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Wavelet Based Surface Decomposition with Boundary Continuity

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ABSTRACT

For the traditional wavelet based surface decomposition, it is difficult to preserve the boundary continuity with adjacent surfaces. Aiming at this problem, this paper proposes a new algorithm for wavelet based surface decomposition while preserving the boundary continuity. First, a B-spline surface is divided into a boundary part and a non-boundary part, and then the non-boundary one is decomposed into a scale part and a wavelet part by wavelet transform. Finally, the T-spline is utilized to reconstruct the boundary one and the scale one. Compared to the traditional wavelet based surface decomposition method, in the new approach the result surface is a T-spline which can preserve the boundary continuity with adjacent surfaces. Several examples and results are given to demonstrate the effectiveness of this approach.

Keywords: wavelet transform, B-spline surface, boundary continuity, CAGD.

1. INTRODUCTION

B-spline surfaces are a powerful mathematical representation of freeform surfaces and also play an important role in computer-aided design (CAD) and computer aided manufacturing (CAM), especially in the area of industrial design such as ships, aircrafts and cars [4]. In addition, surface decomposition operations are often used in CAD systems to get surface simplification and fairing.

B-splines that provide a unified geometry representation of conic sections and freeform shapes have been widely used in CAD and CAM. However, for B-spline surfaces, all control points must lie topologically in a rectangular mesh resulting in failing to allow local refinement. To solve this problem, T-splines that are defined on a T-mesh and allow T-junctions in the control mesh were proposed [6,7] (see Fig. 1). A T-junction is a control point which terminates a partial row or column in the control mesh. Unlike B-spline surfaces, T-splines allow local refinement, and have good properties in surface merging and model simplification [6,7].

Wavelet analysis is a relatively new mathematical tool, which was introduced in the 1980s, and widely applied in the field of signal processing, computational geometry, and data compression due to its filtering property. In recent years, wavelet technology has been applied into the field of Computer Aided Geometric Design (CAGD) [5]. Later, B-spline wavelets were proposed and the relevant decomposition and reconstruction algorithms were also given for curves and surfaces, especially in model simplification and fairing [1,2,8]. In the field of CAGD, B-spline wavelets are also used in the editing of curves and surfaces [3,9]. The surface wavelet decomposition aims to eliminate undesired features from the surface's shape. For the traditional method, an original surface can be decomposed into a scale part and a wavelet part by wavelet transform. However, there is a problem for the surface wavelet decomposition, that is the original surface and its scale part cannot preserve the boundary continuity. In the early research, Cho proposed a method for fairing the surfaces while preserving the boundary continuity [1]. However, this method can not preserve the continuity with adjacent surfaces via wavelet transform directly. The boundary problem has affected the surface wavelet decomposition for many years and is still an open research area. Aiming at this problem, in this paper we will discuss how to preserve the boundary continuity for wavelet based surface decomposition.

This paper presents a new decomposition algorithm for B-spline surfaces with boundary continuity. First, a B-spline surface is divided into a boundary part and a non-boundary part. Then the non-boundary one is decomposed into a scale part and a wavelet part. At last, the T-spline is utilized to reconstruct the boundary one and the scale one. In the new decomposition approach, the result surface is a T-spline which can preserve the boundary continuity.

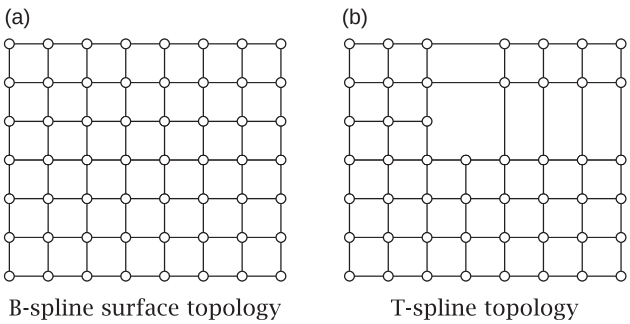

Fig. 1: B-spline surface and T-spline.

This paper is organized as follows. First, B-spline wavelets are reviewed in Section 2. T-splines are then described in Section 3. Section 4 presents a new decomposition algorithm for B-spline surfaces with boundary continuity. The effectiveness of this new algorithm is then demonstrated in Section 5. For simplicity, this paper focuses on bi-cubic B-spline and T-spline surfaces, although the concepts can be generalized to arbitrary degree.

2. B-SPLINE SURFACE WAVELETS

A B-spline surface can be represented as

$$S(u,v) = \sum_{i=0}^{m}\sum_{j=0}^{n} C_{i,j}B_i(u)B_j(v) \tag{1}$$

where $C_{i,j}$ are control points, and $B_i(u)$and $B_j(v)$ are the B-spline basis functions defined on the knot vectors U and V, respectively. For the B-spline surface, all control points lie topologically in a rectangular mesh. Then, Eq. (1) can also be rewritten as a matrix representation

$$S(u,v) = \phi_u(u)\mathbf{C}[\phi_v(v)]^T \tag{2}$$

where $\phi_u(u) = [B_0(u), B_1(u), \cdots, B_m(u)]$, $\phi_v(v) = [B_0(v), B_1(v), \cdots, B_n(v)]$, and $\mathbf{C}$ is a matrix consisting of control points $C_{i,j}$.

According to wavelet transform for curves, B-spline basis functions $\phi_u(u)$ can be represented by two-part basis functions $[\phi'_u(u), \psi'_u(u)]$, which are the scale part and the wavelet part, respectively. Therefore, it will consists of four-part basis functions for surface wavelet transform through the multiplication of two vectors $[\phi'_u(u), \psi'_u(u)]$ and $[\phi'_v(v), \psi'_v(v)]$, which can be described as

$$\begin{bmatrix}\phi'_u(u)\\ \psi'_u(u)\end{bmatrix}[\phi'_v(v), \psi'_v(v)] = \begin{bmatrix}\phi'_u(u)\phi'_v(v) & \phi'_u(u)\psi'_v(v)\\ \psi'_u(u)\phi'_v(v) & \psi'_u(u)\psi'_v(v)\end{bmatrix} \tag{3}$$

Corresponding to the four parts in Eq. (3), the control points of the surface by wavelet transform can be represented as

$$\begin{bmatrix}\mathbf{C}_{\phi'_u(u)\phi'_v(v)} & \mathbf{C}_{\phi'_u(u)\psi'_v(v)}\\ \mathbf{C}_{\psi'_u(u)\phi'_v(v)} & \mathbf{C}_{\psi'_u(u)\psi'_v(v)}\end{bmatrix} \tag{4}$$

The procedure for decomposing the initial control points into four-part ones which contain $\mathbf{C}_{\phi'_u(u)\phi'_v(v)}$, $\mathbf{C}_{\phi'_u(u)\psi'_v(v)}$, $\mathbf{C}_{\psi'_u(u)\phi'_v(v)}$ and $\mathbf{C}_{\psi'_u(u)\psi'_v(v)}$ is called surface wavelet decomposition (see Fig. 2).

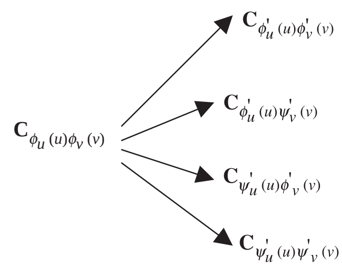

Fig. 2: Surface wavelet decomposition.

where $\mathbf{C}_{\phi'_u(u)\phi'_v(v)}$ is corresponding to the scale part; $\mathbf{C}_{\phi'_u(u)\psi'_v(v)}$, $\mathbf{C}_{\psi'_u(u)\phi'_v(v)}$ and $\mathbf{C}_{\psi'_u(u)\psi'_v(v)}$ are corresponding to the wavelet parts.

Similarly, the procedure for recovering the initial control points from the four-part ones is called surface wavelet reconstruction (see Fig. 3).

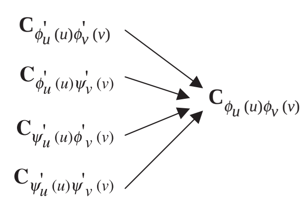

Fig. 3: Surface wavelet reconstruction.

3. T-SPLINE SURFACES

The explicit formula for a T-spline surface can be expressed as [6]

$$S(u,v) = \sum_{i=0}^{n} C_i T_i(u,v) \tag{5}$$

where $\{C_i\}$ are control points, and $\{T_i(u,v)\}$ are blending functions. $T_i(u,v) = N[\mathbf{u}_i](u)N[\mathbf{v}_i](v)$, $N[\mathbf{u}_i](u)$ and $N[\mathbf{v}_i](v)$ are B-spline basis functions defined by the local knot vectors $\mathbf{u}_i = [u_0, u_1, u_2, u_3, u_4]$ and $\mathbf{v}_i =$

$[v_0, v_1, v_2, v_3, v_4]$, respectively. The local knot vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ for each blending function are inferred from a T-mesh according to some rules [7]. The topological relations of the T-spline control points are also provided by the T-mesh, and the parametric length of each edge in the T-mesh is represented by the knot interval [7]. Based on whether the blending functions can provide a partition of unity, T-splines can be divided into three types: standard, semi-standard and non-standard [6,7]. A standard T-spline is one for which

$$\sum_{i=0}^{n} T_i(u, v) = 1 \tag{6}$$

A semi-standard T-spline is one for which

$$\sum_{i=0}^{n} \omega_i T_i(u, v) = 1 \tag{7}$$

where not all $\omega_i = 1$. A non-standard T-spline is one for which there does not exist a set of weights to satisfy Eq. (7). In this paper we mainly focus on decomposing a B-spline surface into a T-spline surface. B-spline surfaces are a special case of T-splines in which the T-mesh does not contain T-junctions. According to the T-spline classification, it also can be seen that the B-spline surfaces belong to a special case of standard T-splines. Their relation is given as follows (see Fig. 4).

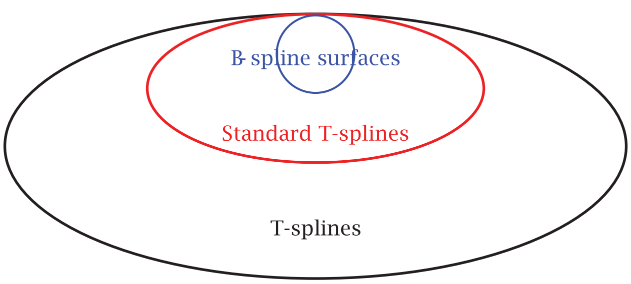

Fig. 4: Relation of B-spline surfaces and T-splines.

4. WAVELET BASED SURFACE DECOMPOSITION WITH BOUNDARY CONTINUITY

The algorithm is summarized into the following three steps.

(Step 1) According to the corresponding conditions for certain boundary continuity, the original surface is divided into two parts: a boundary part and a non-boundary part; (For example, Fig. 5.a shows an initial B-spline surface, where the red control points in 5.b are the corresponding boundary part with C0 boundary continuity, and the white ones in Fig. 5.b are the corresponding non-boundary part. For C1 boundary continuity, the corresponding boundary part is the red control points as shown in Fig. 5.c.)

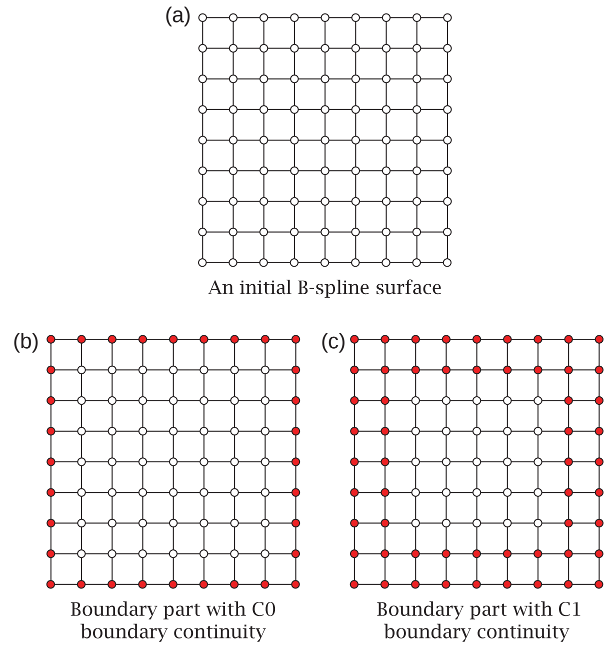

Fig. 5: Surface decomposition with boundary continuity.

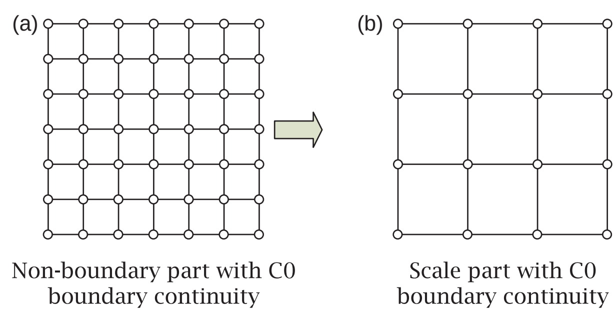

Fig. 6: Wavelet decomposition of the non-boundary part.

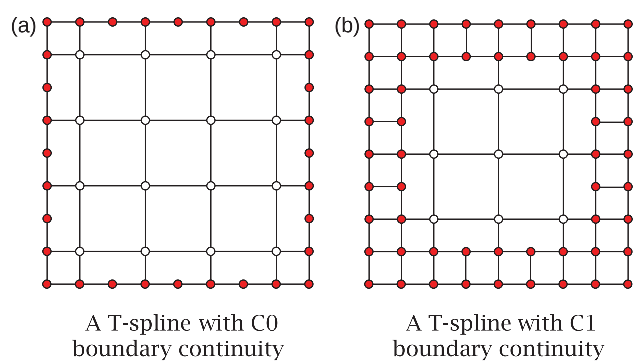

Fig. 7: Surface reconstruction with boundary continuity.

(Step 2) Decompose the non-boundary part of the initial surface into a scale part and a wavelet part; (For example, the non-boundary part (see Fig. 5.b) is decomposed into a scale part (see Fig. 6) and a wavelet part)

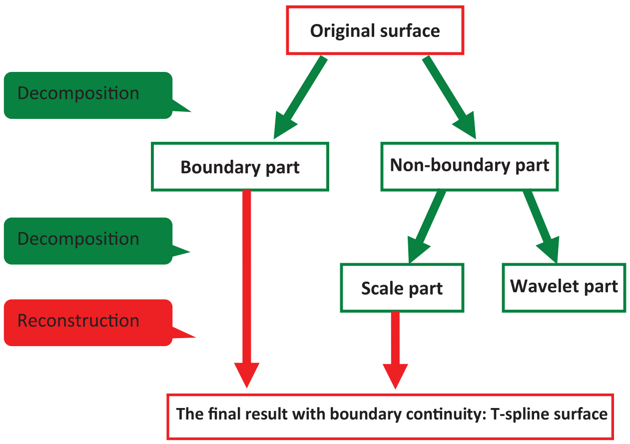

Fig. 8: The new decomposition algorithm.

(Step 3) Reconstruct a new surface by the scale part and the boundary part. (For example, the boundary part (see Fig. 5.b) and the scale part (see Fig. 6.b) can be reconstructed by a T-spline surface (see Fig. 7.a), which can preserve the C0 boundary continuity with the original surface.)

The final result of the reconstructed surface in Step 3 is a T-spline surface, and the boundary continuity with adjacent surfaces is automatically preserved in this algorithm. The main advantage of the new algorithm is that it preserves the boundary continuity which can not be preserved in the traditional surface wavelet decomposition.

Discussion: In our surface decomposition algorithm, the T-spline is used to preserve the boundary continuity. First select the scale part, the wavelet part and the boundary part corresponding to the original surface, then reconstruct a new surface by synthesizing the scale part and the non-boundary part (see Fig. 8). Compared to the result obtained from the traditional surface wavelet decomposition method, our algorithm could preserve the boundary features by reconstructing a T-spline.

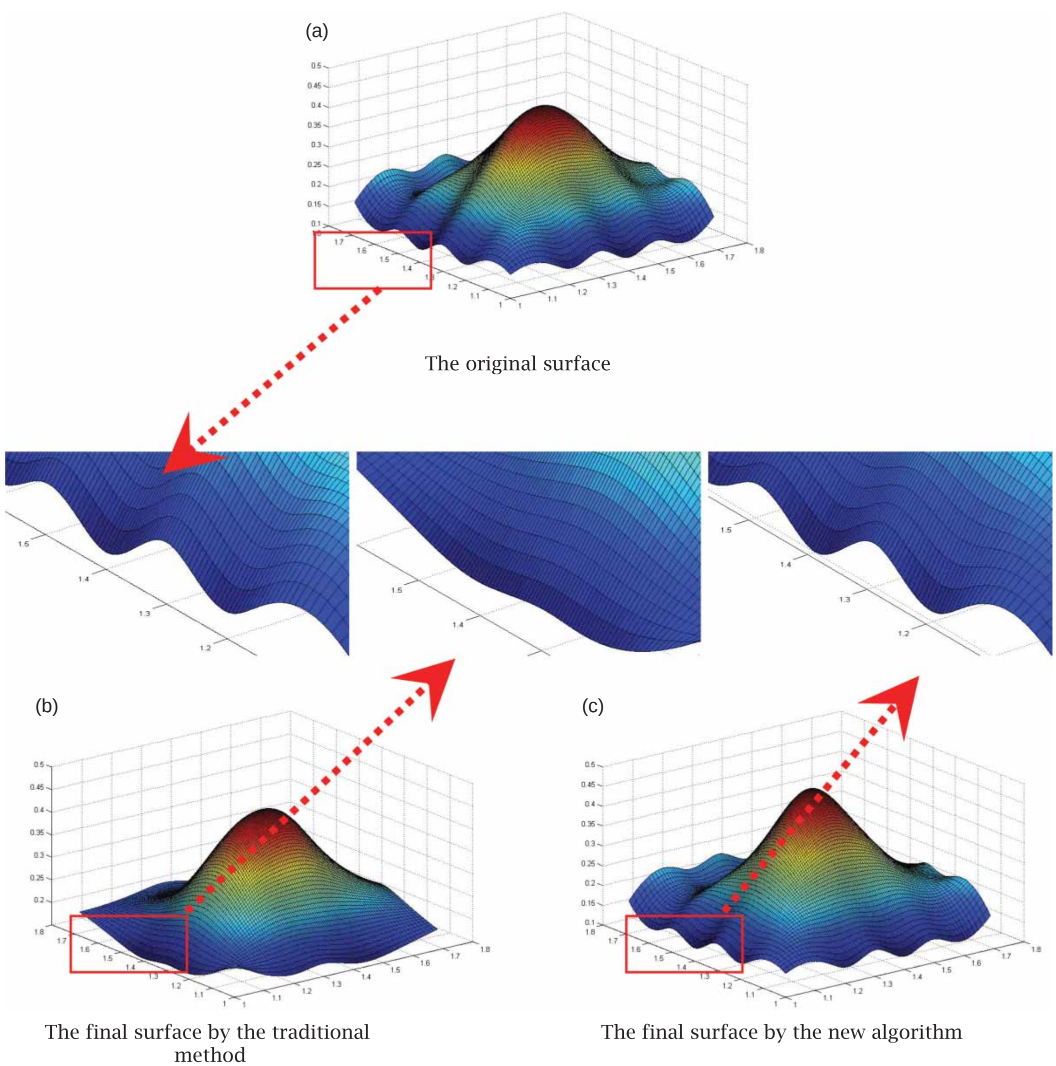

Fig. 9: An example of a hat surface.

5. EXAMPLES

In this section, numerical experiment results are presented to examine the performance of the new algorithm presented in Section 4. The given example is a surface from a fashionable women hat (see Fig. 9(a)). The original hat surface is a B-spline surface with a wave boundary, which enhance the visual feeling and the aesthetic perception of appearance design (see 9(a)). After the traditional wavelet decomposition [2], the wave boundary part does not appear in the scale part and the continuity at the shared boundary is not preserved (see Fig. 9(b)). But using the new algorithm, the new reconstructed surface has the wave boundary part, and is C0-continuous at the shared boundary with the original surface (see Fig. 9(c)).

The results were computed on a PC with configurations of Intel(R) Pentium(R) D CPU 3.40 GHz and 2GB main memory (3.39 GHz). The new algorithm is more efficient than the traditional wavelet decomposition method because the calculation in decomposition is less time-consuming and less control points are decomposed. However, our approach could preserve all the geometrical features of the boundary part compared to the traditional method.

6. CONCLUSIONS AND FUTURE WORK

This paper describes a novel wavelet based decomposition algorithm for B-spline surfaces with boundary continuity. First, three parts are obtained from a surface, that is the scale part, the wavelet part and the boundary part; then a T-spline for reconstructing the scale part and the boundary part is utilized to preserve the boundary continuity in the new surface decomposition approach. Our algorithm is accurate, automatic, efficient, and can be extensively applied to any B-spline surface model in computer aided design and computation geometry.

The boundary problem plays an important role in the processing of geometrical models. In our future work, the new method could be employed in surface fairing and simplification under the constraint of boundary continuity.

ACKNOWLEDGMENTS

This work was supported by the Natural Science Foundation of China (Project No. 61170198).

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