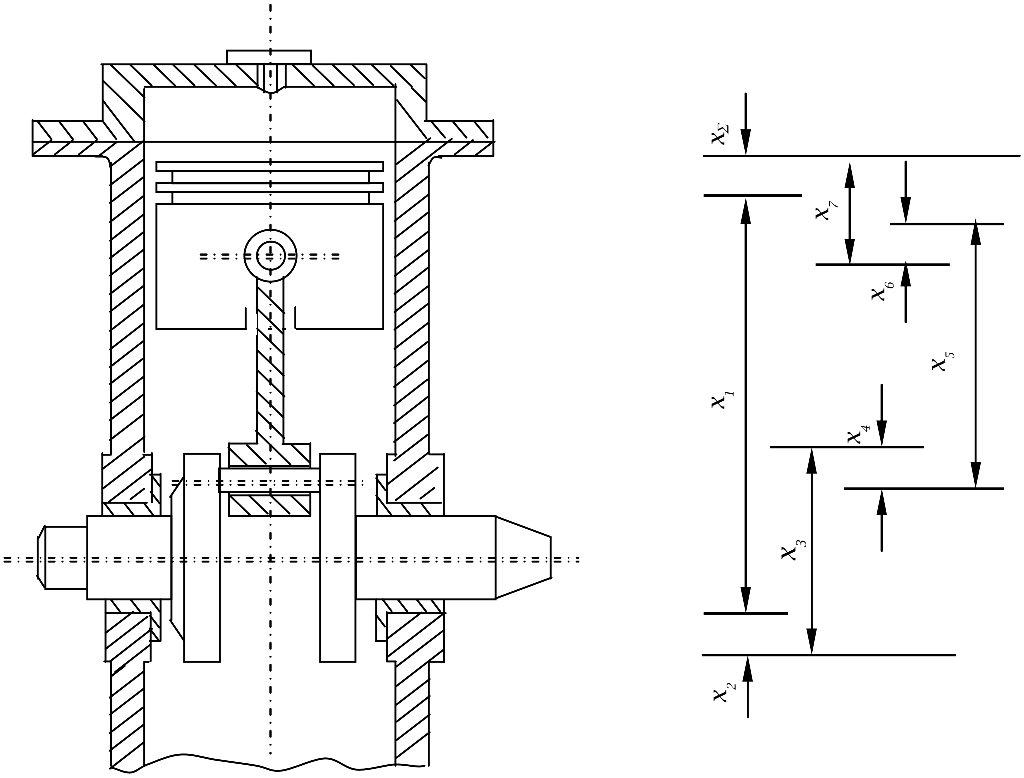


Fig. 2: Flow chart of tolerance design based on SA algorithm.

4. A DEMONSTRATION EXAMPLE

The proposed method has been used to solve a practical problem. A demonstration example is given here. Figure 3(a) shows the simplified partial assembly drawing of the piston and the cylinder of an automotive engine. When the piston reaches the upper dead point, the clearance between the actual position and the design position may vary. This clearance directly affects the compression ratio and its tolerance is set at 0.31 mm.



(a) Simplified assembly draft (b) Assembly dimensional chain

Fig. 3: The assembly drawing of an automative engine and its dimensional chain.

According to the assembly requirement, the dimensional chain is generated as showed in Fig.3 (b). The description of the link in dimensional chain is listed in Table 1. The sketches of main parts related to dimension of component links are shown in Fig. 4.

Link	*Name*	*Nominal Dimension (mm)*	*Link*	*Name*	*Nominal Dimension (mm)*
x_1	Distance between cylinder top surface and centerline of cylinder spindle	292.3	x_{21}	Diameter of crankshaft spindle	70
x_2	Eccentricity between centerline of cylinder spindle and axis diameter	0	x_{22}	Diameter of cylinder spindle	70
x_3	Center distance between crankshaft spindle and crankshaft axle	50	x_{41}	Diameter of crankshaft axle	57

x_4	Eccentricity between centerline of big hole and centerline of crankshaft axle	0	x_{42}	Diameter of connecting rod big hole	57
x_5	Center distance between centerline of big hole and centerline pinhole	174	x_{61}	Diameter of connecting rod pinhole	32
x_6	Eccentricity between centerline of pinhole and centerline of piston pinhole	0	x_{62}	Diameter of piston pinhole	32
x_7	Distance between piston top surface and centerline of piston pinhole	69	x_{Σ}	Distance between piston top surface and cylinder top surface	0.7

Tab. 1: The description of link in dimensional chain.

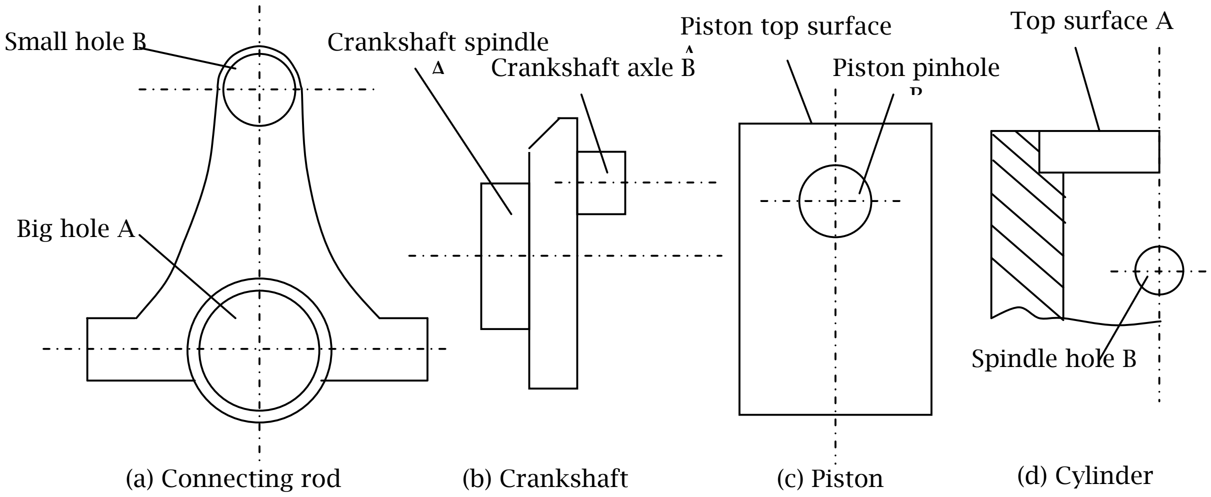


Fig. 4 Sketches of main parts.

The initial tolerance grade for each link in the dimensional chain is first calculated. As shown in [12], the manufacturing features include cylinder, hole and location. The main processes for these features include rough machining, semi-finish machining and finish machining. After choosing the machining methods, the related parameters such as machining cost can be computed. The machining methods and processes are listed in Table 2. Then the optimization model is then established. Finally, using SA, the optimization result is given in Table 2.

Parts	*Process number*	*Machining method and process*	*Limit (mm)*	σ	C_p	*Tolerance (mm)*
Cylinder	1	Rough boring for spindle hole B	0.190-0.460	0.0374	1.1209	0.200
	2	Semi-finishing boring for spindle hole B	0.074-0.120	0.0222	1.1098	0.120
	3	Rough grinding for top surface A	0.020-0.250	0.0452	1.2066	0.225
	4	Finishing boring for spindle hole B	0.030-0.074	0.0187	1.3352	0.084

	5	Finishing grinding for top surface A	0.015-0.045	0.0097	1.3522	0.043
Connecting rod	1	Rough boring for big hole A	0.190-0.460	0.0422	1.1021	0.230
	2	Semi-finishing boring for big hole A	0.074-0.120	0.0158	1.2321	0.077
	3	Finishing boring for big hole A	0.030-0.074	0.0053	1.0001	0.032
	4	Rough boring for small hole B	0.190-0.460	0.0395	1.2543	0.189
	5	Semi-finishing boring for small hole B	0.074-0.190	0.0141	1.2098	0.070
	6	Finishing boring for small hole B	0.019-0.046	0.0091	1.3045	0.042
Crankshaft	1	Rough cutting for outer surface A	0.190-0.460	0.0488	1.1608	0.252
	2	Rough cutting for outer surface B	0.190-0.460	0.0488	1.2138	0.241
	3	Semi-finishing cutting for outer surface A	0.074-0.190	0.0155	1.2075	0.077
	4	Semi-finishing cutting for outer surface B	0.074-0.190	0.0155	1.2075	0.077
	5	Finishing grinding for outer surface A	0.019-0.030	0.0042	1.1052	0.023
	6	Finishing grinding for outer surface B	0.019-0.030	0.0042	1.4917	0.017
Piston	1	Rough cutting for outer surface	0.160-0.390	0.0492	1.2302	0.240
	2	Semi-finishing cutting for outer surface A	0.062-0.160	0.0160	1.2765	0.075
	3	Finishing grinding for outer surface	0.013-0.033	0.0028	1.2933	0.013

Tab. 2: The selection of machining methods and optimizied tolerances.

A comparison between the results shown in Table 2 and the original design [7] shows that the manufacturing cost can be reduced by 14.91%. The saving is caused by maximizing the total process capability when selecting machines while ensuring the product quality.

5. CONCLUSIONS

Based on the discussions above, following conclusions can be drawn:

(a) Unlike the traditional tolerance design methods, which ignore the fuzziness in manufacturing, the proposed method uses the fuzzy theory to select available machines and processes to minimize the total cost. It may result in significant saving.
(b) The proposed method considers both the manufacturing resources (the availability and the accuracy of the machines and processes) and the product quality (the quality lost). With the optimization using SA, it gurantees the tolerance is robust and cost effective.
(c) Based on a practical example, the proposed method saves about 15% of the total cost.

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7. REFERENCES

[1] Bjorke, O.: Computer aided tolerancing, ASME, New York, 1989.

[2] Ngoi, B. K. A.; Ong, C. T.: Product and Process Dimensioning and Tolerancing Techniques, A State-of-the-Art Review, Int. J. of Advanced Manufacturing Technology, 14, 1998, 910-917.
[3] Lee, Y. C.; Wei, C. C.: Process Capability-Based Tolerance Design to Minimise Manufacturing Loss, Int. J. of Advanced Manufacturing Technology, 14, 1998, 33-37.
[4] Lee, Y. H.; Wei, C. C.; Chang, C. L.: Fuzzy Design of Process Tolerances to Maximize Process Capability, Int. J. of Advanced Manufacturing Technology, 15, 1999, 655-659.
[5] Huang, M. F.; Zhong, Y. R.: Optimized Sequential Design of Two-Dimensional Tolerances, Int. J. of Advanced Manufacturing Technology l, 33, 2007, 579-593.
[6] Gao, Y.; Huang, M.: Optimal Process Tolerance Balancing Based on Process Capabilities, Int. J. of Advanced Manufacturing Technology, 21, 2003, 501-507.
[7] Fang, H. F.: A Study on Concurrent Tolerance Design and Methods, PhD dissertation, School of Mechanical Engineering, Zhejiang University, 1997.
[8] Hu, J.; Peng, Y.: Tolerance Modelling and Robust Design for Concurrent Engineering, Proc. Of IMechE. Part C: J. of Mechanical Engineering Science, 221, 2007,455-465.
[9] Willhelm, R. G.; Lu, S. C. Y.: Tolerance Synthesis to Support Concurrent Engineering, Annual of CIRP, 41(11), 1992, 197-200.
[10] Shah J. J.; Ameta G.; Shen Z. S.; Davidson J.: Navigating the Tolerance Analysis Maze, Computer-Aided Design & Applications, 4(5), 2007, 705-718.
[11] Ji, S. P.; Sun. X. H.; Ma, Y. L.: A Statistic Tolerance Analyzing Method Based on Manufacturing Context, Chinese Mechanical Engineering, 10(3), 1999, 249-252.
[12] Mao, J. et al: Manufacturing Environment-Oriented Robust Tolerance Optimization Method, Int. J. of Advanced Manufacturing Technology, 41, 2009, 57–65.
[13] Yang J. X.; Wu Z. T.: A Study on Modelling Techniques for Product Cost-Tolerance, J. of Zhejiang University (Natural Science), 30(5), 1996, 517-522.
[14] Taguchi, G.: On-line Quality Control During Production, Japanese Standards Association, Tokyo, 1981.
[15] Taguchi, G.: Taguchi on Robust Technology Development: Bringing Quality Engineering Upstream, ASME, New York, 1993.
[16] Creveling, C. M.: Tolerance Design, A Handbook for Developing Optimal Specifications, Reading, Addison Wesley, 1993.
[17] Yeo, S. H. et al: Cost-Tolerance Relationships for Non-Traditional Machining Processes, Int. J. of Advanced Manufacturing Technology, 13, 1993, 35-41.
[18] Diplaris, S. C.; Sfantsikopoulos, M. M.: Cost-Tolerance Function, A New Approach for Cost Optimum Machining Accuracy, Int. J. of Advanced Manufacturing Technology, 16, 2000, 32-38.
[19] Yang J. X.; Gu D. Q.; Wu Z. T.: Machining Cost-Tolerance Model Based on Artifical Neural Network, Chinese Mechanical Engineering, 6, 1996, 41-42.
[20] Lin, Z. C.; Chang, D. Y.: Cost-Tolerance Analysis Model Based on a Neural Networks Method, Int. J. of Production Research, 40(6), 2002, 1429-1452.
[21] Cisilino, A. P.; Sensale, B.: Application of a Simulated Annealing Algorithm in the Optimal Placement of the Source Points in the Method of the Fundamental Solutions, Computational Mechanics, 28, 2002, 129–136.
[22] Jiang, L. C.; Liu, J. P.: Revised Simulated Annealing Algorithm, Chinese J. of Engineering Geophysics, 4(2), 2007, 35-40.