



## High-Quality Shape Fitting for Aesthetic Modeling Based on Class A Condition

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### ABSTRACT

In this paper, we propose a method of obtaining high-quality shapes from point clouds. We adopted the “Class A” concept, which has been defined as a shape that is monotonic in its curvature. Such high-quality shapes are preferred in product styling; therefore, an efficient method for their generation will be helpful in the style design process. The proposed method is based on a subdivision surface technique, the Class A definition, and a genetic algorithm. The proposed method is tested on some examples of surface generation, and the results demonstrate the effectiveness of our approach.

**Keywords:** high quality shape modeling, aesthetic design, subdivision, class A.

### 1. INTRODUCTION

The significance of aesthetic aspects in product design has grown rapidly. To survive severe global competition, designers have been urged to generate original and appealing shapes wherever possible. However, the uniqueness of shapes is not necessarily the only interest in industrial design; the quality of shapes also plays an important role in the exterior design of products. Therefore, uniqueness should be harmonized with shape quality even in the early stage of design because it becomes increasingly difficult to modify the shape in later design stages. However, support tools for designers to achieve not only unique but also high-quality shapes have not been sufficiently available. Thus, in this paper, we propose a novel design framework based on the “Class A” concept. Moreover, an application system for reverse engineering situations was constructed and its effectiveness was demonstrated.

There have been some related studies in which the generation of aesthetic shapes has been the main target. Higashi et al. [4] proposed a method of generating smooth surfaces by controlling the variation of curvature. Then, Mineur et al. [6] presented an important definition of a typical curve, which was defined as a form of a planar Bézier curve whose curvature is monotonic. Farin [2] proposed the concept of Class A Bézier curves. The term “Class A surface” had previously meant a high-quality surface that is required

for the outer parts of automobiles. Farin redefined this term as a general form for typical curves and presented mathematical conditions for a 3D Bézier curve to be a Class A curve. Oya et al. [7] presented a design method based on this Class A concept. They constructed a design system in which the Class A matrix was decomposed by singular value decomposition (SVD) and each decomposition parameter was used as a design parameter in the system input. Using this system, shapes can be created intuitively with guaranteed high quality.

In this study, we focus on reverse engineering, which is nowadays a very important process in product design. In a typical reverse engineering scheme, a 3D scanning instrument acquires point data, then appropriate data processing is conducted on CAD software. This process is not a creative activity and considerable time is usually required to recover the original shape. Furthermore, the recovered shape is not necessarily the desired shape even if the noise is completely removed because the modeler creates the model, which is usually a clay mockup, rather than the designers. An interpretation gap generally exists between the shape created by a modeler and the shape intended by a designer. Therefore, extra modification is required to obtain the genuinely desired shape.

The proposed method is based on fitting using a subdivision surface technique, the Class A definition,

and a genetic algorithm (GA). First, we prepare point clouds to be interpolated. Then, the fitting scheme based on subdivision surface is performed so that its limit surface passes through the points smoothly. In this phase, we adopt the Catmull-Clark subdivision scheme because of its rectangular domain, which is preferable in CAD representation, in which quadrilateral surface patches are used. The final output of fitting using subdivision surface is a set of control points of parametric surfaces such as B-spline and Bézier patches. However, unfortunately, the obtained smooth surface is not necessarily the desired one for the reason explained above. In other words, the recovered surface resembles very closely the ideal shape. A new algorithm was constructed to deal with this problem. The Class A definition is applied in the form of conditions on the Bézier control points so that the Bézier curve or surface is monotonic in its curvature. Monotonicity in curvature is a very important concept in this study because it is preferable in industrial design and we desire it in the target shape. It is assumed that designers desire shapes whose curvature is monotonic as an essential part of their designs. Thus, in the next process in our system, we attempt to obtain appropriate Class A matrices by using a genetic algorithm. Here, a Class A matrix is a matrix that determines each control point and satisfies the condition for monotonicity in curvature. The genetic algorithm is applied to modify the components of matrices so that the entire control point network becomes that of a shape whose curvature is monotonic. To avoid excessive modification, a constraint is imposed on the recovered shape so that it remains similar to the scanned shape. This is achieved by evaluating the distance between the original shape and the ideal shape. A designer can choose the preferred shape from various candidates; therefore, the intended high-quality shapes are always obtained without the inefficient manipulation of control points. A schematic illustration of the proposed algorithm is shown in Fig. 1.

## 2. CLASS A THEORY

### 2.1. Fundamentals

First, the condition defining Class A Bézier curves defined by Farin [2] used in our system is briefly described. Here, Class A Bézier curves are curves that are monotonic in both curvature and torsion. The following parametric form can express a 2D or 3D Bézier curve:

$$p(t) = \sum_{i=0}^n b_i B_i^n(t), \tag{1}$$

where  $p(t)$  is a point on the curve,  $b_i$  are the control points, and  $B_i^n(t)$  are Bernstein polynomials. By differentiating this equation with respect to the parameter

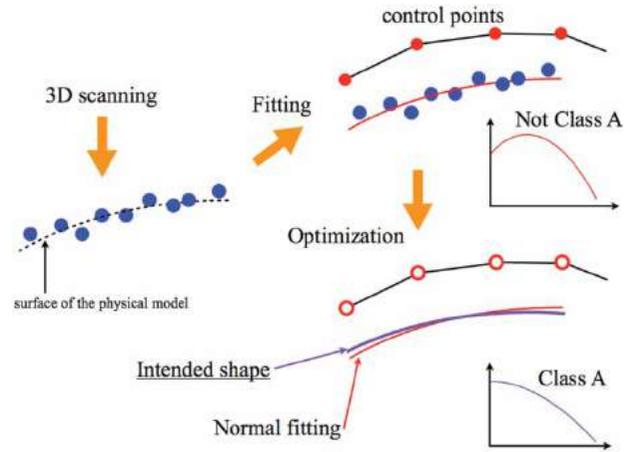


Fig. 1: Schematic illustration of proposed Class A fitting process.

$t$ , we obtain

$$\dot{p}(t) = n \sum_{i=0}^{n-1} \Delta b_i B_i^{n-1}(t), \tag{2}$$

where  $\Delta b_i = b_{i+1} - b_i$  are the forward difference vectors of the Bézier control points  $b_i$ . The derivative of a Bézier curve is also called a hodograph. The forward difference vector is expressed using an arbitrary unit vector  $v$  and a  $3 \times 3$  matrix  $M$

$$\Delta b_i = v_i = M^i v, \quad (i = 0, \dots, n-1) \tag{3}$$

For the unit vector  $v$  and the matrix  $M$ , the following conditions hold:

$$v^T [M^T + M - 2I] v \geq 0 \quad \text{and} \quad v^T [M^T M - I] v \geq 0, \tag{4}$$

$$\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 1 \quad \text{and} \quad \sigma_3^2 \geq \sigma_1, \tag{5}$$

where  $\sigma_i$  ( $i = 1, 2, 3$ ) are the singular values of the matrix  $M$ . Equation (4) describes a geometrical condition for  $v$  and  $M$ ; namely, the angle between  $v$  and  $Mv$  must be less than  $90^\circ$ . The Bézier curve produced by the matrix  $M$  that satisfies the above conditions is a Class A curve; in other words, this curve is monotonic in curvature and torsion. Therefore, a matrix satisfying the Class A condition is called a Class A matrix. In [2], it is stated that a Class A Bézier curve can be generated by a matrix  $M$  that represents a rotation around a particular axis by an angle  $\alpha \leq 90^\circ$  and a scale factor  $s$  when the condition

$$\cos \alpha > \frac{1}{s} \tag{6}$$

holds. In this case, both equations (4) and (5) are satisfied.

### 2.2. Class A B-spline

In the Catmull-Clark subdivision scheme [1], for a regular mesh, its limit surface coincides with a cubic uniform B-spline surface. Thus, its network curves are uniform B-spline curves. In our approach, the Catmull-Clark subdivision surface scheme is adopted as a fitting method owing to its quadrilateral geometry; thus, the Class A technique should be extended to B-splines. By converting the Bézier control points to B-spline control points, we deal with Class A B-spline curves. As shown in Fig. 2, Bézier control points ( $p_0, p_1, p_2, p_3$ ) are converted to B-spline control points ( $q_0, q_1, q_2, q_3$ ) by the following relations.

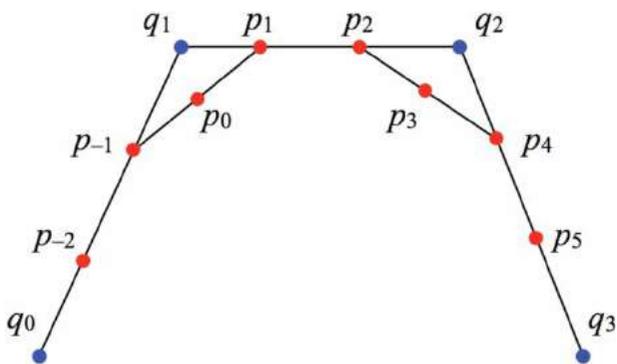


Fig. 2: Control point conversion rule between B-spline and Bézier curves.

$$\begin{aligned}
 q_0 &= p_{-1} - 3(q_1 - p_{-1}), & p_{-1} &= p_0 - (p_1 - p_0) \\
 q_1 &= p_1 - (p_2 - p_1), & q_2 &= p_2 + (p_2 - p_1) \\
 q_3 &= p_4 + 3(p_4 - q_2), & p_4 &= p_3 + (p_3 - p_2)
 \end{aligned}
 \tag{7}$$

Using these conversion relations, it is not necessary to change the definition of Class A Bézier curves.

### 2.3. Class A Surface

Similar to the previous subsection, a Class A B-spline surface is constructed from a Class A Bézier surface. Thus, a method of creating a Class A Bézier surface is described. For simplicity, we start from a translational surface [3] with no twists. The twist of a surface is its mixed partial. Therefore, when all sub-quadrilaterals are parallelograms, the twist vanishes everywhere. Such surfaces are called translational surfaces, which are adopted in this study. Namely, Class A curves are placed in the  $u$ - and  $v$ -directions to form a network of parallelograms. Although the surface shape is constrained, we can avoid considering extra conditions for the twist (2nd partial derivative on each control point). If there are nonzero twists, the monotonic curvature of the surface will be violated.

From the curves in the  $u$ - and  $v$ - directions in Fig. 3, we have

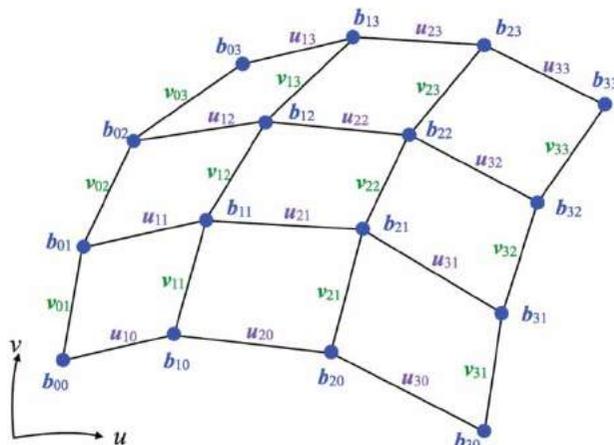


Fig. 3: Control points for Bézier surface.

$$\begin{aligned}
 u_{10} &= u, & u_{20} &= M_u u, & u_{30} &= M_u^2 u \\
 v_{01} &= v, & v_{02} &= M_v v, & v_{03} &= M_v^2 v,
 \end{aligned}
 \tag{8}$$

where  $u, v, M_u,$  and  $M_v$  are the initial vectors and Class A matrices in the  $u$ - and  $v$ -directions, respectively. These relations were introduced based on the definition of Class A curve to generate Class A surface. In the diagonal direction of the surface, the following relations determine each control point.

$$\begin{aligned}
 b_{11} &= b_{00} + u + v \\
 b_{22} &= b_{00} + u + v + M_u u + M_v v \\
 b_{33} &= b_{00} + u + v + M_u u + M_v v + M_u^2 u + M_v^2 v
 \end{aligned}
 \tag{9}$$

Then,  $u_{ij}$  and  $v_{ij}$  are calculated using the following equations.

$$\begin{aligned}
 u_{1j} &= u, & u_{2j} &= M_u u, & u_{3j} &= M_u^2 u \\
 v_{i1} &= v, & v_{i2} &= M_v v, & v_{i3} &= M_v^2 v
 \end{aligned}
 \tag{10}$$

Using these relations, a Class A Bézier surface can be constructed. The control points of four Class A Bézier curves in the  $u$ -direction are converted to those of a B-spline, then the same procedure is subsequently carried out in the  $v$ -direction; in this way, a Class A B-spline can be realized.

### 3. PROPOSED METHOD

Fig. 4 shows a flow chart of the proposed method. The proposed method consists of 3D scanning and point collection, deciding the patch domain, calculation of the B-spline control points, and the Class A modification process. In this section, the early stages of the

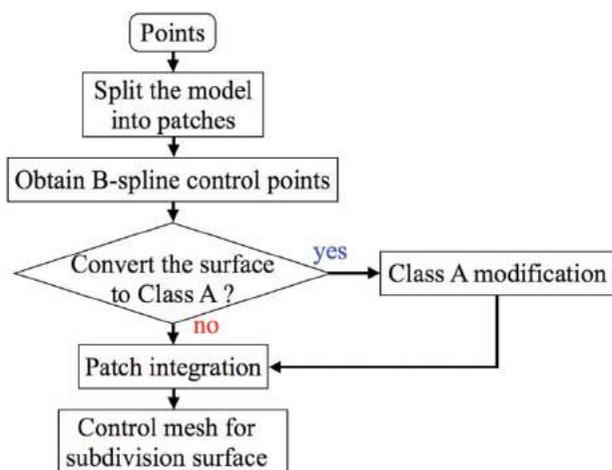


Fig. 4: Flowchart of proposed method.

fitting are briefly explained, and the Class A modification is described in detail. Note that, since this is a fundamental study, the surface patch is assumed to be regular, namely, its valence is four.

### 3.1. Early Stage of Proposed Method

In the proposed technique, point clouds are first divided into arbitrary surface patches. Thereby, the domain in which a designer wishes to generate a Class A surface can be extracted. In this approach, each patch is a tensor product surface represented by a B-spline surface. Specifying four boundary curves to determine the surface parameters results in the generation of a base surface, then, Ma's method [5] is adopted to calculate B-spline control points from the base surface. The base surface is an intermediate surface generated during the Ma's algorithm.

### 3.2. Modification to Class A surface using Genetic Algorithm

The surface generated by the control points as explained above will be a very accurate representation of the scanned object. However, this surface is not necessarily the designer's desired shape because of

scanning noise, subtle warps caused by the handwork of the modeler, and the interpretation gap between the stylist and the 3D CAD modeler and sculptor. Therefore, in this study, a designated domain is modified to form a Class A surface that is assumed to be the designer's ideal shape. We organized this problem as an optimization problem and adopted a genetic algorithm to generate the Class A surface. Although the obtained control points are those of a B-spline surface, it is necessary to convert them into Bézier control points. However, the control points acquired by the previously explained approach are identical to a Bézier surface because of knot multiplication; thus, no subsequent procedure is required.

We adopted a genetic algorithm as the optimization procedure because the objective function is not explicitly obtained and is expected to be a complex-shaped function. As a precondition, we used cubic Bézier curves in the  $u$ - and  $v$ -directions on the surface. Moreover, both terminal points,  $b_0$  and  $b_3$ , coincide with the measured points. Thus, the other control points of the curve,  $b_1$  and  $b_2$ , are determined by the initial vector  $v$  as follows:

$$v = (I + M + M^2)^{-1}(b_3 - b_0), \quad (12)$$

where the matrix  $M$  consists of a scale factor  $s$  and a rotation of angle  $\alpha$ .

The treatment with the GA in this study is next described in detail. In our GA coding, the chromosomes comprise two scale factors and the rotation angles in the  $u$ - and  $v$ -directions on a surface, namely,  $(s_u, s_v, \alpha_u, \alpha_v)$ . These parameters are used to obtain the matrix  $M$  that constructs the control polygon of the Bézier curves. For the first generation, 30 individuals were generated, and random values were assigned to each chromosome within a designated range. The range for the scale factors is  $1 < s \leq 2$ , and that for the rotation angles is  $0 < \alpha < \pi/2$ . To evaluate the individual, the fitness of the chromosomes is determined by the following function:

$$f = \sum_i (S_i - S'_i)^2, \quad (13)$$

where  $S_i$  is the original surface and  $S'_i$  denotes the surface after modification to a Class A surface.

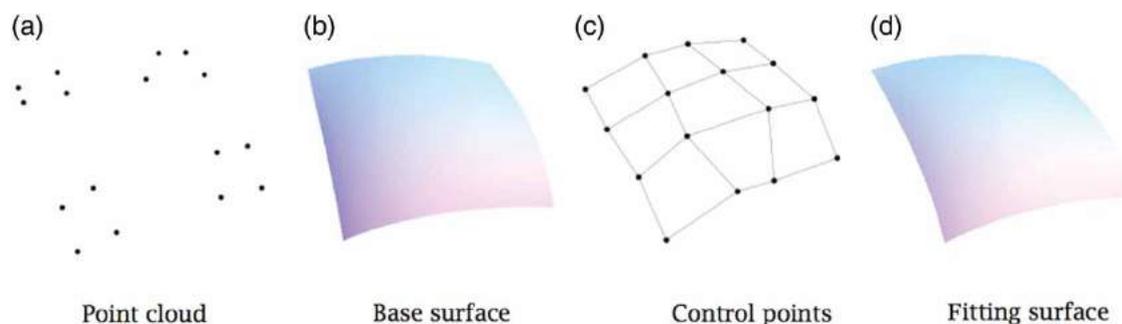


Fig. 5: Process for patch 1.

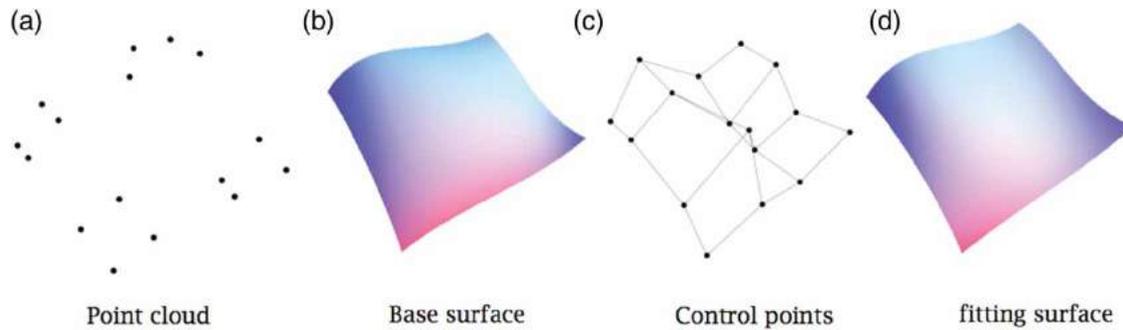


Fig. 6: Process for patch 2.

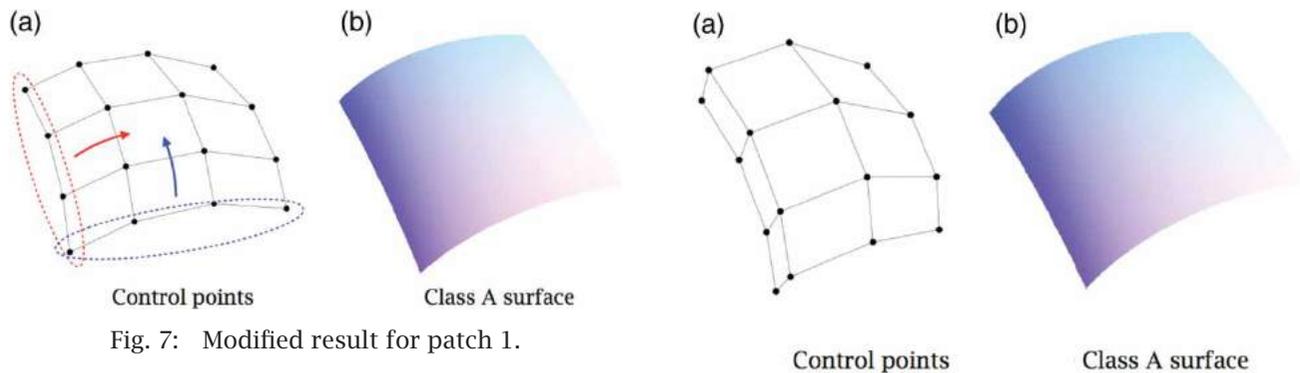


Fig. 7: Modified result for patch 1.

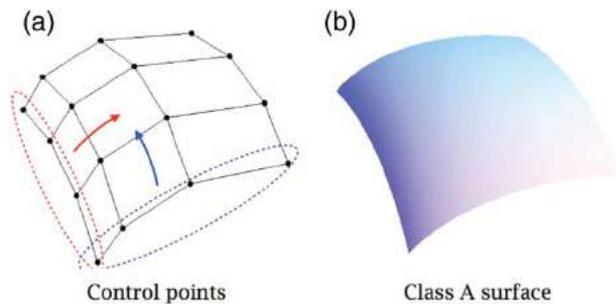


Fig. 8: Modified result for patch 2.

Fig. 9: Another result (B-spline) for patch 1. The control points are those of uniform cubic B-spline.

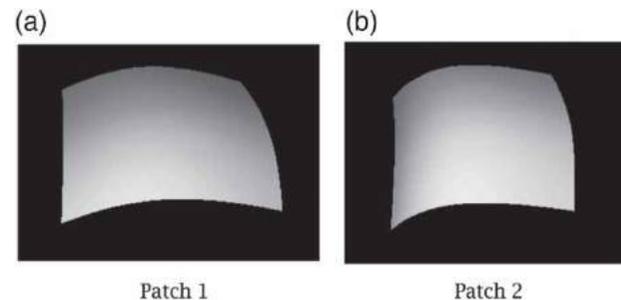


Fig. 10: Resultant surface of subdivision for two models.

Namely, this fitness function is a squared distance of the intended Class A surface from the original data points. Therefore, minimizing the fitness  $f$  means that the ideal surface is found by selecting new control points near the original points. The original surface is constructed from a physical model that is almost precise but possibly with tiny undulations in the curvature profile. It is assumed that designers wish to erase such local undulations so that the original shape is matched as closely as possible.

For the selection rule, the tournament rule is adopted. Two individuals are extracted at random and the better one is selected in terms of the fitness. Repeating this process, two individuals  $A$  and  $B$  are obtained. Next, a crossover is performed by applying the following rules:

$$C_1 = wA + (1 - w)B \quad (14)$$

$$C_2 = (1 - w)A + wB, \quad (15)$$

where  $w$  is a random value between  $1/3$  and  $2/3$ . Two offspring are selected, and the better one survives. In the mutation process, the gene of  $C$  is changed at a rate of  $p_c$ . When the fitness of  $C$  is higher than the average value for the population,  $p_c$  is 0.005. When the fitness of  $C$  is less than the average fitness,  $p_c$  is set to 0.5. The algorithm used to obtain Class A surface control points is summarized as follows.

1. Generate the initial population.
2. Carry out the following loop 250 times. (In our trial, 250 loops were sufficient.)
3. Selection: two individuals are chosen by the tournament rule.

4. Crossover: perform crossover on the chosen two individuals.
5. Mutation: a gene is randomly changed on the basis of the mutation rate.
6. Replacement: replace the worst individual.
7. Loop ends.
8. The best individual with the highest fitness in the final generation is chosen as the optimal solution.

#### 4. EXAMPLES

In this section, examples are shown to demonstrate the effectiveness of the presented method. First, an application to a single patch is shown, then the result for a mouse model is presented. These results are analyzed in terms of the variation of curvature.

##### 4.1. Single-patch Applications

From the point data, base surfaces are generated, which are used to produce B-spline surfaces. Figs. 5 (a) and 6 (a) shows the point clouds for different patch models, namely, patch 1 and patch 2, the results for which are also shown in Figs. 5 and 6, respectively. The base surfaces are shown in Figs. 5 (b) and 6 (b), the control points used for surface subdivision are

shown in Figs. 5 (c) and 6 (c), and the fitting surfaces are shown in Figs. 5 (d) and 6 (d). The final surfaces closely fit the initial points; however, they are not necessarily monotonic in curvature.

These surfaces are modified to Class A surfaces by the proposed method, as shown in Figs. 7 and 8. In Figs. 7 (a) and 8 (a), the modified control points are shown, and in Figs. 7 (b) and 8 (b), the resultant surfaces are shown. The marked control points in these figures are the initial points, where the two boundary curves start. As can be seen in these networks of control points, compared to those of Figs. 5 and 6, irregularities are eliminated and the location of all points are arranged to form Class A surfaces.

Then, each obtained Class A Bézier surface is converted into a Class A B-spline surface. This procedure is conducted by simply calculating the control points of a uniform cubic B-spline from the Bézier surface points. The result for patch model 1 is shown in Fig. 9. Naturally, the result coincides with Fig. 7 (a). Finally, the subdivided surface is generated from these control points, as is shown in Fig. 10.

##### 4.2. Surface Analyses

The surface quality is analyzed to evaluate the proposed method. The modified B-spline surface is

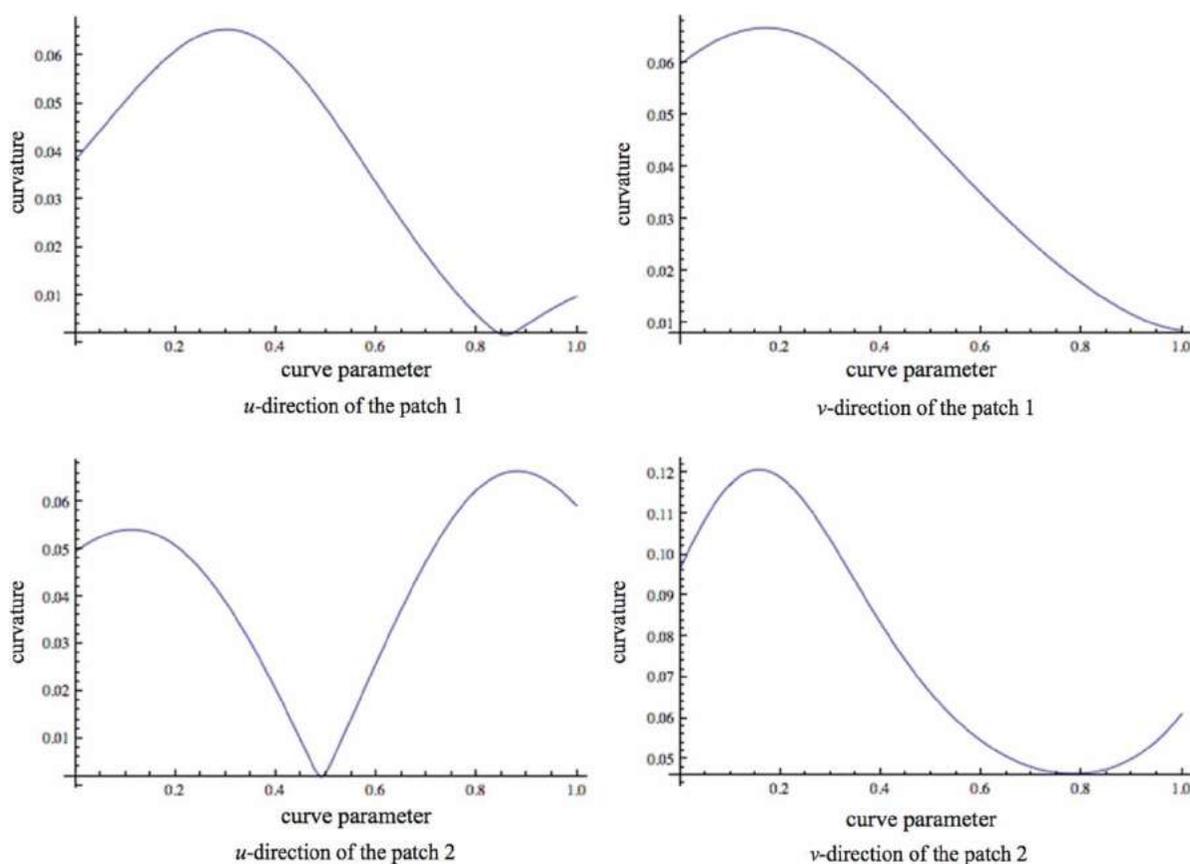


Fig. 11: Curvature profiles of two models before modification.

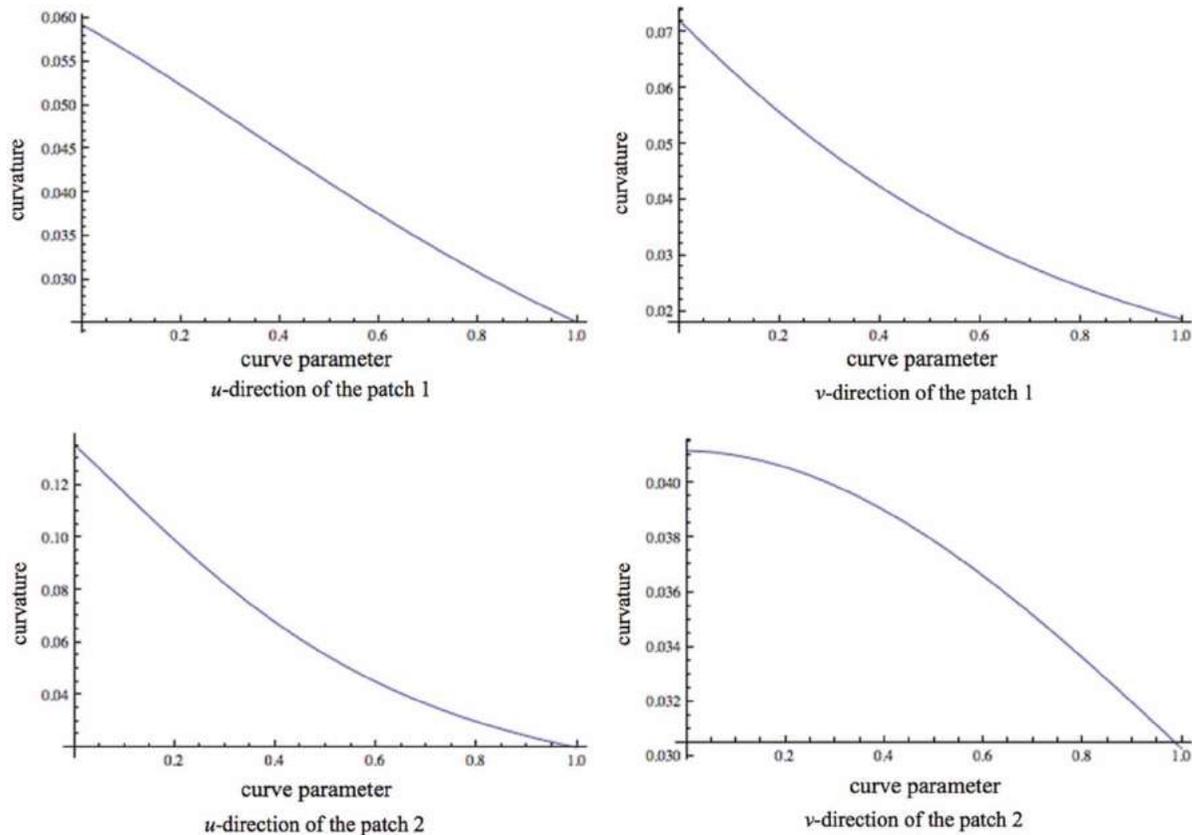


Fig. 12: Curvature profiles of two models after modification.

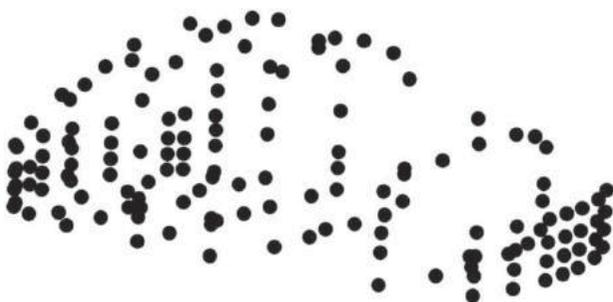


Fig. 13: Pseudo measured points for mouse model.

analyzed for this purpose. The curvature profile is applied to all network curves on the surface to test for monotonicity. Two isoparametric curves,  $u = 0.5$  and  $v = 0.5$ , are used for this test. In Fig. 11, the curvature profiles for the original patch 1 and patch 2 are shown. There are inflection points in both figures, which means that curvature monotonicity is broken and the surface quality is very low. On the other hand, in Fig. 12, the curvature profiles of the modified surfaces are improved and exhibit curvature monotonicity. Although we obtained good results for modification, there are some limitations to the proposed method. That is, in this study, a parallelogram control point network generates the surface;

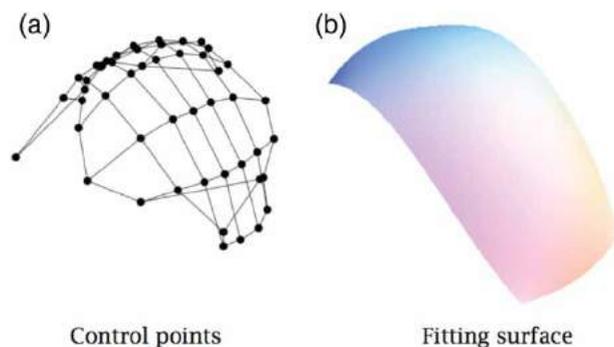


Fig. 14: Control points and fitting surface of the upper surface of mouse model before modification.

this reduces the freedom of surface generation. However, this limitation originates from the immaturity of Class A surface theory; thus, further progress in this research field is required.

#### 4.3. Application to Mouse Model

As a practical application, a computer mouse model was used. Fig. 13 shows the pseudo-measurement point set that was created for this test. First, the point data was split into 16 domains, and B-spline surfaces were generated for these domains. Then,

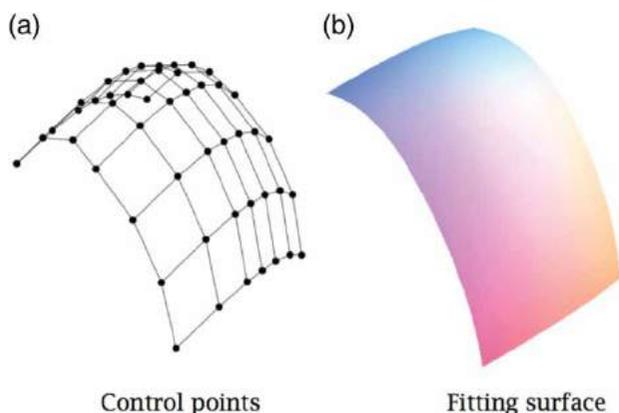


Fig. 15: Control points and fitting surface of the upper surface of mouse after modification.

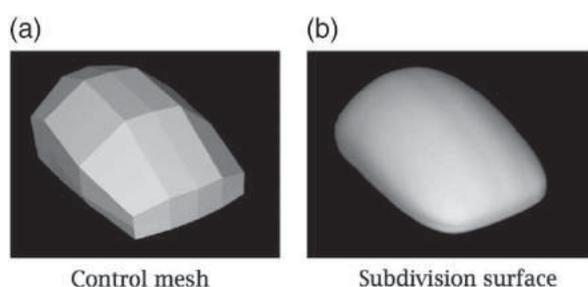


Fig. 16: Control mesh and subdivision surface of obtained Class A model.

the Class A modification process was conducted for each patch. In this case, two boundary curves were located at the center of the top of the mouse model. This was required to eliminate the gaps between the Class A patches. Next, each patch was connected so as to produce a smooth surface. Although the surface almost fitted the points, the control points exhibit a large undulation, as shown in Fig. 14. In contrast, as shown in Fig. 15, the modified control points are located at regular intervals and the surface appears smooth. This surface is close to the measured points and is expected to have monotonic curvature. Fig. 16 shows the control mesh and the subdivision surface of the modified case. As can be seen in the surface figure, there are faint slots at the boundary of each domain. This means that patch connection did not work well, and some unnatural surface discontinuity was introduced. It is presumed that this is because we connected the patches after the Class A modification of each domain, in which the variation

of curvature differs. A theory for Class A B-spline surfaces and their connection techniques have not yet been developed and should be considered in the future.

## 5. CONCLUSION

In this paper, we proposed a method of obtaining Class A shapes from point clouds. Such shapes have been defined as having monotonic curvature, which is preferred in the styling of high-quality shapes. The modification of the digital data acquired in the reverse engineering process requires considerable wasted effort. The proposed method is expected to assist stylists in creative activity by eliminating the need for inefficient digital data manipulation. Our method is based on a subdivision surface technique, a Class A surface, a Class A B-spline, and a genetic algorithm. Some examples that demonstrate the effectiveness of the method are shown. However, there are apparent limitations to this method for more practical use; therefore, the problem of Class A patch connection should be considered as future work.

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