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Automatic construction of structural CAD models from 3D topology optimization

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ABSTRACT

The integration of topology optimization (TO) methods with Computer-Aided design (CAD) arouses a growing interest for mechanical and structural design purposes. However, generating 3D optimized CAD models from raw TO results still remains a tedious task that requires significant experience and user input. This paper presents a fully automated process to generate 3D optimized CAD models from TO results that tend towards beam-like structures. Raw TO results are first derived into an optimized shape as a smooth triangulation. This triangulation is then derived as a curve skeleton, which is finally normalized to generate a CAD model composed with an assembly of standard structural straight beams. 3D beam structures obtained with this automatic process are validated through comparisons between FEA results obtained using mixed-dimensional FEA models and solid 3D tetrahedral FEA models. Efficiency of this automatic CAD model construction approach is demonstrated through applying it on several beam-like TO results. The TO method used in this work is the SIMP method but principles used could be extended to evolutionary (ESO/BESO) and other types of TO approaches.

KEYWORDS

Topology optimization; CAD; integration; SIMP; skeletons; beam structures; mixed-dimensional analysis

1. Introduction

The development of topology optimization (TO) methods [2, 5, 19, 20, 31] has been a very important subject of research and industrial interest for the last 25 years. These methods consist in automatically optimizing the distribution of material along analysis iterations. Applying TO basically automates what is done by engineers when they modify a given version of a geometry with respect to analysis results. The development of most TO methods was first intended for 2D optimization problems, which has then gradually been extended to 3D problems, which then made possible its integration with CAD. The fact that the TO process is now well mastered for 3D geometry and that it is fully automated, starting from CAD models, opens the path for extremely interesting developments in mechanical design and other fields. Indeed automatically optimizing 3D mechanical components and structures can now be foreseen, which can even be extended towards creating components from scratch since the TO process can be applied from extremely rough initial distributions of material. In this latter context, as illustrated in Fig. 1, the TO process actually tends to automate the creation of geometry. Indeed, rough geometry shown in Fig. 1a, along with loads and boundary conditions (BCs) is automatically derived into optimized geometry shown in Fig. 1b though an integration of TO with CAD as described in some of our previous papers [14, 16]. Note that in Fig. 1b *non-design material* is shown in red, while the optimized *design material* is grey. Non-design material refers to material that should not be modified or removed by the optimization process since it is related to other components. Non-design material usually corresponds to geometry on which loads and BCs are applied.

This example illustrates that fully integrating TO with CAD is obviously very interesting and could even be the first step of a new era in the way we design and manufacture products with CAD/CAM technology. Moreover, recent developments related to additive manufacturing (AM) make that the type of geometry generated from 3D TO can directly be manufactured, which was not the case with conventional manufacturing processes. However, AM is expensive and time consuming, which makes that filling the gap between 3D TO results and conventional manufacturing and conventional manufacturing and conventional manufacturing and conventional manufacturing processes still remains a major and very important challenge.

Commercial codes and systems are developed towards this objective (ex.: TOSCA by Dassault Systemes, OptiStruct and solidThinking by Altair Engineering Inc.). However, the research work behind these products is not available in the literature. Moreover, it appears that, in these commercial TO solutions, even if interesting tools exist to facilitate building parts and structures

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Figure 1. Automatic creation of geometry with TO: (a) Initial rough geometry with loads and BCs (b) optimized geometry derived.

from TO results, the gap between TO and CAD is still huge. The approach presented in this paper significantly contributes to filling this gap for TO results that tend towards beam-like structures such as the one shown in Fig. 1b since it is fully automated, which is not the case for commercial TO tools available. Indeed, depending on parameters used in TO, optimized geometry generated from TO can be very different as illustrated Fig. 1.

In the case shown in the figure, the TO method used is the SIMP method [5, 16] and the parameter that is modified is SIMP volume fraction f, which represents the fraction of the initial design material (in %) that is allowed for generating the optimized geometry (see more details in section 3). If f is low (2.5% in Fig. 1(a) and 1.5% in Fig. 2(a)) the optimized geometry tends to beam-like structures, while higher f values generate more massive shapes (20% in Fig. 2(b)). It is obvious that, from these two types of TO results, an engineer would generate very different types of designs. In the first case he would derive the optimized shape as an assembly of structural beams while in the second case he would derive the result as a molded and/or machined solid part.

In this work, we specifically consider TO results that tend towards beam-like structures and the objective is automating the construction of 3D structures from TO results. As described in the next section, the TO method used is a 3D implementation of the SIMP method developed by our team [14, 16], which is inspired from work by Bendsoe and Sigmund [5, 26]. It is important to note that the principles on which the proposed automatic construction approach is based can be successfully applied to other TO schemes such as ESO/BESO [20] or level-set based methods [31].

The objective of this work is fully integrating TO with CAD for beam-like structures. Ideally, the process should start from a rough initial CAD model along with boundary conditions (BCs) and optimization objectives (typically a volume fraction here since the SIMP method is used), and automatically end with a CAD model of the optimized structure that fulfills these objectives, all of this without any other user interaction.

The paper is organized as follows. In the next section (section 2) previous work towards generating CAD models from 3D TO results is synthesized. After briefly presenting the TO method used and how 3D solid CAD models can be derived from it in section 3, section 4 details the proposed approach towards automatically generating beam-like structures from these 3D TO results. section 5 explains how these beam-like structures are validated through finite element analysis (FEA). Several examples are shown to demonstrate the interest, effectiveness and limitations of our approach. The paper



Figure 2. Two types of TO results: (a) A beam-like structure (b) A massive solid shape.

ends with a conclusion about potential further research towards better integrating TO with CAD.

2. Related work

Whatever the TO method used, raw TO results cannot be used as is for reconstructing optimized CAD models. Indeed level-set based methods represent optimized geometry implicitly as a level-set, homogenization based methods as a material and void spatial distribution, SIMP based methods as a relative density field and ESO/BESO methods as a 3D mesh. Thus, integrating TO with CAD, which means deriving CAD models from raw TO results, requires intensive and complex post-processing of these raw results. A few approaches have been proposed in the literature in this direction. Basically, three main strategies have been investigated for post-processing TO results into CAD geometry.

The first strategy towards reconstructing CAD geometry from raw TO results is based on computing isovalue sets (iso relative density for SIMP based methods for example), which leads to discrete representations of geometry such as triangulated surfaces and discretized curves. These discretized representations of geometry can be, in a second step, derived into CAD curves and surfaces. For example, Youn et al [32] extracted CAD curves from 2D TO results based on iso-relative density discretized curves. A similar approach has been applied

to 3D TO results by Hsu et al. [19]. In this work, cross sections of the optimized geometry are computed as 3D iso-relative density discretized curves that are then derived into 3D B-Spline curves. Sweeping through sets of B-Spline cross sections finally allows generating a 3D solid CAD model from 3D TO results. In a similar approach, Tang et al. [28] mixed cross section and surfaces computation to derive 3D TO results into CAD geometry. Koguchi et al. [21] based their CAD reconstruction approach on first using the marching cubes method to obtain an enclosed iso-relative density discrete surface that is then derived into a close set of biquartic surface splines. The second strategy towards reconstructing CAD geometry from raw TO results is based on trying to fit pre-defined shapes, referred to as primitives, to sub-sets of TO results. The approach starts with identifying subsets of the optimized result and comparing and best-fitting these subsets with pre-defined shapes (primitives). Once parameters of these pre-defined primitives are calculated, the optimized CAD model is represented as a Boolean combination of primitives. For example, Lin and Chao [23] used this strategy to parameterize 2D TO results based on using seven predefined 2D primitives. In their work, shape optimization is used to fit actual shapes and dimensions of the optimized result to that of these predefined primitives. Larsen and Jensen [22] created smooth parametric 3D CAD models from TO results based on fitting and sweeping predefined 2D contours to



Figure 3. Flowchart of the approach.

the optimized 3D shape. This last work well illustrates the main drawbacks of this type of strategy since applying it in 3D is limited to specific optimized shapes and requires significant user input, which is inconsistent with the objective of fully automating the reconstruction of CAD models from TO results. A third strategy towards reconstructing CAD geometry from raw TO results is based on interpreting TO results using methods inspired by black and white and grayscale image processing algorithms. Indeed, once a TO result is represented as sets of cells filled of void or solid material or filled with a varying quantity (relative density for SIMP based results) extracting boundaries of optimized results can be considered as quite similar to image segmentation problems. Chirehdast et al. [10] used black and white image segmentation techniques to convert 2D TO results into B-Spline curves. Bremicker et al [7] used similar image segmentation techniques to derive 2D truss structures from black and white type of results in 2D. An interesting aspect of their work is that, after extracting boundaries of optimized results, 2D skeletons are computed from these boundaries using the medial axis transform (MAT). These skeletons are then used as a base for computing 2D bar trusses. This approach is limited to 2D optimization and requires significant user interaction but it is worth mentioning that extending this idea to 3D TO results represents one of the core concepts on which the work presented in this paper is based.

A synthesis of all these methods leads to the conclusion that fully automating reconstruction of CAD models from TO results is a complex challenge that will require significant research efforts in the future. The approach presented along the next sections allows automating this reconstruction in the case of beam-like TO results, as synthesized in Fig. 3.

3. Topology optimization and 3D geometry construction

3.1. The SIMP method

As introduced in the introduction, among several other methods, the SIMP method is used as TO method in this work. SIMP stands for Solid Isotropic Material with Penalization, which is one of the most widely used TO methods. As illustrated in Fig. 4a, our implementation of the SIMP method starts with an initial rough CAD model on which design (colored gray in Fig. 4a) and non-design (colored red in Fig. 4a) sub-domains are specified along with loads and BCs. Overall dimensions are $25m \times 6m \times 5m$, bridge basis is constrained with null displacement and a constant pressure P = 10kPa is applied vertically (Young's modulus is 69 GPa, Poisson's

ratio is 0,33). This CAD model is automatically meshed (see Fig. 4b), with linear tetrahedrons (with constant size $d_g = 275mm$), through a specific adaptation of the advancing front method. As described with details in [14], this adaptation of the advancing front method allows tagging tetrahedrons as part of design and non-design sub-domains and guarantees conformity of the mesh at the interface between sub-domains.

From this starting point, the SIMP TO process is an iterative process in which a relative density field $\rho(x, y, z)$. is updated at each iteration with respect to FEA results (using Code_AsterTM [1]). This relative density varies from 0 (no material) to 1 ("full" or actual material) and it is derived into the distribution of a virtual elastic modulus E(x, y, z), according to the penalisation law $\tilde{E}(x, y, z) = E \rho(x, y, z)^p$ (E is the actual material's elastic modulus and p an integer penalization coefficient that is usually chosen between p = 1 and p = 3). In the following equations mathematical quantities noted using a \square are affected by the relative density field $\rho(x, y, z)$. The SIMP process iteratively searches for a distribution $\rho(x, y, z)$ that minimizes global compliance \tilde{C} (and by the way maximizes stiffness). If the finite element discretization is $[\tilde{K}]$. $\{\tilde{U}\} = \{F\}$, global compliance \tilde{C} is defined as:

$$\tilde{C} = \{\tilde{U}\}^t . \{F\} = \{\tilde{U}\}^t . [\tilde{K}] . \{\tilde{U}\}$$
(3.1)

where $\{\tilde{U}\}$ is the global displacement vector, $[\tilde{K}]$ the global stiffness matrix, $\{F\}$ is the global vector

The local stiffness matrix of element e due to the impact of the relative density ρ_e inside the element:

$$[\tilde{K}_e] = (\rho_e)^p . [K_e]$$
 (3.2)

Thus, if the mesh is composed with N tetrahedrons, the global stiffness matrix is

$$[\tilde{K}] = \sum_{e=1}^{N} [\tilde{K}_e] = \sum_{e=1}^{N} (\rho_e)^p . [K_e]$$
(3.3)

And global compliance \widetilde{C} can be written as:

$$\tilde{C} = \{\tilde{U}\}^{t}.[\tilde{K}].\{\tilde{U}\} = \{\tilde{U}\}^{t}.\left(\sum_{e=1}^{N} (\rho_{e})^{p}.[K_{e}]\right).\{\tilde{U}\}$$
$$= \sum_{e=1}^{N} (\rho_{e})^{p}.\{\tilde{U}\}^{t}.[K_{e}].\{\tilde{U}\}$$
(3.4)

Practically, this global compliance is computed using the total strain energy \tilde{W} as $\tilde{C} = 2.\tilde{W}$.

The SIMP optimization problem is classically formulated as:

ninimize
$$\tilde{C} = \sum_{e=1}^{N} (\rho_e)^p . \{\tilde{U}\}^t . [K_e] . \{\tilde{U}\}$$

r



Figure 4. SIMP optimization: (a) model with BCs (b) mesh of design and non-design material (c) relative density obtained at convergence (d) optimized shape derived.

with
$$\frac{\tilde{V}}{V_d} = f[\tilde{K}] \cdot \{\tilde{U}\} = \{F\} \text{ and } 0 \le \rho \le 1$$

Thus, the volume fraction f is maintained constant along SIMP iterations. It is defined as the fraction between design material \tilde{V} as affected by $\rho(x, y, z)$ and volume of the initial design sub-domain V_d . It is considered that convergence is achieved when the relative difference in global compliance between two successive iterations is less than a given threshold. As described with details in many references [16, 26], checkerboard and mesh sensitivity effects are avoided using filters on the sensitivity $\frac{\partial \tilde{C}}{\partial \rho_e}$ and/or on the relative density distribution $\rho(x, y, z)$. Fig. 4c illustrates the relative distribution $\rho(x, y, z)$ obtained at the end of SIMP iteration for the model and BCs shown in Fig. 4a. This distribution is obtained using f = 4% and filtering relative density $\rho(x, y, z)$ at each SIMP iteration. From this relative density distribution, the optimized shape shown in Fig. 4d is obtained by simply discarding tetrahedrons for which $\rho(x, y, z)$ exceeds a threshold ρ_{th} ($\rho_{th} = 0.2$ in this case).

3.2. Generating a 3D optimized shape from SIMP results

This way of deriving an optimized shape from raw SIMP results is extremely simple. However, its drawbacks are that non-manifold material continuity is obtained is some cases (material connection though a single tetrahedron node or through a single tetrahedron edge) as shown in Fig. 5(a) and that surface boundaries of the optimized shape are represented as a very noisy triangulation (see Fig. 5b). Reference [14] explains with details that non-manifold patterns can easily be removed by reactivating tetrahedrons around non-manifold connections. Processing noisy boundaries is much more challenging since, even if many triangulation smoothing methods are available [9, 11, 24, 29, 30] in the literature, none are really efficient for the type of noisy shapes that we have to process in the context of this work. Indeed most of triangular mesh smoothing methods are designed for removing noise that is present on smooth shapes, due to various sources such as scanning noise or other sources of noise.



Figure 5. Problems related to extracting optimized shapes from TO results: (a) non-manifold connexions (b) extremely irregular boundaries.

In our case as illustrated in Fig. 5b triangulations issued from TO results are in fact extremely irregular, due to the removal of tetrahedrons, which is very different from being noisy. A synthesis of triangular mesh smoothing methods show that some methods only try smoothing noisy triangular surfaces [11, 29, 30] while other [9, 24] also try preserving specific geometric features such as sharp edges.

We have applied many triangular mesh smoothing algorithms on our TO optimized shapes and faced many problems, mainly due to the fact that, as introduced above, these algorithms are not designed for efficiently processing the type of irregular triangulations generated from TO results. At this point of our research, we could not find any satisfying method or combination of methods that is able to handle all types of TO results while meeting all our requirements. Indeed the objective is not only smoothing optimized boundaries but also:

- generating 3D parts that meet the volume fraction objective *f*
- generating continuous optimized material
- generating untangled and good quality triangulations
- preserving features like sharp corners and sharp edges

Indeed, if the smoothing process is too aggressive, it may first affect the optimized part volume and the optimized shape as such, which in fact tends to destroy some of the optimization benefits. This especially occurs when using Laplacian based smoothing algorithms. For example, several iterations of Laplacian-based smoothing as presented in [11] is very efficient in processing tangled and very irregular triangulations but, as well documented in the literature, this tends to significantly shrink the optimized material. In addition, since smoothed

triangulations are then used for building 3D CAD models and FEA models they should feature neither tangled triangles nor triangle with bad quality for FEA calculations. Mesh tangling namely occurs when applying featurepreserving algorithms such as Chen and Cheng's [9] on this type of very irregular triangulations. At this point of our research, the best combination found for successfully processing TO results that tend to beam-like structures, which is the context of this paper, is using a combination of Taubin [29] and Laplacian-based smoothing [11]. Taubin smoothing indeed overcomes a major drawback of Laplacian based smoothing methods referred to as shrinking as illustrated in Fig. 6a. In Fig. 6a 30 iterations of Laplacian type smoothing, as described in reference [11], have been applied on the irregular mesh shown in Fig. 5b.

In Taubin's approach, smoothing is performed as a low-pass filter that decreases curvature variations without shrinking. This is practically obtained by repeatedly applying two consecutive Gaussian smoothing steps, one with a positive scale factor λ that induces shrinking and a second with a negative scale factor μ that induces inflating (see reference [29] for details). As introduced above, in our approach these Taubin smoothing iterations are followed by a couple of Laplacian-type smoothing iterations for obtaining a smoother boundary. This makes that, as illustrated in Fig. 6b a smooth boundary can be obtained without shrinking. Moreover, as shown in the enlarged view in Fig. 6b, resulting triangulation feature excellent element quality, which is mandatory for performing 3D FEA afterwards from it.

Meeting the volume fraction objective is obtained through using appropriate values for both the relative density threshold used ρ_{th} ($\rho_{th} = 0.2$ in this case) and smoothing parameters such as number of Taubin and



Figure 6. problems related to shrinking: (a) Shrinking induced by Laplacian-type smoothing (b) Result obtained with a combination of Taubin and Laplacian-type smoothing.

Laplacian-type smoothing iterations and λ , μ values for Taubin smoothing. In the result shown in Fig. 6b, the actual volume fraction reached after smoothing (using 10 Taubin iterations [29] with $\lambda = 0.33$, $\mu = -0.2$ and 4 Laplacian-type iterations [11]) is $f_{3D} = 4.1\%$ (the objective is f = 4%).

It is worth mentioning that, as also concluded in reference [19], using lower ρ_{th} values is a good strategy for generating smooth optimized shapes that meet volume fraction objectives while avoiding generating

discontinuous distributions of optimized material. This is especially true for low volume fraction objectives, which is the context of this work since it is targeted towards beam-like optimization results. Indeed, for low volume fraction objectives, higher ρ_{th} values easily result in losses in material continuity and, by the way, in inappropriate optimized shapes. Overall, the strategy presented in this paper ensures generating continuous 3D structures that closely meet volume fraction objectives while also generating untangled and good quality triangulations.



Figure 7. 3D FEA results on the optimized part: (a) σ_x stress distribution (b) Von-Mises stress distribution (c) Deformed shape (with 1000 factor applied) and resultant displacement distribution (d) Total strain energy per element distribution.

However, this approach does not also allow preserving sharp edges and corners, which would be particularly useful in the case of TO results tending towards massive 3D shapes like the one shown in b.

3.3. FEA results obtained on the optimized solid shape

The last step in processing these 3D TO results is generating a 3D FEA model from it. This FEA model is generated by filling the optimized solid part obtained with linear tetrahedrons and by applying on it loads and BCs as specified in the TO process itself. The automatic generation of tetrahedrons inside the volume is performed using our CAD/FEA research platform [15]. It is based on an advancing front mesh generation algorithm [14], which is initiated from the smoothed triangulation obtained from the previous steps described above. Once the model filled with tetrahedrons, loads and BCs applied, the FEA model is solved using Code_AsterTM [1]. Note that, as in the case of TO iterations, the FEA formulation used is purely linear. Fig. 7 shows stress, displacement and strain energy distributions issued from the optimized shape illustrated in Fig. 6b.

Note that, for further comparison, total compliance of the optimized part, calculated as twice the total strain energy W_{3D} , is $C_{3D} = 2.W_{3D} = 257J$. These results show that the optimized structure is too stiff and could be further optimized. Indeed, maximum displacement is 0.5 mm (at the middle of bridge deck) and maximum representative Von-Mises Stress is around 1.4 Mpa (along upper horizontal beams). However, it must be reminded that this optimized structure has been created through a fully automatic process and that a lighter structure could easily be obtained using different input data (a thinner non-design domain and a lower value for the volume fraction objective).

In general, optimized parts and structures provided by TO would be extremely difficult, if not impossible, to manufacture as is at a reasonable cost, which means using common manufacturing processes. Since 3D structures are usually assembled using standard straight beams, the next step in our automatic model construction process from 3D TO results is transforming 3D optimized parts into sets of straight standard beams.

4. From 3D topology optimization results to beam structures

4.1. Automatic creation of a curve skeleton

In the approach presented in this paper, starting from 3D beam-like TO results like the one shown in Fig. 6b,

3D structures made of beams are automatically derived through a process that is principally based on shape skeletonization techniques [3, 6, 8, 12, 17, 25, 27]. A survey and thorough classification of these skeletonization methods in general can be found in reference [27]. Skeletons can basically be defined as thin centered structures that approximate topology and geometry of 3D volumes or 2D surfaces. In 3D, they represent concise representations of geometry that can be used, in parallel with classical B-REP and discrete representations, for analyzing, manipulating and processing 3D geometry. One of the interests of skeletonization in the context of this work is that it allows retrieving meaningful information from 3D TO results (topology, number of branches, symmetry, local thickness, local section, etc.). Generating skeletons for 2D shapes is now quite well mastered, which is not the case in 3D. Despite the fact that many methods have recently arouse for generating and processing 3D skeletons, it remains a challenging task, mainly due to potential variety and complexity of 3D shapes in general. In fact, many 3D skeletonization methods are focused on specific applications, such as body movement processing or capturing for example, and cannot be used in a very general context. Indeed, generating skeletons from 3D shapes has been an intensive subject of research investigation for the last 15 years, mainly due to the interest aroused for computer graphics, computer-based animation and video games applications. The skeleton of a 3D shape can either be a surface skeleton, such as those obtained with medial axis transform (MAT) [17, 25, 27] or a curve skeleton [3, 8, 12, 27]. Surface skeletons of 3D shapes are generally defined as medial skeletons, based on the locus of centers of maximally inscribed balls, on Maxwell sets or on grassfire analogy [27], which represent solid theoretical basis for the definition these medial structures. For curve skeletons, which can vaguely be defined as 1D structures that are locally centered in a 3D shape, there is not such a strong theoretical basis for defining these structures in a general context. However, in specific contexts of shapes that may reasonably be assimilated as assemblies of tubular shapes, the definition of curve skeletons is more natural and clear. For such shapes, which is the case in this work since it is focused on beam-like TO results, as listed in reference [27], many interesting approaches (medial surface based methods, distance fields based methods, topology driven methods, contraction methods, etc.) have been proposed and successfully implemented. Among these methods, the skeletonization algorithm used in this work is taken from references [3, 8] and inspired by the fact that Laplacianbased smoothing naturally leads to mesh contraction as illustrated in Fig. 6. Indeed, if many Laplacian-based smoothing iterations are applied, resulting shapes locally



Figure 8. Skeletonization of the 3D optimized solid shape: (a) 3D solid shape after smoothing (b) curve skeleton derived (c) distribution of cross section radius along skeleton branches (d) skeleton after normalization (e) distribution of beam cross section radii (f) 3D CAD model of the optimized beam structure obtained at the end of the process.

tend towards very thin volumes. In this case, an implicit Laplacian smoothing process is constrained in a way that 3D shapes are gradually contracted or thinned while preserving geometry along contraction. After these constrained implicit Laplacian iterations, a skeleton graph is constructed using a farthest-point sampling process, which is finally simplified via topology thinning (see references [3, 8] for details). Thus, following these steps, a curve skeleton can be automatically generated from 3D TO results after smoothing. Fig. 8b illustrates the result of curve skeletonization when applied to the optimized solid shape after smoothing as shown in Fig. 8a.

4.2. Normalization of the curve skeleton and cross section calculations

As mentioned above, one of the interests of computing skeletons from 3D shapes is that it allows assessing local

thickness and local section. In this case, it allows computing distribution of the mean local radius (see Fig. 8c). This radius is calculated along each skeleton branch as the mean local distance between the skeleton and the optimized 3D boundary at this location. The distribution is later used to compute cross section properties of the 3D beam structure generated at the end of the process. Before that, the curve skeleton is normalized as sets of straight lines as illustrated in Fig. 8d. This normalization settles the basis for building the optimized beam structure. Indeed, these straight lines represent neutral fibers of beams of the constructed 3D structure. At this point of our research and as a first approximate, these beams are associated with circular cross sections only and the cross section radius of each branch is considered as constant. For each branch, cross section radius is calculated as the mean value of mean local radius along skeleton branches before normalization (as illustrated in see Fig. 8c). Fig. 8e shows the distribution of beam cross section radii for each branch of the normalized skeleton and Fig. 8f illustrates the 3D beam structure derived.

Beam radii vary from 0.214 m to 0.369 m and a close look at Fig. 8(e) also shows that, even if the initial optimization problem is fully symmetric (both initial geometry and BCs are symmetric in Fig. 4a) the constructed 3D beam structure is not exactly symmetric. Indeed, the beam structure itself even if close to symmetric is not exactly symmetric, which is also not the case for beam cross section radii. For example, the two horizontal beam cross section radii are 0.214 m and 0.245 m. This is not surprising since the 3D optimized shape is not either symmetric because the mesh used for TO itself is not symmetric. Thus, obtaining a symmetric final 3D beam structure would either require constraining the TO process to generate a symmetric result or post-processing the 3D structure to make it symmetric for both geometry and beam cross sections. Nevertheless, this outcome has a great potential since the structure generated can easily be post-processed, modified, and even further optimized through an optimization of beam cross sections.

A first validation of the 3D beam structure created at the end of the process is calculating volume fraction obtained with this structure, referred to as f_{beam} . $f_{beam} =$ 3.9%, which is based on adding beam volumes and dividing the result obtained by design material volume before TO. This shows that volume of the 3D beam structure created is very close to the volume fraction objective (f =4%) and to volume fraction obtained from the 3D optimized shape ($f_{beam} = 4.1\%$). In next section, 3D beam structures generated through the process presented just above are further validated using a comparison between FEA simulations performed on 3D optimized shapes (like in Fig. 7), on the one hand, and on normalized 3D beam structures derived (like in Fig. 8e), on the other hand.

5. FEA validation of 3D beam structures created

5.1. Mixed-dimensional FEA model

FEA on 3D beam structures created through the automatic process presented in section 4 is based on building mixed-dimensional FEA models. Indeed, as illustrated in Fig. 8e, 3D beam structures obtained are composed with a mix of 3D solid geometry (for non-design material) and sets of straight beams with circular cross section properties (for design material). In the FEA validation of 3D structures created, these straight beams are modeled using classical Euler-Bernouilli beam elements [4] while non-design solid geometry is meshed with linear tetrahedrons, which makes the whole FEA model mixeddimensional (a mix of solid and beam finite elements as shown in Fig. 9). The major problem with such mixeddimensional FEA models is connecting beam elements with tetrahedral elements since there is an inconsistency between degrees of freedom (DOF) of these elements (classical tetrahedral elements used in linear elasticity usually feature 3 displacements while 3D beam elements feature 3 displacements and 3 rotations for bending and torsion). In a previous work [13], we have proposed a very simple and efficient solution to overcome these inconstancies between DOF. As shown in Fig. 9b, this solution is based on using mini-beams, which are very stiff beam elements that are automatically generated at the interface between a beam element and a tetrahedral mesh.

These mini-beams locally introduce a rigid connection between a beam element and a tetrahedral mesh.



Figure 9. (a) mixed-dimensional FEA model (b) beam elements and mini beams (c) source of mini-beams distribution at a connexion (d) closer view of this connexion with mini-beams.



Figure 10. (a) resultant displacement distribution for the optimized solid shape (b) resultant displacement distribution for the created beam structure.



Figure 11. Cantilever (a) initial model with BCs (b) optimized solid shape (c) curve skeleton before normalization (d) distribution of cross section radii after normalization (e) (f) resultant displacement distribution for the optimized solid shape and beam structure created.

As shown in Fig. 9b and d, shape of connection surfaces derives from how mini-beams are distributed. As illustrated in Fig. 9c and d, in this work, the shape of connection surfaces is consistent with that of connection surfaces in the optimized solid shape. Fig. 10 illustrates a comparison between the resultant displacement distribution resulting from this mixed-dimensional model (Fig. 10b) and that of the optimized solid shape (Fig. 10a). Note that differences between distributions shown in Fig. 10a and Fig. 7c comes from color scales used, which are not the same. Indeed, for a better comparison between the two displacement distributions shown in Fig. 10, the same color scale is used, as well as the same amplification factor for deformed shapes (1000 times). Maximum displacement for the created beam structure is 0.526 mm while that of the optimized solid shape is 0.462 mm and quantitative difference between these two distributions is quite slight. This slight difference is confirmed by calculating total compliance Cbeam for this beam structure. Total compliance is calculated as twice the total strain energy W_{beam} , which is derived from FEA results obtained with the mixed-dimensional model. $C_{beam} =$ $2.W_{beam} = 288J$, which is slightly higher than that of the optimized solid $C_{3D} = 2.W_{3D} = 257J$.

These results show that the 3D structure created, following the approach presented in this paper, is almost as stiff as the optimized solid shape. This is very interesting since, as introduced above, this 3D structure can easily be edited and modified for improving its performance, which is not the case for the optimized solid shape.

5.2. Examples

The approach presented in this paper is applied to two other TO cases that tend towards beam-like structures. The first example is a classical cantilever case and the second is a L bracket. For both cases Young's modulus is 69 GPa, Poisson's ratio is 0.33 and the objective volume for SIMP iterations fraction is f = 3%.

5.2.1. Cantilever structure

Dimensions of the initial model are $51 \text{ mm} \times 51 \text{ mm} \times 102 \text{ mm}$. It is loaded with a downward vertical force (10N in the Y direction) and null displacements in all directions are imposed at the back (see Fig. 11a). The optimized solid shape, curve skeleton derived before normalization and result of normalization are respectively provided in Fig. 11b,c and d. It can be seen that cross section radii vary from 1.3 mm to 3.7 mm. Resultant displacement distributions for the optimized solid shape and the beam structure created are finally illustrated in Fig. 11e and f using the same color scale and same amplification factor (1000 times) for comparison. Here again,

Table 1. Synthesis of results

	Bridge	Cantilever	L bracket
S	IMP optimization	า	
Element size	275 mm	1.6 mm	40 mm
Nb tetrahedrons	179 167	382 148	345 441
Objective volume fraction f	4%	3%	3%
Nb iterations for convergence	23	21	30
Final compliance \tilde{C}	436 J	7.9 10 ⁻⁵ J	0.21 J
	Solid shape		
Relative density threshold ρ_{th}	0.2	0.25	0.22
Volume fraction <i>f</i> _{3D}	4.08%	3.2%	2.9%
Compliance C _{3D}	257 J	3.5 10 ⁻⁵ J	0.08 J
Maximum displacement δ_{3D}	4.6 10 ⁻¹ mm	3.6 10 ⁻³ mm	4.9 10 ⁻² mm
Ratio f_{3D}/f	102%	107%	96%
	Beam structure		
Volume fraction <i>f</i> _{beam}	3.9%	3.1%	2.8%
Compliance C _{beam}	288 J	4.1 10 ⁻⁵	0.11 J
Maximum displacement δ_{beam}	5.3 10 ⁻¹ mm	4.9 10 ⁻³ mm	9.4 10 ⁻² mm
Ratio f _{beam} /f	97%	103%	93%
Ratio C_{3D}/C_{beam}	89%	71%	72%
Ratio $\delta_{3D}/\delta_{beam}$	87%	73%	52%

the 3D structure created is more flexible than the optimized 3D model but the difference is acceptable (maximum displacement is $4.9 \ 10^{-3}$ mm versus $3.6 \ 10^{-3}$ mm for solid result). A synthesis of results obtained for the 3 examples presented in this paper is provided in Tab 1.

5.2.2. L bracket

In this case, dimensions of the initial model are $2\,\text{m}\times2\,\text{m}\times1$ m. It also is loaded with a downward vertical force (2000 N in the Y direction applied as a constant pressure) and null displacements are imposed at its upper side (see Fig. 12a). The optimized solid shape, curve skeleton derived before normalization and result of normalization are respectively provided in Fig. 12b, c and d. It can be seen that cross section radii vary from 30 mm to 53.2 mm. Resultant displacement distributions for the optimized solid shape and the beam structure created are finally illustrated in Fig. 12e and f using the same color scale and same amplification factor (1000 times) for comparison. In this third example, the 3D structure created is quite more flexible than the optimized 3D model (maximum displacement is $9.4 \, 10^{-2} \, \text{mm}$ versus $4.9 \, 10^{-2} \, \text{mm}$ for solid result). This is notably due to the fact that, even if the optimized solid result is symmetric, both skeletons (before and after normalization) are significantly nonsymmetric. This makes that one side of the L bracket created is stiffer than the other, that structure's stiffness is unbalanced and that resultant displacement distribution is not symmetric.

6. Conclusion and future work

Examples presented in the previous section show that the approach proposed can automatically generate 3D beam structures made of standard beams from TO results



Figure 12. L bracket (a) initial model with BCs (b) optimized solid shape (c) curve skeleton before normalization (d) distribution of cross section radii after normalization (e) (f) resultant displacement distribution for the optimized solid shape and beam structure created.

that tend towards beam-like structures. Results show that these beam structures are slightly more flexible than optimized solid shapes from which they are generated. However, several post-processing procedures can be foreseen to improve quality and stiffness of beam structures created. Constraining beam structures to symmetry, when the input TO problem is symmetric and extending the approach to various other types of beam cross sections are two obvious and short-term improvements. Exploring alternative techniques for the generation of curve skeletons and applying the approach to other types of TO methods are two other short-term research perspectives of our team. Fig. 13 illustrates two types of problems encountered with the method used for automatically generating curve skeletons from optimization results. In Fig. 13b inconsistency is related to a poorly



Figure 13. Problems with curve skeletons (a) side view of cantilever curve skeleton and nodes used for generating it (b) zoom on badly centered curve skeleton at location (c) zoom on the location of a junction that could be improved.

centered branch of the curve skeleton at this location while in Fig. 13b, the problem comes from the location of a junction point, which is not optimal for deriving the 3D structure afterwards.

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