



## Scallop Height of 5-axis Machining of Large Triangles with a Flat End Mill

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### ABSTRACT

In this paper, we study the scallop height when machining an STL file composed of large triangles with a flat end mill on a 5-axis machine. The tool can be placed flat on each triangular face using 5-axis tool positioning methods and machined without scallops. However, when moving across the edge between two triangles, the tool has to lift, which creates a scallop. In this paper, we perform simulations to compute this scallop height numerically as a flat end mill crosses the edge between two triangles.

**Keywords:** CNC machining, 5-axis machining, scallop height.

### 1. INTRODUCTION

STL is a common industry format for specifying surfaces as a set of triangles and is used for a variety of industrial applications including CNC machining [1]. Triangulated surfaces are used in graphics, surface modeling and even CAD. Many software packages (such as Polygonica) are built solely around triangulated representation. A review of recent CNC machining activity including advances in machining of STL surfaces can be found in a review by Lasemi et al. [6]. Traditionally parts defined using STL format are machined on 3-axis CNC machines using ball nosed or spherical end milling cutters. The cutters whose shape resembles a sphere cannot remove all the material from the raw stock to generate the net shape. The material that is left behind must be removed by polishing or other means. Since polishing and other methods of material removal are expensive, it is desirable to minimize the un-machined material left behind after the finishing passes. This un-machined material is called a scallop and is the focus of study in this paper.

To minimize scallops the distance between two parallel passes, side step distance, can be reduced. However, the reduction in side-step distance results in increased tool path length and machining time. Another method of reducing the scallop is the use of 5-axis machining. In 5-axis machining a radiused end mill, also called a toroidal end milling cutter, is inclined so that its cutting surface is close to the

desired surface. This proximity results in a wide strip about the point of contact that meets the user specified tolerance. Owing to the large width of the strip the side step distance can be large. The large side step distance results in shorter tool path lengths, but some of the gains in tool path length are lost to a slower machining speed owing to reduced rigidity of the 5-axis machine. This method applies to flat end mills as well. However, while scallop height of surfaces machined on 3-axis CNC machines is well studied, there are few studies of scallop heights for 5-axis machining of STL files. In this paper, we present such a study, and show that the scallops occur when the tool crosses an edge between adjacent triangles, and that the scallop heights depend on the direction of tool motion relative to the edge.

Traditionally two types of methods are used to study scallop heights. In the first method the edge cut by the moving tool is approximated by a circle or ellipse and projected onto a plane passing through the point of contact, the tool axis and the feed forward direction. This approximation is used extensively in industry to determine the size of the side step distance when planning tool paths for machining a part. The method applies to ball nosed end milling cutter but approximations for flat end milling cutters and radiused end milling cutters are known. The method is approximate at best but is simple to use. In the second method a uniform grid is defined, in a sub-domain of the part, to measure the scallop. Each point

of the grid has a vertical rays shooting from it. The vertical ray is intersected and cut, with the surface swept by the moving tool. Because of this intersection and cutting process this method is commonly called the mow-the-grass method [9]. The surface defined by the grid represents the machined surface and can be used to measure the scallops by comparing it with the desired net-shape. This method depends on the accuracy of the swept surface and the grid size [5].

In 3-axis machining the swept surfaces can be easily and accurately calculated. An accurate swept surface and a fine grid are required for accurate results. However it is only conducive for numerical implementation and cannot be used manually, and it has a few other drawbacks. In particular, the estimation of scallops in vertical or near vertical sections of the part is poor. In our work, we do not look at near vertical sections, and so because of the accuracy offered by the Mow-the-grass (z-map) method it is selected for study in this paper.

5-axis machining of curved surfaces has been well researched. The Principal Axis Method (PAM) uses the surface and properties of generalized toroidal tools to align their curvatures and determine a unique tool position and orientation [8]. This Multi-Point Method (MPM) uses the algebraic definition of surface to determine two points of contact at every tool position. Other methods, including the Rolling Ball Method [3] and the Arc-Intersect Method [4], are numerical implementations of the MPM method with some variations. All the above methods are based on mathematical definition of the surface. As the surfaces become complex the methods become tediously slow and unreliable. One solution to the problem is to deal with simpler surfaces. STL format offers a way to describe any surface as a set of non-intersecting triangles (they do meet at the edges) to any specified accuracy. A 5-axis tool path planning method for STL surfaces would be simpler, faster, robust and reliable. In the literature, one method called the "Drop and Tilt" method (DTM) is used for machining parts defined using STL format [2]. This method is described in Fig. 1. DTM uses a generalized toroidal tool for machining the part. In this method the footprint of the tool path is defined as a zig-zag or parallel set of lines. The foot print is discretized into small steps. At each step the tool is dropped along the z-axis (or other pre-defined direction) until it touches a triangle defining the surface. See A in Fig. 1. The tool in this position sits above the part without gouging it. At the contact point the insert of the toroidal cutter that touches the part is identified. The axis of this insert,  $\hat{u}_1$ , is used to rotate the tool until it touches a second triangle on the part surface. The position B shows the tilted tool in Fig. 1. The tool position and orientation at all the steps along the footprint define the tool path. The DTM is a gouge free tool positioning method that touches the STL surface at two points. A flat end-milling cutter can be used in DTM. The method is based on MPM and it is extrapolated that the scallops produced by DTM are

similar to MPM. This study is the first known study of the scallops produced by DTM.

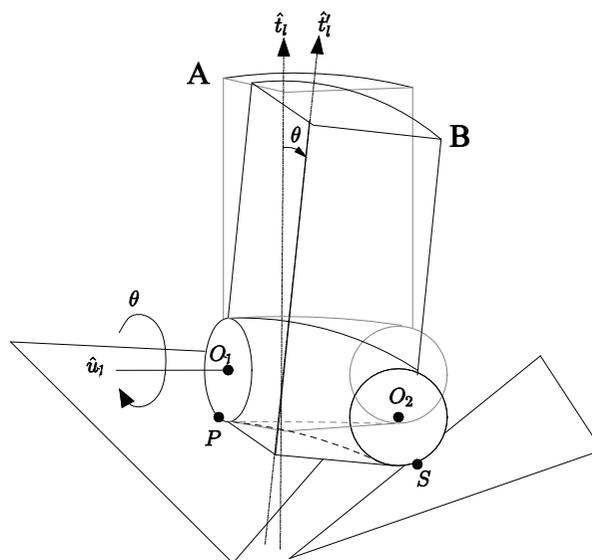


Fig. 1: Drop and Tilt Method: The tool drops along the line  $t_1$  until it touches the point P then it rotates about  $u$  until it touches the second triangle at S.

Scallop heights are the difference between the surface swept by the moving tool and the part definition. In 5-axis machining the tool, whether it is the toroidal end mill or the flat end mill, it moves and tilts as it goes from one tool position to the next. The surface machined by this moving tool is complex and is generated by a collection of curves called the imprint curve. An imprint curve [7] is a curve that lies on the surface of the tool (torus, sphere or cylinder) such that at that instant its velocity (including translation and rotation) is perpendicular to the surface normal. By connecting imprint curves generated in a time sequence the swept surface can be approximated. For flat end milling cutter the imprint curve is the circle described by the cutting corner. By connecting the circle at subsequent tool positions the swept surface is easily generated. In this paper this method of swept surface generation is used. This paper ignores the role of machine kinematics in the generation of the swept surface. This role can be minimized by decreasing the feed forward step or by increasing the discretization of the tool path footprint.

The machining process comprises of a number of steps. In the first step, the appropriate tool path footprint is selected for the given part. In this work the footprint is a series of parallel lines. In step two the tool path footprint is discretized into points separated by the feed forward distance. In step three the tool is dropped at the current point to determine the first point of contact. In step four the tool is rotated based on the DTM concept until it touches the STL surface at a second point of contact. Steps 3 and 4 are repeated until all the tool position and orienta-

tion has been determined for all points on the tool path.

At any tool position the tool is touching the part at two points. Typically these points will lie on two triangles. These triangles will typically be disjointed. The space between them is spanned by a number of triangles. The size, shape and geometry of the in between triangles depends on the part and is not a function of machining parameters such as the side step distance, feed forward distance, tool size, etc. The scallop depends on the two triangles touching the tool and the machining parameters: side step distance, feed forward distance, tool size and direction of tool motion. In reality the scallop will be cut short by other triangles spanning the two points of contact.

When studying scallops, the relative size of the triangles to the tool size has an effect on scallop size. Roughly speaking, there are triangles that are large, medium, or small relative to the tool size. A large triangle would be bigger than the tool diameter, potentially allowing us to move the tool across the face of the triangle with the tool lying flat on the face of the triangle. Medium sized triangles are roughly the size of the tool diameter, and small triangles are smaller than the tool diameter. For machining purposes, STL files are typically composed of many small triangles. However, in this paper, we will study the scallop height of 5-axis machining of large triangles. We do so based on two observations: first, in general the tool will be in contact with at most two triangles at a time regardless of triangle size, and second, when machining small triangles, the scallop heights will be determined by effectively cropping the scallops of “large” triangles, where the large triangles are expanded forms of the small triangles in contact with the tool.

When doing 5-axis machining of large triangles with a flat end mill, when we machine on the interior of a triangle, the tool axis can be oriented so that the tool lies flat on the face of the triangle, leaving no scallop as we move the triangle across the face. The question we address in this paper is what happens as the tool moves across an edge from one triangular face to another triangular face. To determine this, we performed some empirical simulations of a flat end mill as it moves across the edge between two faces along a number of directions, and used the simulations to determine the scallop height. As five-axis machining of triangulated surfaces is a topic that has only recently gained attention of the research community, there is no previous work in 5-axis machining that the presented results can be compared to. Although 3-axis scallop height estimation has been studied it has only historical value to this work.

## 2. SIMULATION DETAILS

Our simulations use a mow-the-grass (commonly known as z-map method) approach to compute the machined surface. We simulated the flat end tool as

a circular disk moving along the tool path. On a triangle interior, the circle will lie flat on the triangle. As we transition from one triangle to the next, the effective cutting edge becomes elliptical. For machining purposes, we need only concern ourselves with the front edge of these circles and ellipses, which act as imprint curves. Connecting these imprint curves between adjacent tool positions gives us a good approximation to the surface swept by the tool.

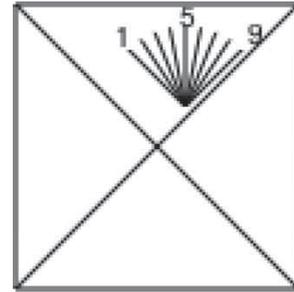


Fig. 2: The four triangles and the directions of tested tool motions, labeled 1-9.

The part being machined in our simulations is an inverted pyramid of size  $100\text{ mm} \times 100\text{ mm}$  with the internal vertex  $20\text{ mm}$  below the other four vertices, as shown in Fig. 2. An inverted pyramid as such has little interest in industry, but was selected it because when a triangulated part is machined, the tool at any given time will contact two points that likely lie on two different triangles. If all other triangles are ignored these triangles will meet along an edge. It is the travel of the tool across this edge that is of interest in this study. Such a part is incomplete and the inverted pyramid is a simple extension that can be represented as a solid. Although the part can be machined with alternate tool paths the study focuses on the point indicated in Fig. 2 with a fan of lines. A small rectangular region is considered around this point. The rectangular region is divided into a number of equally spaced parallel lines typically four. These lines are the footprint of the tool. They are discretized with a feed forward step of  $2\text{ mm}$ . The feed forward step could be smaller, but this results in a prohibitive computation time on the symbolic algebra system used for this work. At each point on these lines the flat end-milling cutter of varying radii ( $3$ ,  $5$  and  $7\text{ mm}$ s) is dropped until a first point of contact is obtained. The tool is then rotated based on DTM, until a second contact point is found. After the tool positions are determined at all the points, the imprint curves are generated and connected to one another to produce the surface swept by the tool as it moves along this tool path. This swept surface is shown superimposed on the part in Fig. 3a. Next the rectangular region is gridded and used for determining the surface left behind after the tool path is executed. The outcome of this

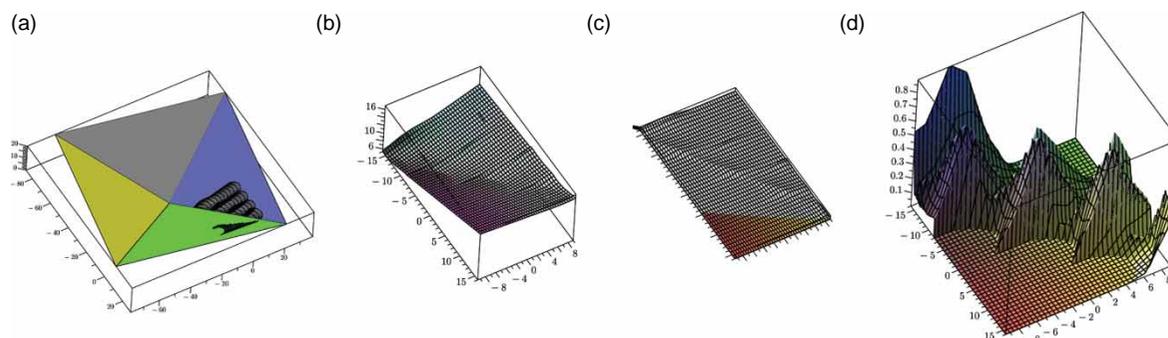


Fig. 3: (a) Illustration of tool path and the swept surface; (b) the results of Mow the grass algorithm applied to the swept surface; (c & d) the un-scaled and scaled scallop map.

step is shown in Fig. 3b. The machined surface is compared with the part geometry to determine the scallop height. The scallop height is much smaller than the part dimensions, so Fig. 3c shows the un-scaled version of the scallop height and the Fig. 3d shows the scaled version of the scallop height. The set shown in Fig. 3 was generated for three tools dimensioned 3, 5 and 7 mms with three varying step sizes of 6, 8 and 11 mms. The feed forward distance was maintained at 2 mm. In this paper only one simulation result is shown for conciseness.

### 3. SIMULATIONS

As a first analysis the maximum scallop height encountered by a 7 mm radius flat end mill is considered. The scallop maps generated above are analyzed. The maximum scallop height for varying side-step distances of 6, 8 and 11 mm were manually extracted for each of the 9 directions shown in Fig. 2. The results are plotted in Fig. 4. The horizontal axis of the plot refers to the tool path direction. Each direction shown in Fig. 2 is labeled from 1 to 9. The maximum scallop height for each of the cases is plotted. The red graph represents the maximum scallop for a tool path with 8 mm side step, the blue represents the maximum scallop height for 11 mm side step and the green represents the maximum scallop height for 6 mm side step. The scallop height for direction 9 can vary greatly depending on where the tool path is located relative to the edge. It is possible to optimally locate the tool path so that it cuts the material on one side of the edge completely. Even in this case if the side step is fixed to one of the values mentioned above a scallop will exist on the neighboring triangle.

The second analysis is based on a study of the tool path direction 1 as indicated in Fig. 2. In this case the tool path is perpendicular to the edge. The behavior of the tool must be studied closely to understand the scallop formation process. This process is shown in Fig. 5. The tool approaching the edge is shown in the left sketch. As it crosses the edge it begins to rise at a rate dependent on the slope of the

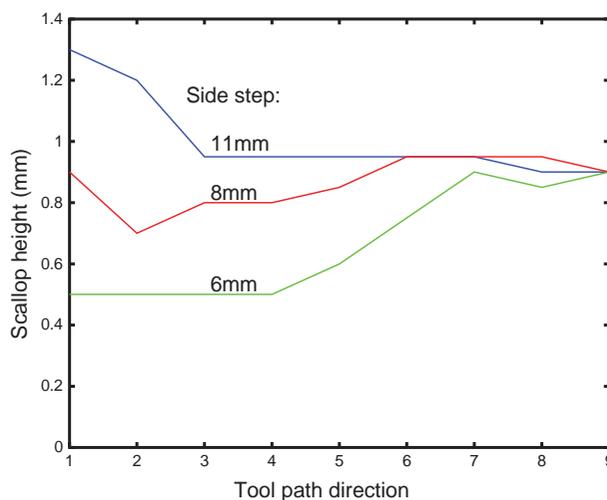


Fig. 4: Maximum scallop height for three different side step distances for a tool radius of 7 mm in nine different directions.

neighboring triangle relative to the first triangle. This rise results in a zero scallop at the leading edge of the tool. However, at other points along the periphery un-machined material is left below the tool. This un-machined material is the cause of the scallop. In the example discussed above, for the 7 mm radius flat end mill, the scallop resulting from this phenomena increase with side step. This result is as expected and is shown in Fig. 4.

The third analysis is for feed directions labeled 2 to 8. The scallop production method is a mixture of the two concepts discussed above. For direction 2 to 5 the lifting of the tool as it crosses the edge plays a stronger role whereas for directions 5 to 8 the tool straddling the edge with the points of contact on either side plays a stronger role. For the directions 2 to 5, let us consider a plane passing through the edge under consideration and equally inclined to the two neighboring triangle. We developed a tool path for the tool crossing the edge along the directions from 2 to 5. For each tool position we project the circular cutting edge onto the plane. The cutting edge for



Fig. 5: As the tool slides from one triangle to the next a scallop is left behind as it crosses the edge separating the two triangles.

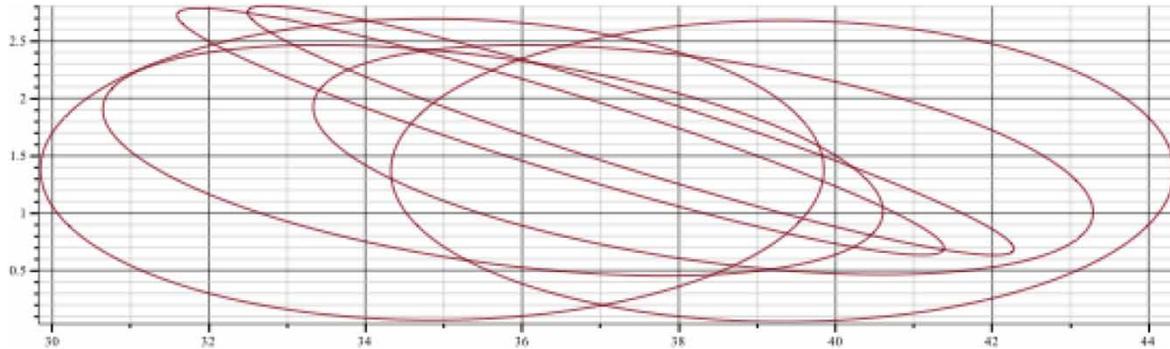


Fig. 6: The projection of the circular cutting edge on the bisecting plane. Note the intersection between the first and last ellipse. Both axes are in mm.

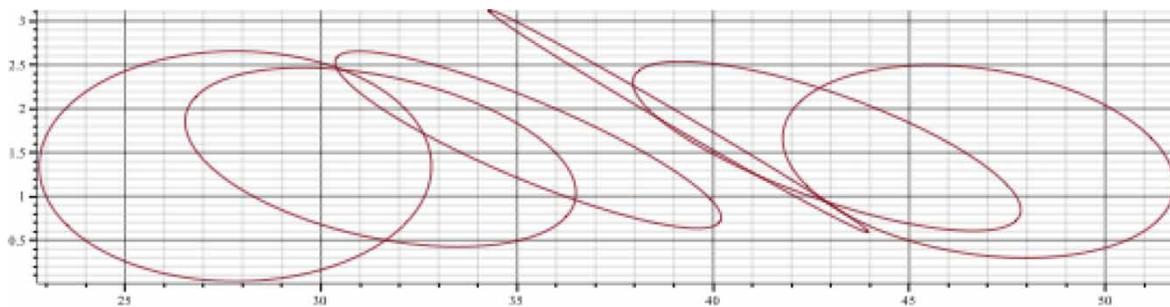


Fig. 7: The projection of the circular cutting edge onto the bisecting plane. Note the first and last ellipses do not intersect. Both axes are in mm.

the tool completely in triangle one and completely in triangle two intersect with one another as shown in Fig. 6. The projections of the tool along the intermediate tool positions show that these positions clean the top of the scallop. The maximum scallop height produced increases from 2 to 5. For the directions 5 to 8, we developed a tool path for the tool crossing the edge along the directions from 2 to 5. For each tool position we project the circular cutting edge onto the plane, as shown in Fig. 7. The cutting edge for the tool completely in triangle one and completely in triangle two do not intersect with one another. The scallop is produced as the tool straddles the edge. The maximum scallop height produced increases from 2 to 5.

To position the tool, we assume a zigzag tool path, and at each position along this tool path we drop the tool along the  $z$ -axis to find a first point of contact with the tool surface, and then tilt the tool to find a second point of contact (we discussed the dependence of the scallop height on the method of tool positioning in Section 1).

The fourth analysis is based on a study of tool size. For three different tool radii tool paths in the nine directions shown in Fig. 2 were generated. The maximum scallop heights were manually observed from mow the grass results similar to Fig. 3. These results are plotted in Fig. 8. The horizontal axis depicts the tool direction as a number 1 to 9. The vertical axis shows the scallop height. While the results in the center (directions 3-8) are predictable, the results at either ends need further discussion. Near the left end, the tool crosses the edge in a direction perpendicular to the edge. In this case, the cutting edge can be divided into two sections. Section 1 surrounds the contact point on either side. Section 2 lies on the outer peripheries of the tool. If the side step distance is small compared to the tool radius, then the scallops are produced by section 1. If the side step is large, then the maximum scallops are produced by section 2. In Fig. 8, for the 7 mm tool, the side step distance is 8 mm, which is small compared to the tool diameter, and the scallop is formed solely by section 1 of the tool. For the 5 mm tool, the side step distance is

again 8mm, which is large compared to the tool diameter, and the scallop height is formed by section 2 (the outer periphery of the tool) and is high. For the 3 mm tool, the side step distance is 5 mm, which is small compared to the tool diameter, and the scallop is formed solely by section 1. This explains why the maximum scallop height graphs for 3 mm, 5 mm, and 7 mm tools are not in order near the left end.

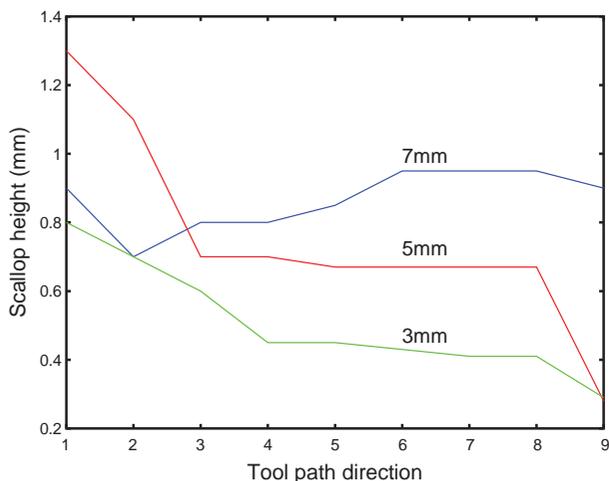


Fig. 8: Maximum scallop height for three different tool sizes in nine different directions. Side step size is 8 mm except for 3 mm tool, which has a 5 mm step size.

#### 4. CONCLUSIONS AND OBSERVATIONS

From our studies, we observed several things about the scallop height generated when a flat end milling cutter traverses across an edge between two triangles. First, although the tool is in contact with one or both triangles at all times, there is still a scallop. Further, of all the tool directions (relative to the edge), the perpendicular direction results in the greatest scallop height.

Second, the scallops are formed from two sections of the tool: an inner section and a pair of outer sections near the periphery of the tool. If the scallop is formed solely from the inner section of the tool, then the scallop is significantly smaller than if it is formed from both sections of the tool. This suggests using a small enough side step distance to ensure the scallops are formed solely from the inner section of the tool.

We note that if there is sufficient clearance, then a cleanup pass may be done by machining so that the tool is tangent to an edge while lying flat on a triangular face. This will remove the scallops along the edges, although there will be additional complexities at the triangle vertices. Further, two passes are needed to fully remove the scallops, with each pass orienting the tool flat along one of the triangles adjacent to the edge. These cleanup passes only make sense for

large triangles, and are likely not relevant to typical STL files.

We have begun looking at using the projection of the circular edges onto the bisection plane passing through the common edge can be used to estimate the scallops with reasonable accuracy. Initial observations indicate that these projections can be used to estimate the scallop heights with reasonable accuracy, eliminating the need for computationally expensive simulation.

Intuition suggests that during the machining of surfaces described with large triangles the tool should cross the edge along a perpendicular direction. However, when you cross an edge in the perpendicular direction the two points of contact lie along the tool path direction, thereby losing the benefits of Multi-Point Machining. This paper shows that the ideal direction to cross the edge is between 30 and 80 degrees. In Fig. 8 the scallops produced by the tool moving in directions 5 to 8 is reasonably flat suggesting that if the edge is crossed by the tool traveling in any of these directions the scallops produced are of the same height. The scallops increase in height as the tool directions goes down to 0. A direction larger than 80 degrees is unpredictable and depends on the tool path footprint relative to the edge. However, as the direction angle increases the scallops are smeared over a larger area.

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