Numerical Analysis of Progressive Cavity Pump Extruder for 3D Printing of High Viscosity Ceramic Paste

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Abstract. Progressive Cavity Pump (PCP), also known as the Moineau pump, is an artificial lift method often used for pumping high viscosity and high solid content fluids from producing wells. It transfers slurry by means of progress, through a pump, of a sequence of small, fixed shape, discrete cavities, as its rotor is turned. Although PCP has been effectively adopted in the oil industry for decades, there is a lack of understanding of the effect of design parameters and operating conditions on the pump performance for a high viscosity paste. In this paper, the objectives of our study are to develop a PCP-based extruder model for paste extrusion additive manufacturing, predict an actual multi-lobe PCP performance and investigate the pump performance by simulating single-phase ceramic paste flow through it. The work primarily focuses on two aspects. In the initial stage, the ceramic paste of different industrial materials such as earthenware is simulated with given boundary conditions in PCP, and the results of ceramic paste simulation in PCP are then compared with the extrusion process. Output parameters such as exit velocity and pressure of the paste in the PCP clearly show that the flow of paste inside the PCP gives better results than compared to the extrusion process of the same material in the paste form.

Keywords: Additive manufacturing, Progressive cavity pump, High viscosity ceramic paste, Material extrusion, Numerical simulation.

DOI: https://doi.org/10.14733/cadaps.2024.171-178

1 INTRODUCTION

Additive Manufacturing, also known as 3D printing, involves the creation of tangible objects using a digital 3D (or CAD) model. In this process, the materials are added in successive layers to form the final 3D object thereby reducing the wastage of material and hence tooling cost. Earlier, 3D printing techniques were considered suitable only to produce prototypes for studies and experimentation. In today’s time, the precision, repeatability, and material range of 3D printing has increased to a great extent. One of the key advantages of 3D printing is that it can produce very complex shapes or
geometries that would be otherwise impossible to construct by conventional methods. Therefore, 3D printing has become one of the widely used manufacturing technologies in the present era.

The Progressive Cavity Pump (PCP) is a type of Moineau pump used in the 3D printing process, that can pump thick products without spoiling them [1]. The Moineau pump is made up of two helical gears that are nested inside each other. The rotor revolves around a longitudinal axis that is parallel to the stator axis. The rotor is designed to keep all of its teeth in contact with the stator at all times. The cavities move without deformation while the rotor rotates inside the stator, allowing fluid to be transferred without pulsation.

Although PCP was invented by René Moineau in 1930, it is still a very young technology that found its use in the late 1970s. They are not as complex as they seem but our knowledge about this technology is still very limited. Due to the limited understanding of this technology, its true potential has not been extracted to date. For many years, various research have been performed to study the effect of pump parameters on the efficiency and performance of the pump. Orisaleye and Ojolo (2019) [6] have carried out a parametric study of solids compaction in the single screw extruder to aid the design of a straight screw extruder for biomass compaction and related applications, while Orisaleye et al. (2019) [5] have proposed the mathematical models to design a screw extruder with consideration for processing slightly non-Newtonian biopolymers. An analytical solution is attempted for the flow of a slightly non-Newtonian shear-thinning material in the screw extruder. Fisch et al. (2020) [2] compared the most commonly used pneumatic extrusion process to a miniaturized progressive cavity pump in terms of their accuracy and precision as well as the compatibility of the extrusion process with bioprinting. The three-dimensional vector theory and the theory of Hypocycloid were developed to give a new modeling insight into the design and performance of PCPs [4]. Although many studies were carried out to examine the mathematical model for the PCPs, to the best of our knowledge, no literature describes the flow of ceramic paste for PCPs. Characteristics of the ceramic paste flow inside PCP have not been studied in depth and the effect of parameters affecting the flow is still unknown. It was also observed that no major work has been done on the simulation of a PCP extruder with high viscosity paste of ceramic as a material and its flow parameters have not been compared with the extrusion process of the ceramic paste. The purpose of this investigation is to assess the dependency of the flow characteristics of ceramic paste on the different parameters of the PCP. For that, they must be studied as they happen i.e., lab-scale else they need to be simulated. Since ceramics have a huge application in the 3D printing process, insight into its flow and also the dependency of flow on the physical characteristics of ceramics would give us a clear understanding of the process.

2 PROBLEM IDENTIFICATION

Upon reviewing the literature on 3D printing and progressive cavity pumps, it was noted that there has been limited exploration into the simulation of a PCP extruder with high-viscosity ceramic paste as a feed material. The use of high-viscosity ceramic paste presents various challenges while flowing through a PCP, such as cavitation and discontinuous flow. Additionally, the gelation process further complicates the flow of the paste. Further research is needed to fully understand and overcome these challenges, as the successful simulation of a PCP extruder with high-viscosity ceramic paste as a feed material could lead to significant practical applications. Therefore, investigating this topic in detail could be beneficial for enhancing the effectiveness and efficiency of the PCP extrusion process, mainly from the perspective of 3D printing of ceramics.

3 NUMERICAL SIMULATION

3.1 Geometric Modeling of PCP

Nguyen et al. (2014) [4] elaborated a detailed mathematical model of the geometry of PCP. They gave an insight into the hypocycloidal model of the PCP. Geometrically, a hypocycloid is a special
plane curve generated by the trace of a fixed point on a small circle (generator circle), that rolls within a larger circle (base circle). As shown in Figure 1, the blue circle represents the base circle, and the red curve shows the profile of PCP generated by the generator circle. Unlike the cycloid, the hypocycloid rolls within a circle instead of along a line. If the smaller circle has a radius of \( r \), and the larger circle has a radius of \( R \), which is equal to \( N \cdot r \), where \( N \) is the number of lobes of the stator. The parametric equations for the curve in 2D can be given as Equations (3.1) and (3.2).

\[
\begin{align*}
  x(\theta) &= r(N-1)\cos \theta + r\cos[(N-1)\theta] \\
  y(\theta) &= r(N-1)\sin \theta + r\sin[(N-1)\theta]
\end{align*}
\] 

(3.1) and (3.2)

**Figure 1**: Method of generating a hypocycloid.

The variable \( r \) in the above equations is the radius of the generator circle when generating a hypocycloid. Figure 2 depicts the profile of the PCP. The radius of the base circle is used to generate the \( N \)-lobe hypocycloid, which is the product \( N \) and \( r \). The eccentricity (\( e \)) of a PCP, is defined as the difference between the radius of the stator and the radius of the rotor. In other words, the eccentricity of a PCP is equal to the radius of the generator circle. The eccentricity is given by Equation (3.3) as,

\[
e = \left[ \frac{D_s - d}{2N} \right]
\] 

(3.3)

where, \( D_s \) is the diameter of the stator, and \( d \) is the diameter of the semi-circular cusp. The diameter of the stator is defined as the sum of the diameter of the base circle and the diameter of the semicircle at the cusp, which is given by Equation (3.4) as,

\[
D_s = (2eN + d)
\] 

(3.4)

The diameter of the rotor \( (D_r) \), which has one lobe less than the diameter of the stator, is defined as given in Equation (3.5).

\[
D_r = \frac{(N-1)D_s}{N}
\] 

(3.5)

In general, a PCP includes an \( N \)-lobe modified hypocycloid stator and an \((N-1)\)-lobe modified hypocycloid rotor. In addition, there will be \((N-1)\) free spaces between the rotor and the stator, where the fluid flows between the rotor and stator. Therefore, the area of the flow of a PCP \( (A_f) \) is given by Equation (3.6) as,

\[
A_f = 2\pi e^2 (N - 2) + 4de
\] 

(3.6)

The relation between the pitch of the rotor \( (P_r) \) and pitch of the stator \( (P_s) \) is given by Equation (3.7), i.e.,
The pump capacity, when the rotor turns with a rotational speed ($\omega$) is given by the flow equation. The flow rate is given by Equation (3.8) as,

$$Q = A_f P_s \omega$$  \hspace{1cm} (3.8)

### 3.2 Validation of the Geometric Model of PCP

A 3D hypocycloidal model has been used to describe the geometry of the PCP. The polylactic acid (PLA) paste flows through the area ($A_f$) having a flow rate ($Q$) with a rotor having rotational speed ($\omega$ rpm) and a stator having the number of lobes as $N$ and pitch length as $P_s$. The semicircle diameter of the rotor is $d$ and the eccentricity of the progressive cavity pump is $e$. Equations (3.3) – (3.8) are used to describe geometry which is validated using MATLAB. The relationship between the flow rate ($Q$) and the rotational speed of the rotor ($\omega$) is shown in Figure 3(a). Figure 3(b) shows the relationship between flow rate and the number of lobes at different values of rotor diameter. From the figures, it is observed that the results obtained are in good correlation with the work done by Nguyen et al. (2014) [4]. Therefore, the equations are validated for different rotational speeds of the rotor and the different number of lobes of the stator.

### 3.3 Solid model of PCP

As mentioned earlier, the ratio of the stator to the rotor is $N:(N-1)$. Also, from the pilot study and results that were validated, the maximum flow rate is achieved when the ratio of the stator to the rotor is 2:1. Therefore, to design the stator, two lobes are considered. However, the rotor is designed keeping in mind the geometry of the stator. With validation from equations, a 3D solid model of the rotor of the progressive cavity pump is created. Input parameters of the stator and rotor are given in Table 1. The solid models of the stator, rotor, and assembly are depicted in Figure 4.
Figure 1: (a) Relationship between flow rate and rotational speed for a Moineau pump, and (b) Relationship between flow rate and number of lobes at different values of rotor diameter.

<table>
<thead>
<tr>
<th></th>
<th>Stator</th>
<th>Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of lobes in the rotor</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Diameter of the rotor (cm)</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Pitch length of the rotor (cm)</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1: Input parameters of stator and rotor.

Figure 4: (a) Stator, (b) Rotor, and (c) Assembly of PCP.

3.4 Materials
For the first simulation, earthenware is used as an extrusion material and simulated to compare its flow with the extrusion process [3]. Also, alumina is simulated to obtain the characteristics of a new flow. The properties of the two materials are given in Table 2.
Table 2: Material properties of Earthenware and Alumina.

<table>
<thead>
<tr>
<th>Property</th>
<th>Earthenware</th>
<th>Alumina</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m$^3$)</td>
<td>1,700</td>
<td>3,690</td>
</tr>
<tr>
<td>Bulk yield value (GPa)</td>
<td>0.07</td>
<td>-</td>
</tr>
<tr>
<td>Wall shear stress (MPa)</td>
<td>0.01</td>
<td>-</td>
</tr>
<tr>
<td>Friction Coefficient</td>
<td>0.147</td>
<td>-</td>
</tr>
<tr>
<td>Specific heat (J/kgK)</td>
<td>-</td>
<td>880</td>
</tr>
<tr>
<td>Thermal conductivity (W/mK)</td>
<td>-</td>
<td>18</td>
</tr>
<tr>
<td>Melting temperature (K)</td>
<td>-</td>
<td>1,973.15</td>
</tr>
</tbody>
</table>

4 RESULTS AND DISCUSSION

4.1 CFD Simulation of Earthenware Ceramic Paste in PCP

In a progressive cavity pump, a paste of earthenware ceramic is made to flow inside the stator. The paste of the ceramic and rotor of PCP are given initial conditions which would yield the outlet velocity and pressure of the paste. For a given rotational speed of the rotor and different input pressures, results are obtained in terms of inlet pressure and outlet velocity. The results are listed in Table 3. The numerical simulation of earthenware in PCP is performed for different pressures ranging from 1 MPa to 2.5 MPa. Simulation results of earthenware in PCP at 2 MPa are shown in Figure 5. Figure 6 shows the pressure and velocity variation of ceramic in PCP and the conventional extruder. From the figure, it is observed that PCP yields a higher outlet velocity and hence a faster rate for the 3D printing process for the same inlet conditions and material.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Inlet Pressure (MPa)</th>
<th>Outlet Velocity in PCP (m/s)</th>
<th>Outlet Velocity in Extrusion (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.035</td>
<td>0.01</td>
</tr>
<tr>
<td>2</td>
<td>1.5</td>
<td>0.048</td>
<td>0.018</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.067</td>
<td>0.03</td>
</tr>
<tr>
<td>4</td>
<td>2.5</td>
<td>0.075</td>
<td>0.037</td>
</tr>
</tbody>
</table>

Table 3: Pressure and velocity at PCP outlet.

Figure 5: Flow velocity of ceramic in PCP at inlet pressure of 2 MPa.

4.2 CFD Simulation of Alumina Ceramic Paste in PCP

The flow of earthenware paste in PCP shows that flow is more favourable in PCP than in the extrusion process. From all the collective data, the exit velocity of alumina in PCP and its variation with change in the rotational speed of the rotor can be found.
Figure 6: Pressure and velocity variation of ceramic in PCP and conventional extruder.

For the numerical simulation, the inlet pressure is kept constant at 2 MPa, and the speed of the rotor changes which led to different exit velocities of alumina. Table 4 gives an idea about the relation between the rotor speed and exit velocity of the alumina. Also, Figure 7 shows the simulation result of alumina flow at 10 rpm.

Table 4: Exit velocity of Alumina at different rotor speeds.

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Rotor Speed (rpm)</th>
<th>Exit Velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>0.043</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.069</td>
</tr>
<tr>
<td>3</td>
<td>30</td>
<td>0.082</td>
</tr>
<tr>
<td>4</td>
<td>40</td>
<td>0.104</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>0.130</td>
</tr>
<tr>
<td>6</td>
<td>60</td>
<td>0.150</td>
</tr>
</tbody>
</table>

Figure 7: Flow of Alumina at a rotor speed of 10 rpm.

5 SUMMARY AND CONCLUSION

Different numerical models of progressive cavity pumps were studied, and the study led to the conclusion that the three-dimensional hypocycloidal model of PCP most appropriately describes the
actual geometry of the PCP. Accordingly, a 3D hypocycloidal model of PCP was developed, and the equations and parameters of the proposed numerical model were validated in MATLAB. Next, industrial ceramics like earthenware, in its paste form, was simulated inside the PCP and the outlet velocity and pressure were compared to that of the extrusion process. This led to the observation that the paste flow in PCP is more favourable than in the extrusion process. The simulation results show that the exit velocity of earthenware paste increases in PCP in comparison to that in the extrusion for the same inlet condition. This is because the rotation of the rotor plays a vital role in increasing the pressure of the paste in addition to the inlet pressure of the paste. Later, alumina in the paste form was simulated with the validated model of the PCP for different speeds of the rotor of the PCP. It gives an insight into the flow of alumina inside the PCP and the dependency of outlet velocity on the rotor speed of the PCP. This helps to characterize the flow of alumina and the method to control the flow velocity of alumina paste with the rotation of the rotor of PCP.

The findings of this work provide insights into the mathematical model of PCP and its advantages over the traditional extrusion process. The higher printing speed resulting from the increased exit velocity of the paste in PCP can significantly reduce the time required for the 3D printing process. Moreover, the validated mathematical model of PCP can be used for the design and optimization of PCP, leading to improved printing quality and increased efficiency. Overall, the work demonstrates the potential of PCP as a promising alternative to traditional extrusion-based 3D printing techniques for materials in the paste form. The work provides a foundation for further research and development of PCP technology for various applications in the field of 3D printing.

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REFERENCES