COMPUTER-AIDED DESIGN & APPLICATIONS, 2018
VOL. 15, NO. 5, 661–666
https://doi.org/10.1080/16864360.2018.1441232

A new formulation of the minimum variation log-aesthetic surface for scale-invariance and parameterization-independence

Sho Suzuki [a], R.U. Gobithaasan [b], Shin Usuki [a] and Kenjiro T. Miura [a]

[a]Shizuoka University, Japan; [b]Universiti Malaysia Terengganu, Malaysia

ABSTRACT

The log-aesthetic curve, which includes the logarithmic (equiangular) spiral, clothoid, and circular involute, achieves control over the curvature distribution by defining its shape as an integral of its curvature, and is expected to be utilized for the field of aesthetic design.

Some formulations of the log-aesthetic surface as extensions of the log-aesthetic curve have been proposed. The minimum variation surface is one of them, and has a feature that it can be used for arbitrary four boundary curves. The minimum variation log-aesthetic surface is defined as a surface which minimizes an objective function. However, it is not scale-invariant and parameterization-independent.

In this study, we propose a new formulation of the minimum variation log-aesthetic surface for scale-invariance and parameterization-independence.

KEYWORDS

Log-aesthetic surface; minimum variation log-aesthetic surface; variational principle; scale invariance; parameterization independence

1. Introduction

Recently, aesthetic design which takes account of designability has become popular. In aesthetic design, the creation of high quality curve and surface models is demanded. However, on current CAD systems, the operator must move control points by trial and error to obtain high-quality curves and surfaces. This incurs high costs and requires a great deal of expertise. Therefore, an efficient method to generate fair curves and surfaces is desirable to achieve high-quality that will satisfy customers' aesthetic requirements.

The log-aesthetic curve was proposed as a curve which satisfies these quality requirements. Harada et al. [1] defined "Aesthetic curves" as curves whose logarithmic distribution diagram of curvature (LDDC) can be approximated by a straight line. In response to this research, Miura et al. [4] derived analytical solutions of the curves whose logarithmic curvature graph (LCG) as an analytical version of the LDDC is strictly given by a straight line and defined the curve as the log-aesthetic curve. For a given curve, the arc length of the curve and the radius of curvature are denoted by s and ρ, respectively. The log-aesthetic curve satisfies the following equation:

$$\rho^{\alpha} = cs + d \tag{1.1}$$

Here, α, c and d are constants. In particular, α is the slope of LCG and a parameter for controlling the impression of the curve. Figure 1 illustrates log-aesthetic curves for various α values. Also, one segment of the log-aesthetic planer curve is uniquely determined by both the endpoints and tangent vectors there [3]. Hence, one can modify the log-aesthetic curve by changing these boundary conditions and α value. Since the log-aesthetic curve is defined by use of curvature as the above equation, its curvature distribution is smooth. In addition, it includes logarithmic (equiangular) spiral, clothoid, and circular involute as well as Nielsen's spiral. For these reasons, it is expected to be utilized in the field of aesthetic design [9].

Although the log-aesthetic curve has a number of good properties, it is difficult to extend it to surfaces because of the complexity of its general equation. As a solution to this problem, the minimum variation log-aesthetic surface [8] was proposed. This surface is defined as a surface which minimizes an objective function and allows the usage of arbitrary boundary curves with tangent continuity. However, the value of the objective function depends on the scale of the model and its parameterization.

In this research, we derive a new formulation of the minimum variation log-aesthetic surface for scale-invariance and parameterization-independence.

CONTACT Sho Suzuki suzuki.sho.14a@shizuoka.ac.jp; R.U. Gobithaasan gr@umt.edu.my; Shin Usuki dsusuki@ipc.shizuoka.ac.jp; Kenjiro T. Miura miura.kenjiro@shizuoka.ac.jp

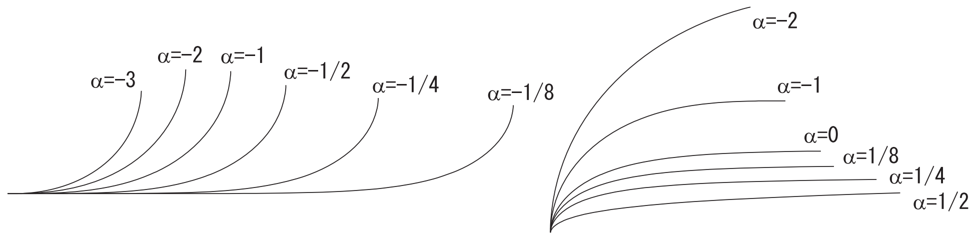

Figure 1. Log-aesthetic curves with various *alpha* values.

2. Related work

As mentioned in the introduction, it is very difficult to extend the log-aesthetic curve to the surface that has similar good properties to the log-aesthetic curve. To solve this problem, two surface formulas besides the minimum variation log-aesthetic surface have been proposed that generate free-form surfaces by sweeping the log-aesthetic curve [2, 7]. The log-aesthetic curved surface [2] is defined as a sweeping surface using two profile curves, which are composed of log-aesthetic curves, and one guide line composed of a non-log-aesthetic curve. The surface guarantees the isoparametric curves parallel to two profile curves become the log-aesthetic curve and the quality along the isoparametric curve is guaranteed. In contrast, the isoparametric curves parallel to the guide line do not become log-aesthetic curves and high quality in this direction cannot be guaranteed. As a solution to this problem, Saito et al. proposed the complete log-aesthetic surface [7]. The complete log-aesthetic surface is defined as a pure sweeping surface with two log-aesthetic curves. This formulation also uses the log-aesthetic curve as the guide line and guarantees that all parametric curves are log-aesthetic. However, for these two formulations, at least one boundary curve cannot be specified. Consequently, the situations where these formulations can be used are severely restricted.

3. Minimum variation log-aesthetic surface

The surface formulations using sweep have a problem that some boundary curve cannot be specified. In contrast, by using variational principle for the surface formulation, the minimum variation log-aesthetic surface can be used for arbitrary four boundary curves with tangent constraints. The minimum variation log-aesthetic surface is defined by reformulating the log-aesthetic curve with the variational principle and extending it to surfaces. In this section, we reformulate log-aesthetic curve with variational principle and introduce the minimum log-aesthetic surface.

3.1. Variational formulation of the log-aesthetic curve

From Eqn. (1.1), when we assume $\sigma = \rho^{\alpha}$ the log-aesthetic curve is given by a straight line connecting two given points (s_1, σ_1) and (s_2, σ_2) in the s-σ plane (aesthetic space) as shown in Fig. 2, where the horizontal and vertical axes are arc length s and σ, respectively. Therefore, from variational principle, the log-aesthetic curve is reformulated as a curve that minimizes the following energy J_{LAC}.

$$J_{LAC} = \int_{s_1}^{s_2} (1 + \sigma_s^2) ds \tag{3.1}$$

The Euler equation of Eqn. (3.1) is as follows.

$$\sigma_{ss} = 0 \tag{3.2}$$

Obviously, Eqn. (3.2) is equivalent to the second derivative of Eqn. (1.1). Furthermore, the Euler equation of Eqn. (3.1) is equivalent to that of the following equation K_{LAC}.

$$K_{LAC} = \int_{s_1}^{s_2} \sigma_s^2 ds = \int_{s_1}^{s_2} \alpha^2 \rho^{2\alpha-2} \rho_s^2 ds \tag{3.3}$$

Finally, Eqn. (3.3) is represented by arc length parameter s, and we rewrite Eqn. (3.3) using a general parameter t

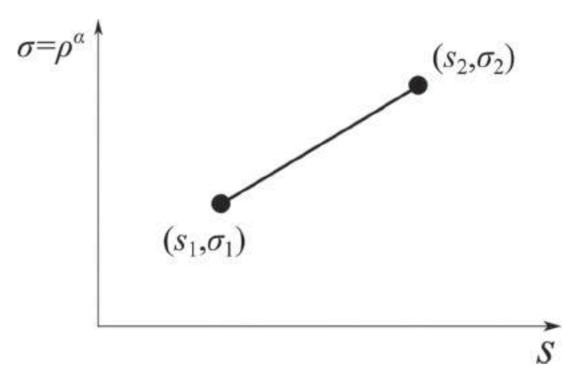

Figure 2. A straight line connecting two given points (s_1, σ_1) and (s_2, σ_2) in the s-σ plane (aesthetic space).

and obtain the following expression:

$$K_{LAC} = \int_{t_1}^{t_2} \frac{1}{\left\| \frac{dC}{dt} \right\|} \alpha^2 \rho^{2\alpha-2} \rho_t^2 dt \tag{3.4}$$

where, $||dC/dt||$ represents the norm of the first derivative of curve C with respect to a general parameter t. We use Eqn. (3.4) as the objective function of the log-aesthetic curve. By minimizing the functional value of curves, one can generate the nearest curve having the property (such that Eqn. (1.1)) of the log-aesthetic curve out of the shapes expressible under the given conditions.

3.2. Variational formulation of the log-aesthetic surface

The objective function of the log-aesthetic surface is derived by extending the objective function of the log-aesthetic curve K_{LAC} to surfaces. The objective function is defined so that minimizing the objective function transforms isoparametric curves into log-aesthetic curves. We obtain the following objective function J_{LAS} by applying Eqn. (3.4) to both of the directions of the surface and define the minimum variation surface as minimizing this function:

$$J_{LAS} = \int_{u_1}^{u_2} \int_{v_1}^{v_2} \left\{ \frac{1}{\sqrt{E}} \alpha^2 (\rho^u)^{2\alpha-2} (\rho_u^u)^2 + \frac{1}{\sqrt{G}} \beta^2 (\rho^v)^{2\beta-2} (\rho_v^v)^2 \right\} dv du \tag{3.5}$$

Here, E and G are elements of the first fundamental form and are given by $E = \partial S/\partial u \cdot \partial S/\partial u$ and $G = \partial S/\partial v \cdot \partial S/\partial v$, respectively. ρ^{u} and ρ^{v} are the radii of curvature of isoparametric curves in the u and v directions, respectively. In the integral of Eqn. (3.5), the first term is the optimization term of the isoparametric curve in the u direction and the second term is the optimization term of the isoparametric curve in the v direction.

As isoparametric curves become log-aesthetic curves, this minimum variation surface is equivalent to the complete log-aesthetic surface. However, as this formulation defines the surface by minimizing the objective function, the minimum variation surface is remarkably different from the complete log-aesthetic surface. That is, the minimum variation surface can specify an arbitrary boundary curve, and hence the objective function can be used for generating the surface. In many boundary cases, surfaces in which all of the isoparametric curves become log-aesthetic curves cannot be generated.

4. Scale-invariance

Moreton and Sequin [6] introduced the minimum variation surface (MVS) functional that measures curvature variation by integrating the principle curvature's squares of derivatives in its principle directions. They derived its scale invariance [5]. Multiplication of the area term is used for scale invariance and scale invariance of the MVS functional is given by the following expression:

$$E_{MVS} = \int \{(\kappa_{\mathrm{max}}^{\mathrm{max}})^2 + (\kappa_{\mathrm{min}}^{\mathrm{min}})^2\} dA \int dA \tag{4.1}$$

where $\kappa_{\mathrm{max}}^{\mathrm{max}}$ and $\kappa_{\mathrm{min}}^{\mathrm{min}}$ are derivatives of principle curvatures in its principle directions.

In this section, we will perform a similar modification for K_{LAC} expressed in Eqn. (3.3) to make it scale-invariant and extend it to surfaces. First, we consider a curve whose arc length is equal to 1. If the curve is log-aesthetic, i.e. $\sigma = \rho^\alpha$ is a linear function of arc length s, there is a constant c such that

$$c = \sigma_s = \sigma_{end} - \sigma_{str} \tag{4.2}$$

Here, σ_{end} and σ_{str} are the values of σ at both end points.

Then, Eqn. (3.3) becomes

$$K_{LAC} = \int_{s_1}^{s_2} c^2 ds = c^2 = (\sigma_{end} - \sigma_{str})^2 \tag{4.3}$$

On the other hand, if we consider to introduce scale factor r and a curve which is scaled by that scale factor r. Then, the arc length of the curve is equal to r and $\sigma = \rho^\alpha$ becomes $\sigma' = (r\rho)^\alpha = r^\alpha \rho^\alpha$.

There is a constant c' similarly to Eqn. (4.2) such that

$$c' = \sigma_s' = \frac{r^\alpha(\sigma_{end} - \sigma_{str})}{r} = r^{\alpha-1}(\sigma_{end} - \sigma_{str}) \tag{4.4}$$

Then the value of the objective function in Eqn. (3.3) becomes

$$K_{LAC}' = \int_{s_1}^{s_2} c'^2 ds = c'^2 r = r^{2\alpha-1}(\sigma_{end} - \sigma_{str})^2 \tag{4.5}$$

Therefore, by replacing scale factor r with the arc length of curve h, the scale-invariant objective function of Eqn. (3.3) is given by

$$K_{LAC-SI} = \frac{K_{LAC}}{h^{2\alpha-1}} \tag{4.6}$$

Based on the curve case, we define the scale-invariance objective function of the surface. First, as in the curve case, we consider a surface whose area is equal to 1. we separate the two terms in Eqn. (3.5) into two integrations in the u and v directions as follows:

$$J_{LAS} = \int K_{LAC\_u} dv + \int K_{LAC\_v} du \tag{4.7}$$

where

$$K_{LAC\_u} = \int_{u_1}^{u_2} \frac{1}{\sqrt{E}} \alpha^2 (\rho^u)^{2\alpha-2} (\rho_u^u)^2 du \tag{4.8}$$

$$K_{LAC\_v} = \int_{v_1}^{v_2} \frac{1}{\sqrt{G}} \beta^2 (\rho^v)^{2\beta-2} (\rho_v^v)^2 dv \tag{4.9}$$

Note that K_{LAC_u} and K_{LAC_v} indicate the objective function of the log-aesthetic curve in Eqn. (3.4) with respect to iso-parametric curves in the u and v directions, respectively.

Next, we consider the case that the surface is scaled by scale factor r (such that the area of the surface become r^2). Then, from the discussion of the curve case, K_{LAC_u} and K_{LAC_v} are scaled to $r^{2\alpha-1}$ and $r^{2\beta-1}$times, respectively. Therefore, we obtain the following equation:

$$J_{LAS}' = r^{2\alpha-1} \int K_{LAC\_u} dv + r^{2\beta-1} \int K_{LAC\_v} du \tag{4.10}$$

Finally, we obtain the scale-invariant objective function of Eqn. (3.5) by comparing Eqn. (4.7) with Eqn. (4.10) as follows

$$J_{LAS-SI} = \frac{\int K_{LAC\_u} dv}{A^{\alpha-1/2}} + \frac{\int K_{LAC\_v} du}{A^{\beta-1/2}} \tag{4.11}$$

where, $A = r^2$ are the area of the surface.

5. Parameterization independence

The objective function of minimum variation log-aesthetic surface in Eqn. (3.5) is defined so that the isoparametric curves of the minimized surface become log-aesthetic curves. However, this formulation includes surface parameter u, v and depend on its parameterization. Thus, we achive parameterization-independence by using principle radius of curvature $\rho^{\max}$ and $\rho^{\min}$. We define the scale invariant objective function of the minimum variation log-aesthetic surface by extending Eqn. (3.3) with principle curvatures as follows

$$J_{LAS-PI} = \int \Big\{ \alpha^2 (\rho^{\max})^{2\alpha-2} (\rho^{\max}_{\max})^2 + \beta^2 (\rho^{\min})^{2\beta-2} (\rho^{\min}_{\min})^2 \Big\} dA \tag{5.1}$$

Note that we use area microelements dA instead of $dudv$, furthermore, $\rho^{\max}_{\max} = \partial\rho^{\max}/\partial e_{\max}$ and $\rho^{\min}_{\min} = \partial\rho^{\min}/\partial e_{\min}$ where $e_{\max}$ and $e_{\min}$ are unit vectors. Hence, the coefficient such as $||dC/dt||$ in Eqn. (3.4) dose not appeared in Eqn. (5.1).

Especially, when α and β = -1, from $\rho = 1/\kappa$ and $\rho_t = d/dt(1/\kappa) = -\kappa_t/\kappa_2$, (5.1) becomes:

$$J_{LAS-PI}|_{\alpha=\beta=-1} = \int \Big\{ (\kappa^{\max}_{\max})^2 + (\kappa^{\min}_{\min})^2 \Big\} dA \tag{5.2}$$

Eqn. (5.2) is equivalent to the objective function of the minimum variation surface [6].

Additionally, we consider scale invariance of Eqn. (5.1) with paying attention the fact that scaling surface by scale factor r, the direction orthogonal to the principle direction evaluated by each term in Eqn. (5.1) also scale to r in area microelements. By the same discussion in the previous chapter with the above fact, we obtain the following scale-invariant objective function:

$$J_{LAS-PI-SI} = A^{-\alpha} \int \alpha^2 (\rho^{\max})^{2\alpha-2} (\rho^{\max}_{\max})^2 dA + A^{-\beta} \int \beta^2 (\rho^{\min})^{2\beta-2} (\rho^{\min}_{\min})^2 dA \tag{5.3}$$

Especially, when α and β = -1, we obtain the following:

$$J_{LAS-PI-SI}|_{\alpha=\beta=-1} = A \int \Big\{ (\kappa^{\max}_{\max})^2 + (\kappa^{\min}_{\min})^2 \Big\} dA \tag{5.4}$$

Eqn. (5.4) is equivalent to the scale-invariant objective function of the minimum variation surface in Eqn. (4.1).

6. Results

In this section, we adopt the objective function given in Eqn. (5.3) for B-spline surfaces and optimize the position of the control points of the surface by minimizing the objective function. At that time, we impose constraints on coordinates of and tangent vectors across the boundary curves as boundary conditions. Hence, we fix two control points from the boundary to keep the shape of the boundary curves and tangent vectors across them and input these control points. We used the downhill simplex method for optimization.

We applied our method to complete log-aesthetic surfaces [7] to which noise had been added. One of the formulations of a complete log-aesthetic surface is given by the following equation.

$$\begin{pmatrix} x(\theta,\phi) \\ y(\theta,\phi) \\ z(\theta,\phi) \end{pmatrix} = R_z(\phi) Sc(e^{b_g \phi}) \begin{pmatrix} t_r + e^{b_p\theta}\cos\theta \\ 0 \\ t_z + e^{b_p\theta}\sin\theta \end{pmatrix} \tag{6.1}$$

where θ and ϕ are surface parameters, b_p, and b_g are shape parameters, t_r t_z are offset parameters, $R_z(\phi)$ is

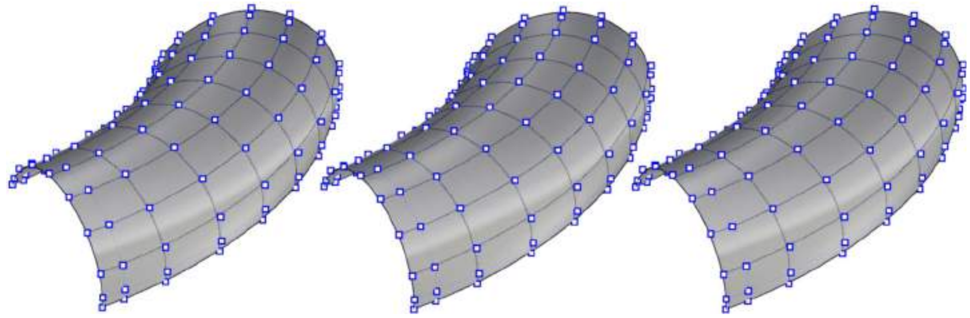

Figure 3. Generated surfaces. Left: before optimization. Middle: surface with added noise. Right: after optimization.

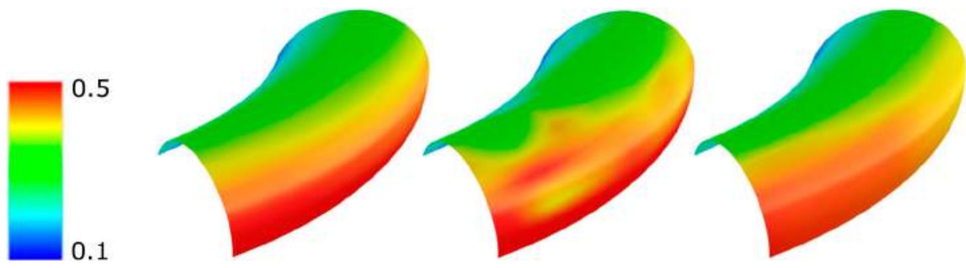

Figure 4. Mean curvature distribution. Left: before optimization. Middle: surface with added noise. Right: after optimization.

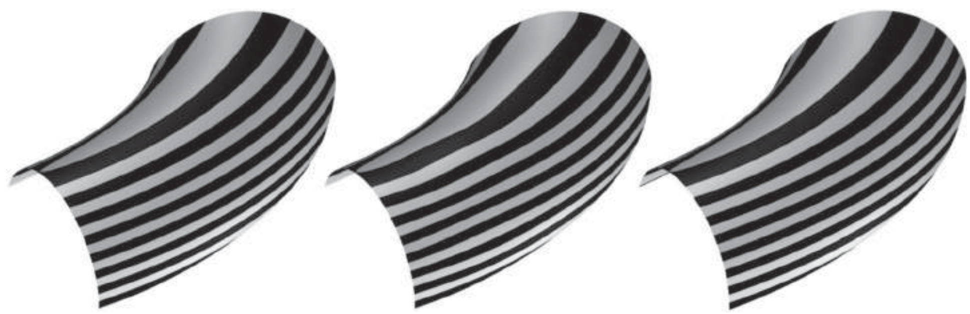

Figure 5. Zebra map. Left: before optimization. Middle: surface with added noise. Right: after optimization.

a rotation function around the z axis, and $Sc(e^{b_g\phi})$ is the scaling function. First, we generate a complete log-aesthetic surface with $b_p = 0.2$, $b_g = 0.2$, $t_r = 5$, and $t_z = 3$. Next, we cut a part of the surface and approximate this surface with bicubic B-spline, which has 10×10 control points. Finally, noise is added to the surface and our objective function is applied (i.e., we optimize the inner 6×6 control points). We used a PC with a Core i7-7700 3.60 GHz CPU.

Figure 3 shows generated surfaces. In the figure, the original surface is shown on the left, the surface with noise is shown in the middle, and the surface optimized by our method is shown on the right. The processing time for optimizing surfaces is about 140 [s]. Figure 4 and 5 show the mean curvature distribution and zebra map of these surfaces. These results showed that the surface with added noise is remarkably deteriorated. In contrast, after optimization, the surface is smooth and has almost the same quality as the original surface.

7. Conclusions

In this research, we have derived a new formulation of the minimum variation log-aesthetic surface for scale-invariance and parameterization-independence. Furthermore, we have generated surfaces by minimizing our newly defined objective function. The results indicate that we can obtain free-form surfaces of high quality. However, the processing times for relatively large surfaces are expected to be very long. Therefore, in future work, we will use GPU processors to reduce the processing time and hope to achieve a real time.

Acknowledgements

This work was supported in part by JSPS Grant-in-Aid for Scientific Research (B) Grant Number 25289021, JSPS Grant-in-Aid for Challenging Exploratory Research Grant Number 26630038 and FRGS grant(FRGS:59265) provided by University Malaysia Terengganu and Ministry of Education, Malaysia.

ORCID

Sho Suzuki http://orcid.org/0000-0002-2607-7204
R.U. Gobithaasan http://orcid.org/0000-0003-3077-8772
Shin Usuki http://orcid.org/0000-0003-1165-5507
Kenjiro T. Miura http://orcid.org/0000-0001-9326-3130

References

[1] Harada, T.; Yoshimoto, F.; Moriyama, M.: An aesthetic curve in the field of industrial design, Proc. of IEEE symposium on visual language, 1999, pp38-47. https://doi.org/10.1109/vl.1999.795873

[2] Inoue, J.; Harada, T.: An algorithm for generating log-aesthetic curved surfaces and the development of a curved surfaces generation system using VR, Proc. Int'l. Assoc. Soc. of Design Research, 2009.

[3] Meek, D.S.; Saito, T.; Walton, D.J.; Yoshida, N.: Planar two-point G1 Hermite interpolating log-aesthetic spirals. Journal of Computational and Applied Mathematics, 236(17), 2012, 4485-4493. https://doi.org/10.1016/j.cam.2012.04.021

[4] Miura, K.T.: A general equation of aesthetic curves and its self-affinity. Computer-Aided Design & Applications, 3(1-4), 2006, 457-464. https://doi.org/10.1080/16864360.2006.10738484

[5] Moreton, H.; Sequin, C.: Scale-invariant functional for smooth curves and surfaces, Geometric Modelling. Springer Vienna, 1995, 303-321.

[6] Moreton, H.; Sequin, C.: Functional optimization for fair surface design, ACM SIGGRAPH, 1992, 167-176. https://doi.org/10.1145/133994.134035

[7] Shikano, K.; Saito, T.; Yoshida, N.: Complete Log-Aesthetic Surfaces by Logarithmic Helical Sweep. SIAM Conference on Geometric Design(GD/SPM13), 2013.

[8] Suzuki, S.; Gobithaasan, R.U.; Salvi, P.; Usuki, S., Miura, K.T.: Minimum Variation Log-aesthetic Surface, Asian Conference on Design and Digital Engineering(ACDDE), Jeju, Republic of Korea, 2016.

[9] Ziatdinov, R.; Yoshida, N.; Kim, T.: Analytic parametric equations of log-aesthetic curves in terms of incomplete gamma functions. Computer Aided Geometric Design, 29(2), 2012, 129-140. https://doi.org/10.1016/j.cagd.2011.11.003