www.cadanda.com
Computer-Aided Design AND APPLICATIONS

Shape Optimization Definiteness using NURBS Curves and Genetic Algorithm

Miri Weiss Cohen[1], João Batista Q. Zuliani[2], Frederico Gadelha Guimarães[c3]

[1]Braude College of Engineeringl, miri@braude.ac.il
[2]Centro Federal de Educação Tecnológica de Minas Gerais, joaoz@ymail.com
[3]Universidade Federal de Minas Gerais, UFMG, fredericoguimaraes@ufmg.br

Corresponding author: Miri Weiss Cohen, miri@braude.ac.il

Abstract. A Shape Optimization method performed on multi material Topology Optimization results is presented. The approach is based on two major phases, first, parametrization the material boundaries and second, optimizing those boundaries to definite shapes. The parameterization process involves identifying the boundaries of different materials and representing the boundaries by NURBS curves. This approach conveys the ability to define a new parametrized internal boundary profile, which is much more flexible. Once a parametrized model is defined, the optimization is performed using the parameters of the boundary curves as the optimization variables. A Genetic Algorithm (GA) is used to optimize the best variables (control points and weights) for achieving the best coordinates of the boundary curves, sufficing the fitness function criteria. The method was tested on a C-Core magnet, and results are presented.

Keywords: Shape Optimization, NURBS curves, Genetic Algorithm
DOI: https://doi.org/10.14733/cadaps.2019.67-78

1 INTRODUCTION

Topology optimization (TO) tries to find the best distribution of materials inside a region of interest that meets certain desirable criteria fixed a priori. Although TO methods are effective in determining a good initial distribution of material in the design domain, they are cell-based representation of topology, which in many cases lead to irregular and vague boundary layouts. A post processing procedure is applied to refine the results obtained and increase the efficiency of the TO layout. This is usually done by the parameterization of the topology defined by the initial distribution of material found, leads to the so-called Shape Optimization (SO) [4, 11]. Both TO and SO problems can involve multiple constraints and multiple objectives, reflecting design specifications and requirements [13, 1]. Some of the TO methods result an image of irregular and diffuse contours [4]. In these cases it is necessary to interpret the layout obtained before converting it into a parameterized model. This interpretation of the layout can be done directly by the designer using his knowledge of the problem under study or by an automated process. If the number of layouts is very large, it becomes impracticable to perform this task directly by the designer, thus requiring an automated method. The automated processing of parameterization is usually performed using image preprocessing mechanisms and on-board identification techniques [5, 14, 21].

In the Shape Optimization process various methods are used to find the best geometric definitions of the boundaries found in the initial layout, while configurations such as holes and regions of different materials, must remain unchanged. Many attempts to smoother and definite material boundary have been studied, most of them done by the parameterization of the TO initial distribution. Campelo et. al. [5] proposed an approach that uses a Clonal Selection Algorithm for TO (TopCSA), followed by a parameterization routine providing a parametric model defined by B-splines curves and the Real-coded Clonal Selection Algorithm. Another work, done by Tang and Chang [21] proposes using the HM or SIMP method to solve the TO problem forwarded by a boundary smoothing method using B-spline curves. Lin and Chao [14] developed an integrated TO and SO by way of an automated image interpretation that generates the parameterized model for the SO stage. The parameterization is done by identifying the holes with template matching in the topology obtained by Homogenization method and solving the SO problem using standard nonlinear programming techniques. Given the complexity of the task, SO is still of valid research topic.

2 PROPOSED APROACH

The general single/multiobjective TO problem can be defined as finding the optimal distribution of material in the cells of the design region that minimizes the single/multiple objective functions while satisfying the constraint functions. Mathematically, the optimization problem can be defines as:

$$\min \quad \Big(f_1(\xi), \ldots, f_m(\xi)\Big) \in \mathbb{R}^m \tag{1}$$

$$s.t.: \quad \begin{cases} g_i(\xi) \leq 0, i = 1, \ldots, p \\ \xi \in \mathcal{S}^{|\tilde{\Omega}|} \end{cases} \tag{2}$$

The design region is a closed and bounded subset $\Omega \subset \mathbb{R}^2$ of the 2-dimensional geometric space. The discrete description of Ω into an array of cells is represented by $\tilde{\Omega}$. Let n be the number of different material properties available to define the layout and consider a soft background material that fills the space around the structure of interest (e.g., air).

There are $n+1$ possible states $\mathcal{S} = \{0, 1, 2, \ldots, n\}$ for any given cell $c \in \tilde{\Omega}$. The vector ξ assigns a state $i \in \mathcal{S}$ for each $c \in \tilde{\Omega}$. Therefore, the objective function can be defined as $f_r(\xi) : \mathcal{S}^{|\tilde{\Omega}|} \mapsto \mathbb{R}$. The problem constraints $g_i(\xi) : \mathcal{S}^{|\tilde{\Omega}|} \mapsto \mathbb{R}$ are mathematical representations of the system requirements or limitations.

The defined SO problem for a given topology involves the same objective and constraint functions of the TO formulation. The only change is on the optimization variable and search domain, i.e. the SO is done with respect to the control parameters of the topology parameterization [3, 6]

In this work the parameterization process involves identifying the boundaries of different materials and defining these boundaries. In order to obtain a flexible solution, NURBS curves are defined for sections that form the internal boundaries. It is natural that a proper representation of the boundary shape is essential for efficient optimization. The shape optimization is performed using the parameters of the boundary curves as the optimization variables. In the associated SO problem the optimization variables represent the coordinates of the control points and their weights, both partially defining the NURBS curve. A Genetic Algorithm (GA) [20] is used to optimize the variables (control points and weights) for achieving the best boundary curves, sufficing a fitness function criteria. The phases in this proposed approach are depicted in Figure 1, and done sequentially. This work is an extension of the TO optimization phase (on the left of the figure) which is presented and detailed in [23, 22].

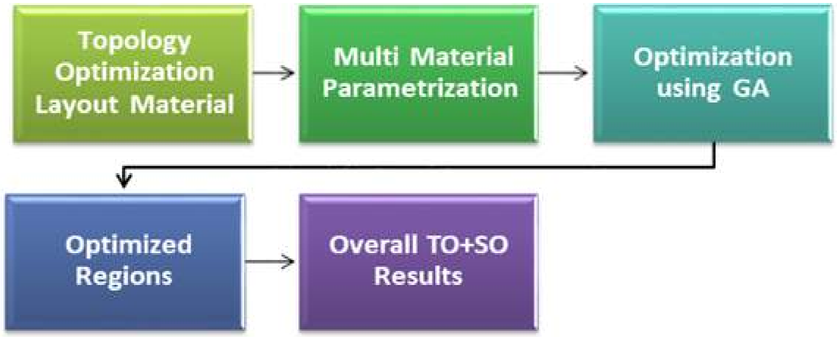


Figure 1: Proposed approach phases from Topology Optimization output to final Shape Optimization results

3 SHAPE PARAMETRIZATION

The Shape parameterization process involves identifying the boundaries of different materials and describing these boundaries by NURBS curves . Our approach achieves the ability to define the internal and external boundary profiles to have more flexibility in handling the boundaries found in TO. This phase is the initial and basic step allowing to obtain an efficient SO.

As in other geometric modeling tasks, NURBS curves offer many advantages in handling analytical and parametric representations, thus making it a usable tool for a SO. Some of the major advantages using NURBS curves [16, 17] include: simple Processing; computationally stable; invariable and precise mathematical representation for free forms, and obtain local control properties.

In the process of shape parametrization we defined two main constraints:

- To ensure the maintenance of the TO characteristics of the solution even with changes. In particular, control of the intersections of the contours which serve as internal borders
- Maintaining the TO as such that, no new regions are created.

hese two challenges were solved by the following:

- Considering vertices at the outer edge of the design region, and intersecting vertices of lines of the internal border, as initial or final points of the NURBS curves.
- Choosing fixed ranges of values, for the variables related to the coordinates of the control points and weights values.

The Shape parametrization phase begins with a refinement of the grids representing the discretized space. This is done in order to have a suitable number of candidate points accounting for vertices representing the two distinct extreme points. An additional process is performed by an automatic identification of polygons which form the inner borders of material regions. Figure 3 depicts the three main stages performed. Given a the following output, the vertices defining the cells of the design space are classified according to the number of edges belonging to the internal boundary.

For each vertex we assign a value of 0, 1, 2, 3 or 4. Figure 2 depicts some examples of vertices which were assigned different values. Vertex X_0 has the value 0 because it is on the interior of a region. Those on the inner border are classified from 1 to 4: vertices classified as 1 are also over the external border of the design space what make that only one edge of the inner border is connected to it, as an example see vertices X_1, X_3, X_5, X_6 and X_7; vertices classified as 2 are those that belong to a line separating only two different material region, see vertex X_8; in the same manner, we assign a value of 3 to vertex X_4, and to vertex X_2 a value 4.

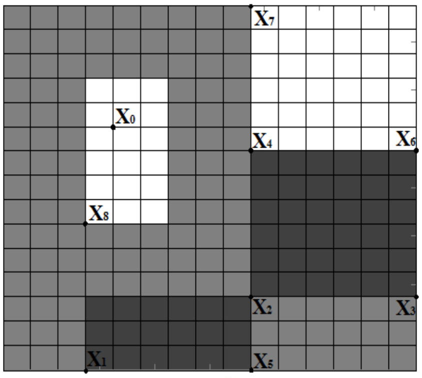
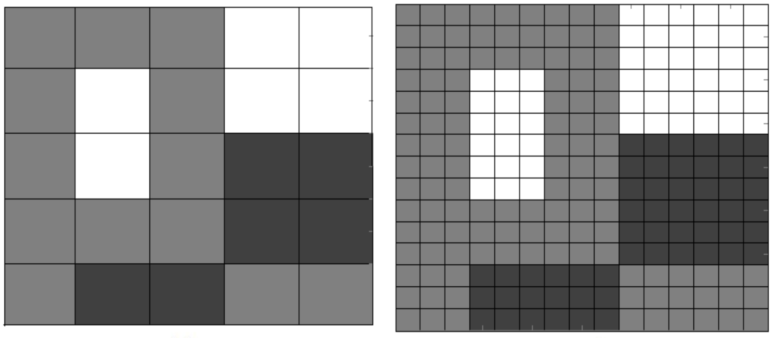


Figure 2: Vertices classification

Before defining the control points of a curve that parameterizes a span of the inner border, a set of points belonging to the inner border is chosen. Starting from a vertex with classification 1, 3 or 4 the procedure searches for the vertices connected to it, by an edge in the inner boundary. Then, choosing a new vertex, (not yet chosen) as the next vertex in the polygon edge. The process continues selecting connected vertex by edges in the inner border until reaching a vertex with classification 1, 3 or 4.



$$\begin{bmatrix}
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0\\
0&0&0&2&2&2&2&0&0&2&0&0&0&0&0&0\\
0&0&0&2&0&0&2&0&0&2&0&0&0&0&0&0\\
0&0&0&2&0&0&2&0&0&2&0&0&0&0&0&0\\
0&0&0&2&0&0&2&0&0&3&2&2&2&2&2&1\\
0&0&0&2&0&0&2&0&0&2&0&0&0&0&0&0\\
0&0&0&2&0&0&2&0&0&2&0&0&0&0&0&0\\
0&0&0&2&2&2&2&0&0&2&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0\\
0&0&0&2&2&2&2&2&2&4&2&2&2&2&2&1\\
0&0&0&2&0&0&0&0&0&2&0&0&0&0&0&0\\
0&0&0&2&0&0&0&0&0&2&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0
\end{bmatrix}$$

(c)

Figure 3: Example:(a) Discretized design space; (b) Discretized design space with grid refinement; (c) Vertices classification matrix

For example, the vertices on the polygon edge in the inner border that connect points X_1 and X_2 in Figure 2. Considering vertices with classification 1, 3 or 4 as extreme points will provide the characterization of intersection points on the inner border. This process contributes to the preservation of the original topology, avoiding the merge of different material regions or creation of new regions with no assigned material. If there are more than four consecutive vertices on a line segment, only the extreme points of the segment and its neighbor are saved in the set (see Fig. 2).
If the number of selected vertices is insufficient to define a p-degree NURBS curve, a midpoint of each two consecutive points are added to the set of points. These added points do not change the polygon shape under consideration.
The following procedure is detailed in Algorithm 1, which is detailed for open and closed curves.

Algorithm 1: Determination of control points used by interpolating of the NURBS curves

```
Input: Discretized topologies ξ
begin
    foreach vertex v_ij ∈ ξ do
        q_{v_ij} ⟵ quantity of edges of the inner border that v_ij is a endpoint;
    end
    repeat                                                          /* open curves */
        select a vertex with classification 1, 3 or 4;
        repeat
            select a neighboring vertex of the inner border connected to the previous vertex;
        until the selected vertex is classified as 1,3 or 4;
    until all vertices classified as 1, 3 or 4 are used as often as their classification;
    repeat                                                          /* closed curves */
        if exists unselected vertex classified as 2 then
            select one of these vertices;
            repeat
                select a neighboring vertex of the inner border connected to the previous vertex;
            until the selected vertex is the initial;
        end
    until all vertices are selected;
    map all the vertices into the world coordinates of the design domain
end
Output: Sets of interpolating points defined as the inner border
```

After all control points are defined, all the weights are initially set equal to 1, and will be adjusted in the optimization phase. The vertices are then mapped into the coordinates in the design space and interpolated by a p-degree NURBS curve. As depicted in figure Fig. 4, the initial intersection points and the number of lines passing through these points are maintained. As indicated before, this constraint must be satisfied to progress to the next stage of optimization

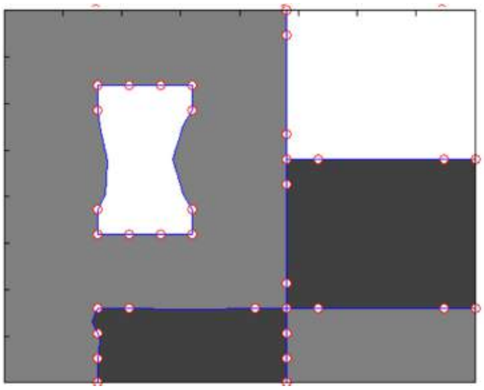


Figure 4: Examples of Selected vertices of the polygonal on the inner border and its interpolated NURBS curves.

In Tables 1 and 2, the whole process is demonstrated. Starting from the TO layout, control points configuration and definition, proceeding to defining the inner and exterior NURBS curves resulting new multi material regions.

Table 1: Shape Parametrization Stages from TO results to Parametrization

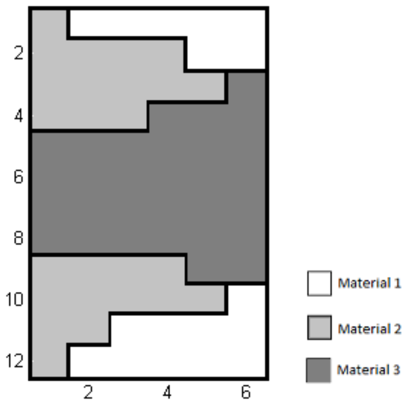
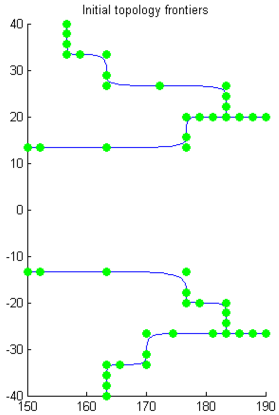
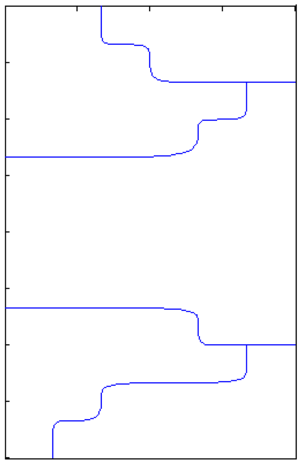
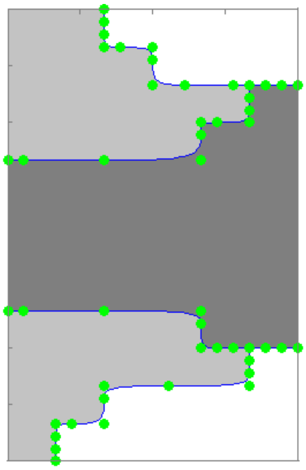
Topology Optimization	Control Points
	

Table 2: Shape Parametrization Stages from TO results to Parametrization

NURBS Curves	CP+ Parametrization
	

4 SHAPE OPTIMIZATION USING GENETIC ALGORITHM

Once the parametrized process is finalized, shape optimization can be performed using the definition parameters as optimization variables. As indicated earlier, the same objectives and constraints of the TO will be maintained. Hence, considering the most popular meta-heuristics to solve optimization problems, it was decided to apply an evolutionary algorithm to define the best geometric forms of the internal boundaries of the layouts. The motivation for using Genetic Algorithm (GA) is its advantage as a stochastic search algorithm and a problem-solving methodology [19]. It has assets such as flexibility, adaptation, global search capability, and its suitability for parallel computation.

GAs have been used to solve complex problems with objective functions that are multi-model and multi-constrained. A wide range of genetic operators can be used to perform the crossover, mutation, reinsertion of potential solutions, and generation the offspring population [10]. Some encoding may cause the algorithm to create unfeasible solutions, or may change and enlarge the search space, making it difficult to converge[12]. Work by [7, 8] has suggest applying GA to boundary conditions, resulting structural quantification for design.

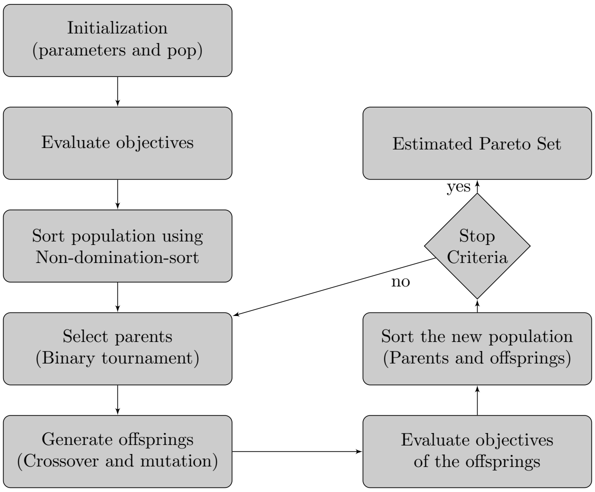


Figure 5: Flow chart of the Optimization process using Genetic Algorithm

In this work, we used GA to optimize the NURBS curve shape to convey the parameter values resulting the best value of the fitness function. The obvious degrees of freedom, defining the curve are the control points, weights and knot vector values. We have chosen only the control points coordinated and the weights for the optimizing GA process, This was done for two reasons: First, to limit the search space. Second, a sufficient genotype can be represented by the coordinates of the control points and the weights. The process of GA used, is depicted in Figure 5. For the optimization process to be effective, it is desirable that the determination of the internal borders of the solution represent approximately the layout obtained by the TO. Two approaches for parameterization of the internal boundary were tested: one in which the NURBS curve are approximated, and the second, in which the NURBS curve are interpolating a subset of the vertices. we have chosen the interpolation approach achieving better results of the borders representation. Moreover, a smaller number of control points were used, and consequently a smaller number of decision variables for the optimization of shape.

Configured with binary tournament selection of the parents chromosome and the genetic operators are the polynomial mutation and the SBX crossover, see [20] .

The GA parameters have the following values:

- The population size *pop=40*
- Crossover *rate 0.8*
- Pool size equal to half the population size;
- Number of generations *gen=200* (serves as stop criteria);
- range of the weights variable was from 0 to 20;
- binary tournament selection of the parents chromosome

As a result of these criteria and values, the coordinates of the control points found by the GA process, maintained the topological properties of the majority cases of layouts found by the TO stage. The range defined for the coordinates of control points take into account the distance to the closest control point. The range for the weights is defined as a fixed range. The values proposed for upper and lower limits of the variables,maintained the constraints. During the optimization process for almost every solution generated by genetic operators, an applicable solution was evolved.

5 EXPERIMENTAL RESULTS

The proposed approach was applied to a C-core magnetic actuator. Optimization of Electro magnetic design is a well researched area. Some works include [2],[18] and [9]. The C-core magnetic actuator is composed of three main parts: the armature and the yoke solid blocks of ferromagnetic material; and the design domain, which was discretized in a 20×10 square grid, see Figure 6. Each cell within the design domain can assume three states, corresponding to three different materials: air, iron or a magnetic material (for this example, *NdFeB* magnets were used). Figure 6, depicts the C-Core magnet and it parts.

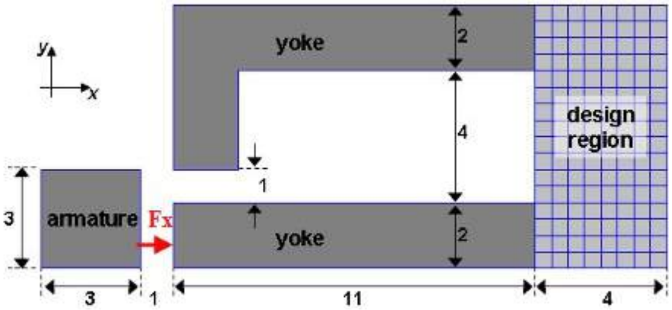


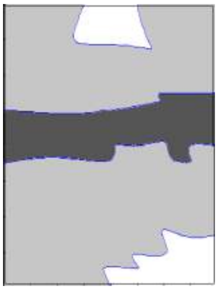
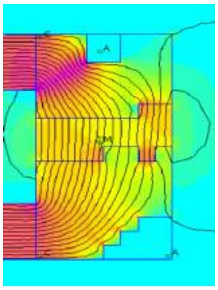
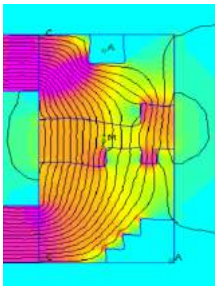
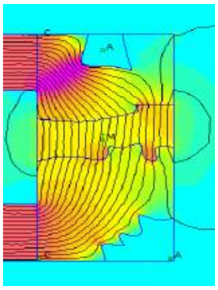
Figure 6: C-core magnetic actuator (values in cm)

The goal of the design of the c-core is to find an optimal material distribution that maximizes the x-directional attractive force F_x on the armature or equivalently minimizes the negative of this force, while minimizes the volume Vol_{PM} of permanent magnet material (PM) in the design region. The cost of the permanent magnet material is related to the volume of PM, and it accounts for the majority part of the cost of the PM machine due to the high price of the rare earth material. Then the optimization problem can be stated as:

$$\begin{cases} \textbf{min} \left(-F_x(s), Vol_{PM}(s) \right) \in \mathbb{R}^2 \\ \textbf{s. t.:} s \in S^{|\Omega|} \end{cases} \tag{3}$$

where $S^{|\Omega|}$ is the set of all layouts on the discretized region. The design domain was discretized in a 20×10 square grid. In this study case, the output force F_x was calculated by FEMM [15] using nonlinear finite element analysis.

Table 3: TO its parameterization, Shape Optimization and respective field lines.

Topology Optimization	Parameterized Solution	Shape Optimization
		
		

The upper part of Tables 3 and 4 and 4, (the tables contain figures) depicts the changes in material distribution during the optimization stages proposed in this work. The results for an implementation of the Parametrization stage, and the SO.In the lower part of the table, the figure depicts the magnetic flux density. Numeric results are detailed in Table 5. Most of the magnetic flux is going through the magnetic material, with just few field lines going through air. This is an accomplishment of the goal. The differences from the topology design to the final design (SO) are visually small but it is possible to see that the field contours had been smoothed after shape optimization. Another major result is the value of the calculated force. Table 4 present a comparison to the values of the objectives for the solution s_1, which has the highest value of F_x obtained by $MOACO_{11}$. The values presented are the obtained in the TO stage,that after the parameterization process and that in the SO stage.

Table 4: Shape optimization results of $Sample_1$.

	TO	Parameterized	Optimized
$F_x\ (N/m)$	1,041.5	-1,050.5	-1,062.6
$Vol_{PM}(\%)$	0.420	0.428	0.439

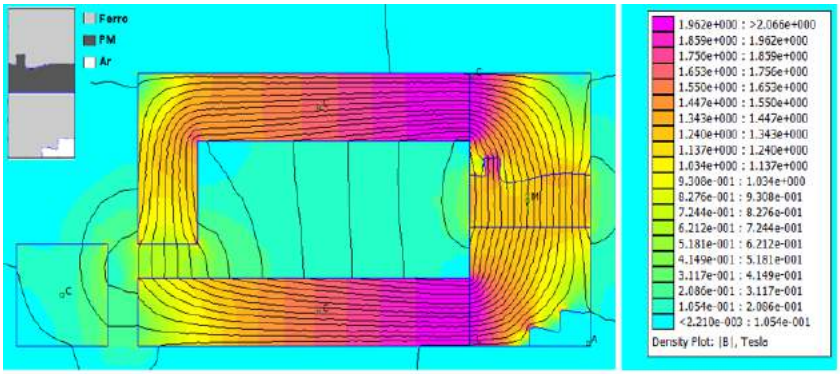


Figure 7: C-core magnetic actuator SO results and respective fields lines

Figure 7 depicts a full representation of the C-core magnet. Comprising the static part and the design space result after the shape optimization. On the upper left corner the multi material representation of the design space is represented in gray-scale.

6 CONCLUSIONS

This work presents an integrated approach of an automated multi-material process of SO with parameterization of the topologies found by TO. Using NURBS curves and GA optimization conveyed leading to a refine tuning of the internal boundaries separating regions of different materials. The proposed approach has been tested in the design of a C-core electromagnetic actuator with two objectives: the maximization of the attractive force on the armature and the minimization of the volume of permanent magnet material. The experiments show the adequacy and edibility of the proposed method, since initial coarse designs in the TO stage were further refined by SO.

ACKNOWLEDGEMENTS

This research was partially funded by *Marie Sklodowska-Curie International Research Staff Exchange Scheme Fellowship* within the 7th European Community Framework Program (FP7), SRG-Bruade (grant 500.383.12) and Brazilian agencies: FAPEMIG, CNPq.

ORCID

Miri Weiss Cohen, http://orcid.org/0000-0001-5250-1016
João Batista Q. Zuliani, http://orcid.org/0000-0003-4011-5980
Frederico Gadelha Guimarães[c], http://orcid.org/0000-0001-9238-8839

REFERENCES

[1] Asadpoure, A.; Tootkaboni, M.; Valdevit, L.: Topology optimization of multiphase architected materials for energy dissipation. Computer Methods in Applied Mechanics and Engineering, 325, 314–329, 2017.

[2] Batista, L.S.; Campelo, F.; Guimaraes, F.G.; Ramirez, J.A.: Multi-domain topology optimization with ant colony systems. COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, 30(6), 1792–1803, 2011. http://doi.org/10.1108/03321641111168101.

[3] Bendsøe, M.P.; Kikuchi, N.: Generating optimal topologies in structural design using a homogenization method. Comput. Methods Appl. Mech. Eng., 71(2), 197–224, 1988. ISSN 0045-7825. http://doi.org/10.1016/0045-7825(88)90086-2.

[4] Bendsøe, M.P.; Sigmund, O.: Topology Optimization: Theory, Methods and Applications. Berlin, Springer, second ed., 2003.

[5] Campelo, F.; Ota, S.; Watanabe, K.; Igarashi, H.: Generating parametric design models using information from topology optimization. Magnetics, IEEE Transactions on, 44(6), 986–989, 2008. http://doi.org/10.1109/TMAG.2007.916350.

[6] Céa, J.; Garreau, S.; Guillaume, P.; Masmoudi, M.: The shape and topological optimizations connection. Computer Methods in Applied Mechanics and Engineering, 188(4), 713 – 726, 2000. http://doi.org/10.1016/S0045-7825(99)00357-6. IVth World Congress on Computational Mechanics. (II). Optimum.

[7] Cervera, E.; Trevelyan, J.: Evolutionary structural optimisation based on boundary representation of nurbs. part i: 2d algorithms. Comput. Struct., 83(23-24), 1902–1916, 2005. ISSN 0045-7949. http://doi.org/10.1016/j.compstruc.2005.02.016.

[8] Cervera, E.; Trevelyan, J.: Evolutionary structural optimisation based on boundary representation of nurbs. part ii: 3d algorithms. Comput. Struct., 83(23-24), 1917–1929, 2005. ISSN 0045-7949. http://doi.org/10.1016/j.compstruc.2005.02.017.

[9] Choi, J.S.; Yoo, J.: Simultaneous structural topology optimization of electromagnetic sources and ferromagnetic materials. Computer Methods in Applied Mechanics and Engineering, 198(27–29), 2111 – 2121, 2009. ISSN 0045-7825. http://doi.org/http://dx.doi.org/10.1016/j.cma.2009.02.015.

[10] Chuang, Y.C.; Chen, C.T.; Hwang, C.: A real-coded genetic algorithm with a direction-based crossover operator. Information Sciences, 305, 320–348, 2015.

[11] Haslinger, J.; Mäkinen, R.: Introduction to Shape Optimization: Theory, Approximation and Computation, 2003.

[12] Haupt, R.L.; Haupt, S.E.: Practical genetic algorithms. John Wiley & Sons, 2004.

[13] Lee, J.S.; Göransson, P.; Kim, Y.Y.: Topology optimization for three-phase materials distribution in a dissipative expansion chamber by unified multiphase modeling approach. Computer Methods in Applied Mechanics and Engineering, 287, 191–211, 2015. http://doi.org/10.1016/j.cma.2015.01.011.

[14] Lin, C.Y.; Chao, L.S.: Automated image interpretation for integrated topology and shape optimization. Structural and Multidisciplinary Optimization, 20(2), 125–137, 2000. ISSN 1615-147X. http://doi.org/10.1007/s001580050144.

[15] Meeker, D.: Finite element method magnetics. version 4.2. Available online: http://www.femm.info, 2010.

[16] Piegl, L.: On NURBS: a survey. Computer Graphics and Applications, IEEE, 11(1), 55–71, 1991. ISSN 0272-1716. http://doi.org/10.1109/38.67702.

[17] Piegl, L.; Tiller, W.: The NURBS book (2nd ed.). Springer-Verlag New York, Inc., New York, NY, USA, 1997. ISBN 3-540-61545-8.

[18] S. Batista, L.; Campelo, F.; G. Guimarães, F.; A. Ramírez, J.; Li, M.; A. Lowther, D.: Ant colony optimization for the topological design of interior permanent magnet (ipm) machines. COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering, 33(3), 927–940, 2014.

[19] Simon, D.: Evolutionary optimization algorithms. John Wiley & Sons, 2013.

[20] Sivaraj, R.; Ravichandran, T.: A review of selection methods in genetic algorithm. International journal of engineering science and technology, 1(3), 3792–3797, 2011.

[21] Tang, P.S.; Chang, K.H.: Integration of topology and shape optimization for design of structural components. Structural and Multidisciplinary Optimization, 22(1), 65–82, 2001. ISSN 1615-147X. http://doi.org/10.1007/PL00013282.

[22] Zuliani, J.B.Q.; Batista, L.S.; Guimaraes, F.G.; Weiss Cohen, M.; Li, M.; Lowther, D.A.: Multiobjective topology optimization with ant colony systems in applied electromagnetics. In Proceedings of the 19th International Conference on the Computation of Electromagnetic Fields, PC4–1. International COMPUMAG Society, Budapest, 2013.

[23] Zuliani, J.B.Q.; Weiss Cohen, M.; Batista, L.S.; Guimaraes, F.G.: Multi-objective topology optimization with ant colony optimization and genetic algorithms. Computer-Aided Design and Applications, 0(0), 1–9, 2015. http://doi.org/10.1080/16864360.2015.1033328.