



## Implementation of Biomechanical Mating Conditions in CAD

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### ABSTRACT

The design of customizable products that interface with the body can be designed using a design strategy based on top-down assembly methods. In this strategy, the human surface serves as the base part and the product is designed in reference to the body. Because traditional assembly design defines mating conditions for prismatic parts only, additional mating conditions that address the interface of complex curves and surfaces associated with the body have been defined. This paper presents methods for the implementation of bio-interface mating conditions in CAD.

**Keywords:** mass customization, mating conditions, biomechanical, interface design.

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### 1. INTRODUCTION

The design of customizable products that interface with the body pose unique problems due to the complexity of human surfaces, the need for parametric strategies, and undeveloped CAD tools specific to biomechanical design. In [3], the authors have introduced design strategies for the articulation of customizable products that interface with the human body through top-down assembly modeling and the use of geometric references attached to human scan data. In their research, the authors have not found other research utilizing the modeling of products that interface with the body in reference to scan data or using assembly model theory. This paper presents constraint equations that would allow the implementation of these strategies and techniques in CAD.

Current CAD tools for mechanical design focus on the design of prismatic parts and have assembly modeling operations for implementing mating conditions between these parts in CAD. These tools and CAD operations allow designers to specify mating conditions between prismatic faces, axes, and edges. However, in the design of products that interface with the body, the prismatic mating conditions rarely apply to the human-product interfaces which generally consist of mating conditions between complex surfaces, curves, and points on the body.

While the case studies and examples in [3], [7], and this paper were modeled using traditional CAD tools and in reference to human data, the parametric modeling process for products that interface with the body would be greatly simplified with the implementation of biomechanical mating conditions for human surfaces, curves, and points. The purpose of this paper is to identify the strategies and methods for implementing these mating conditions and creating product geometry that interfaces with human references in CAD. Although this paper will not identify the solution for every case, the solution for convex objects using the NURB representation used in CAD is presented.

Many of the biomechanical mating conditions involve specifying the distance between the human surfaces and product geometry, and offsetting geometry to create a product that fits the body. Research in calculating distances between points and B-spline curves and surfaces can be found in [12], [6], [1], and [16] where different techniques and algorithms were developed to determine distances between different geometric entities. A number of researchers have developed methods for offsetting geometry with applications in machining, robotics, and CAD. [10], [8], and [4]

demonstrate surface offset research for machining applications and use algorithms involving offsetting vertices, using tangent plane methods for point cloud data, and methods using bi-cubic surface patches, surface conversion, and subdivision algorithms. [9] provides a good review of offsetting methods in CAD as well as provides accurate algorithms for offsetting NURBS curves and surfaces.

While this paper does not go into detailed algorithms for each biomechanical mating condition implementation, it gives an introduction to the different biomechanical mating conditions, the applications of these mating conditions in design, and generalized methods and techniques for implementing the mating conditions in CAD. Further and more detailed development of the algorithms for each individual mating condition is left for future work.

## 2. BACKGROUND OF BIOMECHANICAL DESIGN METHODOLOGY BASED ON ASSEMBLY MODEL THEORY

The development of these constraint equations resulted from research in the development of a methodology for the design of customizable products that interface with the body. This design methodology incorporates top-down assembly model theory, biomechanical mating conditions. This section will discuss assembly model theory, the biomechanical mating conditions [3], and the bio-interface design methodology [7], [3].

### 2.1 Assembly Model Design

Assembly models are mock-ups of a mechanical product in assembly form. In CAD assembly models, the parts of the product are modeled and assembled together to create the assembly model. This allows designers to evaluate the interactions between the parts, review assembly procedures, analyze the relative motion of parts, and simulate product appearance. To assemble parts into an assembly model, mating conditions are specified and dictate how geometry or reference datums are constrained in order to hold the product together in a logical fashion.

In biomechanical design, products interface and mate with the human body. Because of this, the authors introduced a design methodology based on top-down assembly techniques where the human body serves at the base part of the assembly, and the product is designed in reference to the human body [3]. The product is modeled by bringing scan data into a CAD system, defining geometric references based on anatomical features or surfaces on the body, and modeling the product geometry in reference to these anatomical datums. This process is shown in Figure 1. Although products that interface with the body can be modeled using these techniques and traditional CAD tools, the process would be simplified with the definition and implementation of biomechanical mating conditions in CAD.

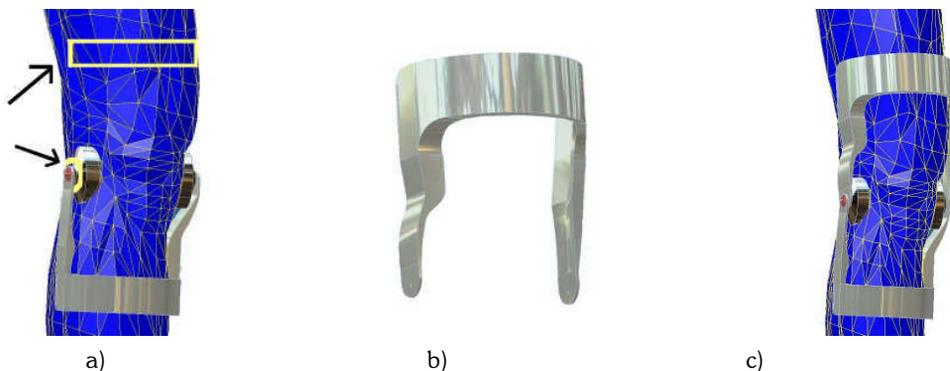


Fig. 1: Top Down Assembly in modeling a knee brace. a) Mating surfaces are identified on the existing parts and body for the top section of the brace (marked in yellow). b) The top brace is modeled off of the interface by using parameters from the assembly model. c) The top brace is added to the assembly. [2]

#### 2.1.1 Traditional and Biomechanical Mating Conditions

Traditional assembly design specifies the mating conditions for mechanical parts and are generally based upon shared axes, faces, or distances between two parts. Mating categories in traditional mechanical assembly modeling include contact, angular, or distance conditions. Each of these mating categories contains subclasses that further specify the

constraint. Contact constraints involve defining planes or axes on two separate parts that touch each other. Distance constraints confine parts to be a specified distance apart. Finally, angular constraints define an angle between key features on two parts. Generally, several of these constraints are used per part to fully define that part's position in an assembly.

In traditional mechanical design, the specification of mating conditions is limited to these prismatic surfaces. However, because human surfaces and biomechanical products often involve the mating of complex surfaces and points that are not prismatic, mating conditions for these additional interfaces were developed [3].

In the contact class, biomechanical additions include a surface contact, where two complex surfaces come into contact with each other, a curve contact, where a narrow surface or curve on the product contacts the body, and a point contact, where a point or small area on the product contacts the human surface. Generally, the angular subclasses consist of an axis or face and an angle. No additional biomechanical mating conditions were needed since the traditional align subclasses could be applied to reference datums and axes of the body. The final mating condition class is the distance class. In traditional design, this mating condition specifies the distance between two prismatic faces or the distance a datum reference. In biomechanical design there are additional distances that need to be specified in order to define the assembly of a product to the human body. These conditions include the following distances:

- Point to point
- Point to curve
- Point to surface
- Curve to curve
- Curve to surface
- Surface to surface

These mating conditions are shown in Table 1.

<b>Mating Condition Classes</b>	<b>Traditional Mating Condition Subclasses</b>	<b>Biomechanical Mating Condition Subclasses</b>
Contact	Mate	Surface Contact
	Align	Curve Contact
	Tangent	Point Contact
Angular	Parallel	
	Perpendicular	
	Custom Angle	
Distance	Center	Point-to-Point
		Point-to-curve
		Point-to-surface
	Distance	Curve-to-curve
		Curve-to-surface
		Surface-to-surface

Tab. 1: Summary of traditional and biomechanical mating conditions.

**2.2 Bio-Interface Design Methodology**

Whitney et al. [15] describe a method for designing assemblies that uses geometric relationships and assembly features to achieve kinematic constraints over an entire assembly. In this process, geometric relationships referred as Key Characteristics (KCs) are defined between parts and illustrated in a liaison diagram. KCs define important relationships between parts where failure could result if not followed [5], [14], [15]. This theory lends itself well to the biomechanical assembly design process where key characteristics are defined between the human scan data and the product. These key characteristics ultimately show where mating conditions occur between the product and the body.

These methods and the mating conditions described in Section 2.1 are incorporated into the traditional design method defined by Ulrich and Eppinger [13] to create a design strategy for articulating customizable products that interface with the body. This strategy is stated below and illustrated by a body armor example [11]:

1. Planning
2. Concept Development
  - a. Geometric Configuration
3. System Level Design
  - a. Liaison Diagram
  - b. Mating conditions
  - c. Parametric Strategies
  - d. Feature Structure
4. Detailed Design
  - a. Bio-reference definitions
  - b. Product geometry creation
  - c. Parameterization
  - d. Mass customization
5. Testing and Refinement
6. Production Ramp Up

2.2.1 Planning and Concept Development

In the planning and concept development phases, the business strategy is identified, needs of the target market are determined, and the product concept is selected. In biomechanical design, these steps do not change. In the concept development phase for this assembly model method, the interaction of the product components to the body and each other is defined in the geometric configuration, shown in Figure 3.

2.2.2 System Level Design

In system level design, the geometric configuration is further developed to specify the key characteristics of the assembly. These KCs are then represented in a liaison diagram. Following the articulation of the liaison diagram, the diagram is analyzed in order to define the mating conditions, parametric strategies, and bio-references used to define the feature structure of the product. The parametric strategies define how the product geometry will reconfigure to fit a different set of scan data, generally where the product mates with the human scan data. Figure 4 illustrates the liaison diagram for the body armor example. Tables 2 and 3 specify the key characteristics, mating conditions and parametric strategies for the body armor.

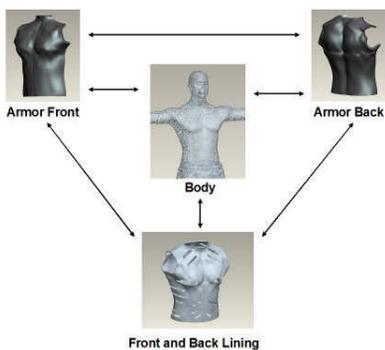


Fig. 3: Geometric Configuration for Body Armor.

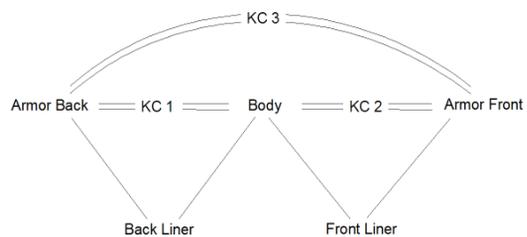


Fig. 4: Liaison Diagram for Body Armor.

KC	Description	Mating Condition
1	Back armor to body	Surface-to-surface contact
2	Front armor to body	Surface-to-surface contact
3	Back to front	Contact (traditional)
4	Back liner to back	Surface-to-surface contact
5	Front liner to front	Surface-to-surface contact

Tab. 2: Description of key characteristics and identification of associated mating conditions.

KC	Mating Condition	Parametric Strategy
1	Surface-to-surface contact	Create surface by: 1. creating curves referenced to key bio-points on scan data surface to make several cross-sections. 2. create surface through cross-sections using swept blend. As key bio-points change, cross sections will adjust to fit 3. Negative ext
2	Surface-to-surface contact	Create surface by: 1. creating curves referenced to key bio-points on scan data surface to make several cross-sections. 2. create surface through cross-sections using swept blend. As key bio-points change, cross sections will adjust to fit 3. Negative ext
3	Contact (traditional)	Negative extrusion attached to coronal plane
4	Surface-to-surface contact	Offset surface from back to ensure parts fit together properly
5	Surface-to-surface contact	Offset surface from front to ensure parts fit together properly

Tab. 3: Description of the mating conditions and associated parametric strategies.

With the parametric strategies defined, the designer can then identify the geometric references attached to scan data that the product geometry must be referenced to create a parametric product. With these references identified, the feature structure can be specified. The feature structure decomposes the product into the basic operations required to create the geometry in CAD. Figure 5 shows the feature structure for the body armor where the product is created using the following steps. In 5(a) datum points at locations on scan data where there are significant changes in geometry is created, 5(b) shows the creation of datum planes through datum points 5(c) cross section sketches for body armor on datum planes are created and referenced to scan data, 5(d) shows the creation of a swept blend through cross sections. 5(e) creates negative extrusions for arm holes and lining, 5(f) Creates negative extrusion referenced to coronal plane to remove material on alternating sides of plane for front and back halves. After the feature structure is defined, the detailed design phase of the methodology begins.

2.2.3 Detailed Design

The detailed design phase of the design process includes bio-interface design, geometry creation, parameterization, and mass customization. In bio-interface design, the critical interfaces identified in the liaison diagrams by key characteristics are further developed and defined in order to create the appropriate bio-datums and references on the scan data for geometry creation. With the definition of bio-references attached to scan data, the product geometry is created, shown in Fig. 6 for the body armor example.

Because the geometry is referenced to key bio-points bio-datum planes the product should regenerate as the points and datum planes are moved to create a customizable product. However, because current CAD programs are designed for traditional mechanical parts, the creation of these mating conditions and parametric models are not easily changed to allow for the reconfiguration of the product to fit a new individual. This paper presents the constraint equations that would allow the user to define the biomechanical mating conditions in CAD.

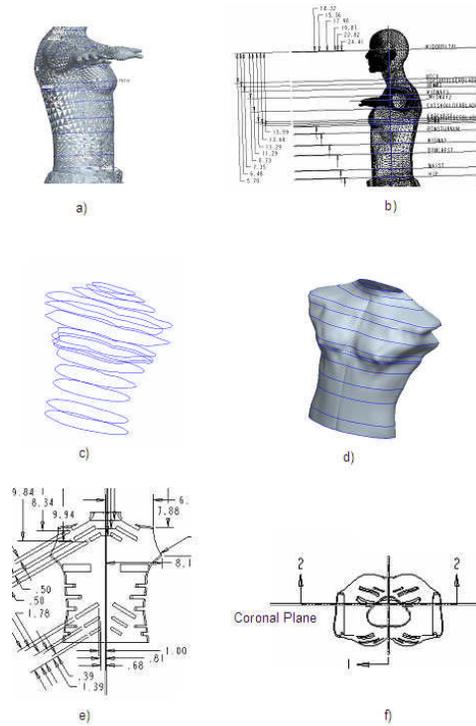


Fig. 5: Feature Structure for Body Armor Case study.

**3. CONSTRAINT EQUATIONS FOR BIOMECHANICAL MATING CONDITIONS**

[3] and [7] outline a method to design products that interface with the body through the identification of key interfaces, the classification of mating conditions for these interfaces, and 3D parametric constraints for the product. However, current CAD systems do not support the discussed mating conditions for bio-interface design. CAD programs allow the specification of mating conditions for prismatic parts, but do not support the mating of complex surfaces, curves, and points. This paper identifies strategies and methods for implementing these mating conditions in CAD programs.

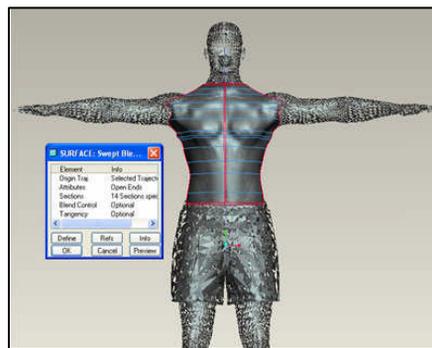


Fig. 6: Geometry Creation in body armor.

To implement bio-interface mating conditions in CAD, constraint equations for bio-interfaces must be developed in a form used in CAD systems. The mating conditions and constraint equations must follow the method used by CAD systems to calculate geometry of mating parts. This means the constraint equations must use points, lines, planes, and NURB curves and surfaces.

A NURB curve can be represented by Equation (3.1).

$$C(t) = \frac{\sum_{i=1}^n N_i^k(t) w_i P_i}{\sum_{i=1}^n N_i^k(t) w_i} \quad (3.1)$$

Where  $t$  is the interval value,  $C(t)$  is the NURB curve as a function of  $t$ ,  $P_i$  is the  $i^{\text{th}}$  control point,  $w_i$  is the weight for the  $i^{\text{th}}$  control point,  $k$  is the order of the NURB,  $n$  is the number of control points, and  $N_i^k(t)$  are the normalized B-spline blending functions equal to:

$$N_j^0(t) = \begin{cases} 1 & \text{if } t_i \leq t < t_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (3.2)$$

For a degree zero B-spline curve. For higher degree B-spline curves, the recursive formula found in Equation (3.3) is used:

$$N_j^k(t) = \frac{t - t_j}{t_{j+k} - t_j} N_j^{k-1}(t) + \frac{t_{j+k+1} - t}{t_{j+k+1} - t_{j+1}} N_{j+1}^{k-1}(t) \quad (3.3)$$

A NURB surface can be represented by Equation (3.4).

$$S(u, v) = \frac{\sum_{i=0}^m \sum_{j=0}^n N_i^k(u) N_j^l(v) w_{i,j} P_{i,j}}{\sum_{i=0}^m \sum_{j=0}^n N_i^k(u) N_j^l(v) w_{i,j}} \quad (3.4)$$

Where  $S(u, v)$  is the point on the surface at  $(u, v)$ ,  $P_{i,j}$  is a control point in the control point array,  $k$  is the order of curves in the  $u$  direction,  $l$  is the order of curves in the  $v$  direction, and  $N_i^k(u)$  and  $N_j^l(v)$  are the normalized B-spline blending functions in the respective  $u$  and  $v$  directions on the surface.

The mating constraints for bio-interface design include:

- Point-to-Point
- Point-to-Curve
- Point-to-Surface
- Curve-to-Curve
- Curve-to-Surface
- Surface-to-Surface

Contact conditions are distance constraints where the distance value is zero. Therefore, the mating constraint equations for the above mating conditions also address the surface, curve, and point contact mating conditions in bio-interface design. Distance values in all cases can be zero, for direct contact, a positive value for distances away from the body, or a negative value, when a force is exerted on the human surface.

It is important to note that for each mating condition type, the necessary points, curves, and surfaces for the mating condition must be defined parametrically to scan data. This allows the points, curves or surface to adjust and update to fit the new scan data. Although not further developed in this paper, methods for allowing this capability include

landmarking and template models. With landmarks, points, curves, or surfaces on the scan data can be defined in terms of anatomical landmarks on the body. With a template model, the reference elements are defined in absolute terms. The template updates to fit the scan data, allowing all incoming scan data to have the same number of points at the same location as the template model, but follow the new scan surface.

The following sections define methods for the numeric implementation of the bio-interface mating conditions in a CAD system. To discuss this implementation, the theoretical basis behind the numerical method is also presented. For each mating condition, the description of the constraint, the strategy behind the constraint implementation, and the numerical implementation in the CAD system is presented. It is important to note that in some cases, as in the modeling of traditional mechanical parts in CAD, some of these mating conditions may be contradictory with each other when several mating conditions must be implemented at the same time. In these cases, the engineer will need to determine the precedence of mating conditions and eliminate the lower precedent conditions to complete the design of the product.

### 3.1 Point-to-Point Mating Condition

The point-to-point constraint equation allows a point on the product geometry to interface with a specified point on the human scan data. This interface can be either direct contact, where the distance between points is zero, or a specified distance. Potential applications for a point-to-point constraint in products that interface with the body are:

- knee brace hinge with knee joint
- surgical stereotaxy devices
- elbow brace

#### 3.1.1 Theoretical Basis for Point-to-Point Constraint Equation Implementation

In designing a point-to-point mating condition, the parametric relationship between the human surface and the product geometry must first be defined. In the case of a point-to-point mating condition, the parametric relationship is the definition of the distance between the two points. This distance can be positive, zero, or a negative value if a force is desired at a location on the body.

The constraint equation for a point-to-point mating condition utilizes the equation for the distance between two points, shown in Equation 3.1.1

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (3.1.1)$$

Where  $d$  is the distance between points  $P_1 = (x_1, y_1, z_1)$  and  $P_2 = (x_2, y_2, z_2)$ .

If the scan data, or human surface, is approximated by a NURB surface  $S(u, v)$ , a selected point on the human surface that interfaces with a point on the product geometry can be represented Equation 3.1.2:

$$P_2(x, y, z) = S(u_0, v_0) \quad (3.1.2)$$

Where  $P_2(x, y, z)$  represents a point with  $x$ ,  $y$ , and  $z$  coordinates at  $(u_0, v_0)$  on the human surface. Thus, the distance equation for a point-to-point mating condition would be:

$$d = \sqrt{(S(u_0, v_0)_x - x_1)^2 + (S(u_0, v_0)_y - y_1)^2 + (S(u_0, v_0)_z - z_1)^2} \quad (3.1.3)$$

Where  $S(u_0, v_0)_x$  is the  $x$  coordinate,  $S(u_0, v_0)_y$  is the  $y$  coordinate, and  $S(u_0, v_0)_z$  is the  $z$  coordinate at point  $(u_0, v_0)$  on the NURB surface.

#### 3.1.2 Numeric Implementation of Point-to-Point Constraint Equations

The numeric implementation of the point-to-point constraint in a CAD system requires the system to allow the user to select a point on the human scan data. This point must be fully defined in terms of either anatomical landmarks, bio-references, or in absolute terms to allow for a parametric model.

Generally, the point on the product geometry will be located on a normal vector from the human data, however, the user may specify the direction to the product point with respect to a reference datum, axis, or created product

geometry. With the specification of the directional vector and the distance of product point from the human surface, the product point can then be created by projecting the point along the directional vector at the specified distance.

The directional vector can also be specified with respect to a curve on the human scan data, or the human surface. This results in a point-to-curve and the point-to-surface constraint equation. The next sections further develop the theoretical basis and numerical implementation for these constraints.

### 3.2 Point-to-Curve Mating Condition

The point-to-curve constraint equation allows a point on the geometry to interface with a specified curve on the human scan data. Like the point-to-point constraint equation, this interface can either be direct contact or a specified distance. Potential applications of this mating condition in products are:

- Halo Brace
- Foot Orthotic
- Posture Corrector

The next sections describe the strategy and implementation of the point-to-curve constraint equation.

#### 3.2.1 Theoretical Basis for the Point-to-Curve Constraint Equations

The point-to-curve constraint equation can be represented using the offset curve equation. If the curve on the human surface is approximated with the NURB  $C(t)$  and the point on the NURB curve where the interface is to occur is at  $t_0$ , the equation for the new point can shown by Equation 3.2.1:

$$P(x, y, z) = C(t_0) + dn_0 \quad (3.2.1)$$

Where  $P(x, y, z)$  represents the point on the product geometry that interfaces with the curve on the human surface at  $t_0$ ,  $d$  is the distance for the point-to-curve interface, and  $n_0$  is the directional vector with respect to the curve at  $C(t_0)$ .

In creating the product geometry, the user will generally know the NURB approximation of the curve on the human surface, as well as the distance to the point on the product geometry. The steps for creating a point that interfaces with a curve on the scan data is:

1. Define the parametric relationship between the curve on the human surface and the point on the product.
2. Create guide curve on scan data, defined in reference to scan data
3. Specify direction and distance of product point from guide curve (distance function).
4. Using the distance function, calculate the new point on the geometry the specified point on the guide curve using Equation (3.2.2).

#### 3.2.2 Numerical Implementation of Point-to-Curve Constraint Equations

The Point-to-curve constraint equation requires that the CAD system allow the user to create a curve on the scan data. This curve must be fully defined in absolute terms, in reference to bio-reference planes, or with respect to anatomical landmarks. Generally, the curve would be created through a series of points on the scan surface. The steps for numeric implementation are:

1. Create the guide curve through a series of points on the scan surface
2. Specify the distance function for projecting the product point from the guide curve. In general, the scan data will be represented by a polygon. The directional vector can be calculated in terms of the polygon plane.
3. At the specified point on the guide curve, calculate the distance function.
4. Construct the point on the product.

### 3.3 Point-to-Surface Mating Condition

The point-to-surface constraint equation allows a point on the geometry to interface with a specified surface on the human scan data. Like the point-to-point and point-to-curve constraint equations, this interface can either be direct contact or a specified distance and utilizes the distance equation between two points. However, in calculating the

directional vector from which to project the new point and the specified distance, the vector direction will be calculated in respect to the human surface. Potential applications of this mating condition in products are:

- Safety glasses frame to side of head
- Orthotic devices
- Radiation therapy masks

### 3.3.1 Theoretical Basis for Point-to-Surface Constraint Equation Implementation

If the human surface is approximated with the NURB surface  $S(u,v)$  and the point on the NURB curve where the interface is to occur is at  $(u_0, v_0)$ , the equation for the new point represented by Equation 3.3.1:

$$P(x, y, z) = S(u_0, v_0) + d\bar{n}_0 \quad (3.3.1)$$

Where  $P(x, y, z)$  represents the point that interfaces with  $S(u_0, v_0)$ ,  $d$  is the distance for the point-to-surface interface, and  $\bar{n}_0$  is the directional vector to the surface at  $S(u_0, v_0)$ .

Generally, the user will know  $S(u_0, v_0)$ , the point on the human surface for the interface, and the distance to the point on the product. Therefore the steps for implementation are:

1. Define the parametric relationship between human surface and point on the product.
2. Create guide point on scan data that interfaces with the part. Define point in reference to scan data
3. Specify direction and distance of product point from guide point
4. Using the distance function, calculate the new point from the guide point on the human surface.

### 3.3.2 Numeric Implementation of the Point-to-Surface Constraint Equation

The Point-to-Surface constraint equation requires that the CAD system allow the user to create a reference surface on the scan data. The steps for numeric implementation are:

1. Create the guide surface through a series of points on the scan surface
2. Specify the distance function for projecting the product point from the guide surface. This distance function will be defined by the parametric relationship that defines the type of point-to-surface constraint. This can be in the form of the normal vector or a directional vector. In general, the scan data will be represented by a polygon. The directional vector can be calculated in terms of the polygon plane
3. At the specified point on the guide curve calculate the distance function
4. Construct the relative point of the product.

## 3.4 Curve-to-Curve Constraint Equation

The curve-to-curve mating condition occurs when a curve on the human surface interfaces with a curve on the product geometry. This interface can either be a direct contact mating condition or a distance that must be maintained between the body and the product. Potential applications of this mating condition include:

- Palm Protectors
- Forearm crutches
- Traction devices

There are three types of curve-to-curve constraint equations which include:

- Offset Curve-to-Curve
- Projected Offset Curve-to-Curve
- Random Offset

The types of curve-to-curve are a result of the different parametric relationships required between the human body and the designed product. The offset curve-to-curve constraint maintains a constant distance from the curve on the body to the product geometry and results in a constant force. The projected offset curve-to-curve constraint is a scaling of a

curve on the body geometry to create the product curve. Finally, in the random offset curve-to-curve constraint, the product curve does not maintain a constant distance from the body curve. This can result in a force on the body. Illustrations of these three curve-to-curve constraint relationships can be found in Figure 7.

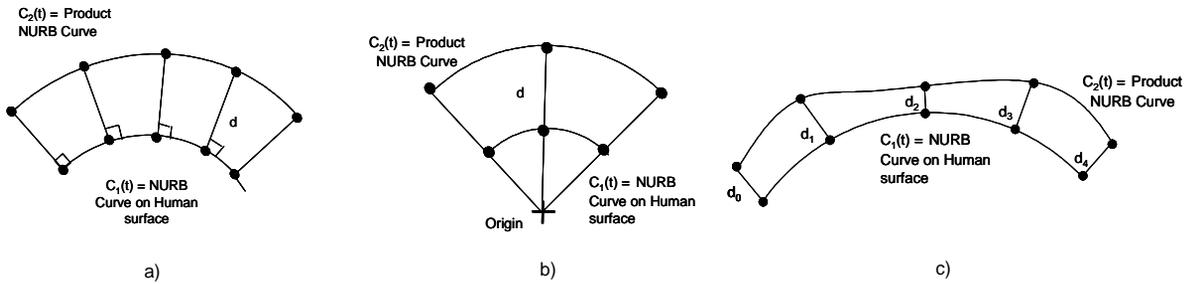


Fig. 7: The Three Types of Curve-to-Curve Constraint Relationships. a) Offset Curve-to-Curve Constraint. b) Projected Offset Curve-to-Curve Constraint c) Random Offset Curve-to-Curve Constraint.

3.4.1 Theoretical Basis for Implementing Curve-to-Curve Constraint Equations

In developing the curve-to-curve constraint equation, the curve on the human surface must be approximated with a NURB curve equation and must be defined in terms of anatomical landmarks and bio-references or in absolute terms. In addition, the parametric relationship, which determines whether the constraint equation is the offset, projected, or random curve-to-curve must also be defined. The strategy behind these constraint equations utilizes the geometric offset curve equation shown in Equation 3.4.1.

$$C_2(t) = C_1(t) + dn \tag{3.4.1}$$

Where  $C_2(t)$  is the offset curve,  $C_1(t)$  is the original curve,  $d$  is the distance between the curves, and  $n$  is the normal or directional vector to the curve  $C_2(t)$ .

The offset curve-to-curve constraint maintains a constant distance from the curve on the body to the product geometry. This allows the offset curve equation to be used. Points on the product curve can be generated by using Equation (3.4.1) at different  $t$  values on the curve. Equation (3.4.2) shows the calculation of a point on the product geometry at  $t_0$  for an Offset Curve-to-Curve interface.

$$P(x, y, z) = C(t_0) + d\bar{n}_0 \tag{3.4.2}$$

Where  $P(x, y, z)$  represents the point on the product curve at  $t_0$ ,  $C(t_0)$  represents the point on the curve at  $t_0$ ,  $d$  is the distance to the product curve, and  $n_0$  is the normal at  $t_0$ .

The Projected Offset Curve-to-Curve constraint equation scales the body curve to create the product curve. This equation uses an origin create the directional vector and scale the curve, shown in Equation (3.4.3).

$$C_2 = C_1 + \|C_1 - origin\|d \tag{3.4.3}$$

Where  $C_2$  represents the curve on the product geometry,  $C_1$  is the curve on the human surface, and  $d$  is the distance from the curve on the product geometry and the curve on the human surface.

The random offset curve allows the distance between the product geometry and the human surface to vary, allowing for forces to be exerted on the body. The equation for the random offset curve is shown in Equation (3.4.4).

$$C_2(t) = C_1(t) + d(t)\overline{n(t)} \tag{3.4.4}$$

The construction of this interface requires the definition of a curve on the human surface as well a function specifying the distance from the guide curve to the curve representing the product geometry. This projection can be done using a ruled surface, which uses a linear blend in one direction, and represents the parametric relationship and distance function between the curve on human surface and product geometry. The distance function used depends on the type

of curve-to-curve constraint, either offset, projected, or random offset. The process of constructing a guide curve projecting the curve using a ruled surface is illustrating in Figure 8.

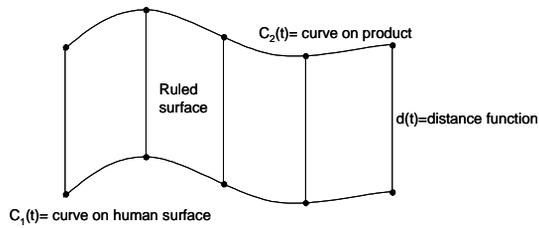


Fig. 8: Construction of Ruled Surface for a Curve-to-Curve Mating Condition.

The theoretical process for implementing a curve-to-curve mating condition is summarized in the following steps:

1. Define the parametric relationship between the curve on the human surface and product curve.
2. Create a NURB guide curve on scan data, defined in reference to scan data. Calculate the points on the NURB curve using the NURB equation.
3. Specify direction and distance of product curve from guide curve
4. Using the distance function, calculate the points on the product geometry from each point on the NURB guide curve.
5. Interpolate to determine the control points of the curve on product geometry using the matrix operations on the curve matrix form in Equation (3.4.5).

$$[X] = [B][P] \tag{3.4.5}$$

Where P represents the matrix of control points, B represents the blending functions, and X represents the matrix of values on the curve.

Matrix operations on Equation (3.4.5) results in Equation (3.4.6) and allows the calculation of the control points of the curve. With the theoretical basis defined, the numerical implementation of the curve-to-curve interface can be defined.

$$P = [B^T B]^{-1} B^T X \tag{3.4.6}$$

### 3.4.2 Numerical Implementation for the Curve-to-Curve Constraint Equation

The Curve-to-curve constraint equation requires that the CAD system allow the user to create a curve on the scan data. The steps for numeric implementation are:

1. Create the guide curve through a series of points on the scan surface
2. Specify the distance function for projecting the product curve from the guide curve. This distance function will be defined by the parametric relationship that defines the type of curve-to-curve constraint, either offset, projected, or random offset. This can be in the form of the normal vector or a directional vector. In general, the scan data will be represented by a polygon. The directional vector can be calculated in terms of the polygon plane
3. At each point on the guide curve calculate the distance function
4. Construct the relative points of the product curve.

### 3.5 Curve-to-Surface Mating Condition

In the curve-to-surface mating constraint, a curve on the designed product is maintained at a specified distance from the surface of the body. This distance can either be a positive value, negative value, or zero. Potential applications of this constraint equation are:

- Edge of safety glasses to forehead

- Edge of orthotic to surface of foot
- Edges of prosthetic device to human surface

The implementation of this mating condition relies on the specification of parametric relationships between a curve, representing an edge of the product, and a human surface, represented by scan data. The next sections discuss the theoretical basis of the curve-to-surface implementation and a numerical method for implementing this constraint equation in the CAD system.

### 3.5.1 Theoretical Basis for Curve-to-Surface Constraint Equation Implementation

In developing the Curve-to-Surface constraint equations, parametric relationships between the curve and surface must first be defined. This relationship or interface can be a direct contact interface, a curve at a constant distance from the surface, or a curve with a distance function from the surface. As in the curve-to-curve mating condition, the curve-to-surface also can have three different types of interfaces: the offset, projected offset, and random offset. The surface of the scan data and the curve must be approximated with a NURB surface and NURB curve, respectively.

In the construction of this interface, this application requires a guide curve be defined on the human surface as well a function specifying the distance from this guide curve to a projected curve representing the edge of the product geometry. This projection can be done using a ruled surface, as in the curve-to-curve constraint equation. The process of constructing a guide curve on a surface and projecting the curve using a ruled surface is illustrated in Figure 9. Because the guide curve is constructed in to create the product curve, the curve-to-curve constraint equations in section 3.4.1 remain the same in the theoretic implementation of the curve-to-surface interface. To be parametric, the guide curve must be defined in reference to anatomical landmarks and bio-references through landmarks or absolute terms.

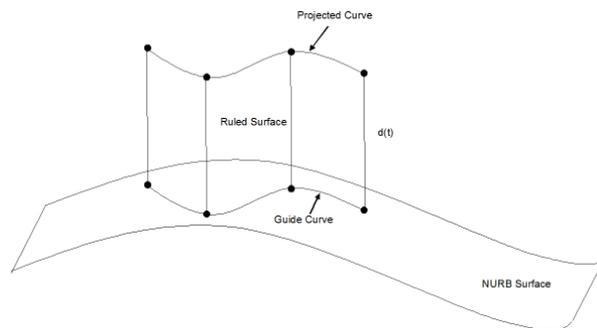


Fig. 9: Method of Construction for Product Curve in a Curve-to-Surface Interface.

The theoretical process for implementation of the curve-to-surface mating condition is summarized in the following steps:

1. Define the parametric relationship between human surface and product curve.
2. Create a NURB guide curve on scan data, defined in reference to scan data
3. Specify direction and distance of product curve from guide curve
4. Using the distance function, calculate the new points from each point on the guide curve.
5. Interpolate to determine the control points of the product curve using Equation (3.4.5).

### 3.5.2 Numerical Implementation of Curve-to-Surface Constraint Equation

In the numerical implementation, the ability to define a guide curve on the scan data through a series of points on the scan surface must be provided in the CAD system. The steps for numeric implementation are:

1. Create the guide curve through a series of points on the scan surface
2. Specify the distance function for projecting the product curve from the guide curve. This can be in the form of the normal vector or a directional vector. In general, the scan data will be represented by a polygon. The directional vector can be calculated in terms of the polygon plane
3. At each point on the guide curve calculate the distance function

4. Construct the relative points of the product curve.

As mentioned in the theoretic basis for this constraint equation, the construction of relative points must be defined parametrically to scan data such that when new scan data is used, the curve will be able to adjust and update to fit the new scan data.

**3.6 Surface-to-Surface Mating Condition**

The surface-to-surface constraint equation exists when a product surface interfaces a human surface. This interface can either be through direct contact or through the specification of a distance that is to be maintained between the surfaces. Potential applications of this constraint equation include:

- helmets
- shin guards
- body armor

The next sections discuss the theoretical basis and numerical method for implementation of this constraint.

*3.6.1 Theoretical Basis for Implementation of Surface-to-Surface Constraint Equations*

Because NURB surfaces are an extension of NURB curves, the theoretical basis for implementing a surface-to-surface mating condition is similar to that of the curve-to-surface interface. In developing the curve-to-surface constraint equations, parametric relationships between the product surface and the human surface must first be defined. This relationship or interface can be a direct contact interface, a product surface at a constant distance from the human surface, or a product surface with a distance function from the human surface. As in the curve-to-curve and curve-to-surface mating conditions, the surface-to-surface mating condition can also be classified into three types: offset surface, projected offset surface, and random offset surface. The surface of the scan data must be approximated with a NURB surface.

The offset surface-to-surface constraint maintains a constant distance from the surface of the body to the product geometry surface. This allows the offset surface equation to be used, shown in Equation (3.6.0).

$$S_2(u, v) = S_1(u, v) + d\bar{n} \tag{3.6.0}$$

Where  $S_2(u,v)$  represents the product surface,  $S_1(u,v)$  represents the human surface,  $d$  is the distance between the surfaces, and  $n$  is the directional vector. Equation (3.6.1) shows the calculation of a point on the product geometry for an Offset Surface-to-Surface interface.

$$P(x, y, z) = S(u_0, v_0) + d\bar{n}_0 \tag{3.6.1}$$

Where  $P(x,y,z)$  represents the point on the product surface at  $(u_0,v_0)$ ,  $S(u_0,v_0)$  represents the point on the human surface at  $(u_0,v_0)$ ,  $d$  is the distance from the human surface to the product surface, and  $n_0$  is the directional vector at  $(u_0,v_0)$ . This directional vector is specified with respect to the human surface.

The Projected Offset Surface-to-Surface constraint equation scales the body surface to create the product surface. This equation uses a directional vector from a specified origin to the point on the human surface to scale the curve, shown in Equation (3.4.3).

$$S(u, v)_2 = S(u, v)_1 + \|S(u, v)_1 - origin\|d \tag{3.6.2}$$

Where  $S_2$  represents the product surface,  $S_1$  is the NURB human surface, and  $d$  is the distance between the product and human surfaces. The product surface can be calculated similar to Equation 3.6.1 where points on the product surface were calculated by specifying a location  $(u_0, v_0)$  on the human surface.

The random offset curve allows the distance between the product geometry and the human surface to vary, allowing for forces to be exerted on the body. The equation for the random offset surface is shown in Equation (3.6.3). Points on the product geometry can be calculated at various  $(u,v)$  locations.

$$S_2(u, v) = S_1(u, v) + d(t)\overline{n(t)} \tag{3.6.3}$$

In the construction of the surface-to-surface interface, a guide surface must be defined on the scan surface as well a function specifying the distance from the guide surface to a projected surface, which represents the product geometry. This projection can be done using a ruled solid, which uses a linear blend in the direction of the distance function, and represents the parametric relationship between the human surface and product geometry. This process of constructing a guide surface and projecting the surface using a ruled solid is illustrated in Figure 10.

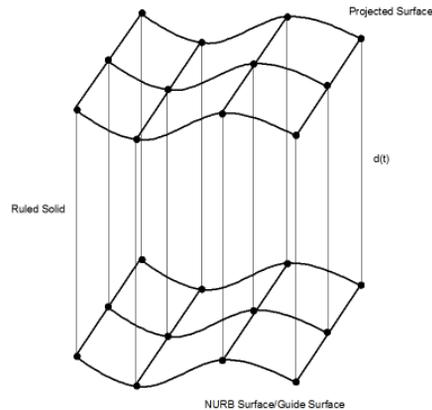


Fig. 10: Method of construction for Product Surface in a Surface-to-Surface mating condition.

As in the curve-to-surface interface, to be parametric the guide surface must be defined in reference to anatomical landmarks and bio-references. This can either be through the definition of landmarks locations on the scan data, or in absolute terms using a template model.

The theoretical process for implementing the surface-to-surface mating condition is summarized in the following steps:

1. Define the parametric relationship between the human surface and the product curve.
2. Create a guide NURB surface on the scan data, defined in reference to the scan data.
3. Specify the direction and distance of the product surface from the guide surface.
4. Using the distance function, calculate the new points from each corresponding point on the guide surface.
5. Interpolate to determine the control points of the projected NURB surface using Equation (3.4.5) for surfaces.

### 3.6.2 Numerical Implementation of Surface-to-Surface Constraint Equation

In the numerical implementation, the ability to define a guide surface on the scan data through a series of points on the scan surface, specify reference curves, and create a surface from these reference curves must be provided in the CAD system. The steps for numeric implementation are:

1. Create the guide surface by creating several reference curves through a series of points on the scan surface.
2. Specify the distance function for projecting the product surface from the guide surface. This can be in the form of the normal vector or a directional vector. In general, the scan data will be represented by a polygon. The directional vector can be calculated in terms of the polygon plane
3. At each point on the guide surface calculate the distance function
4. Construct the relative points of the product surface.

As mentioned in the theoretic basis for this constraint equation, the construction of relative points must be defined parametrically to scan data such that when new scan data is used, the curve will be able to adjust and update to fit the new scan data.

## 4. CONCLUSIONS

This paper presents generalized theoretical and numerical methods for the implementation of the bio-interface mating conditions. These methods allow the constraints to be implemented in a CAD system which would simplify the creation of these mating conditions using solid modeling operations in the detailed design phase of the methodology. In

addition, this implementation would also help in creating parametric models in CAD since the mating interfaces are also the surfaces that must reconfigure during the customization process.

While this paper identifies strategies and methods for implementing biomechanical mating conditions, it does not identify solutions for every geometric case. Further work needs to be done to identify solutions for mating conditions other than surface patches defining convex objects. In addition, methods for checking the robustness should also be researched. These methods could include techniques for checking product geometry in reference to body dynamics, motion, physiological factors, and manufacturing tolerances.

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