Data Compression Method for Trimmed Surfaces Based on Surface Fitting with Maintaining G^1 Continuity with Adjacent Surfaces

Yuta MURAKI[1], Katsutsugu MATSUYAMA[2], Kouichi KONNO[3] and Yoshimasa TOKUYAMA[4]

[1]Iwate University, muraki@iwate-u.ac.jp
[2]Iwate University, matsuyama@eecs.iwate-u.ac.jp
[3]Iwate University, konno@eecs.iwate-u.ac.jp
[4]Tokyo Polytechnic University, tokuyama@mega.t-kougei.ac.jp

ABSTRACT

Trimmed surfaces are one of the versatile shape representations. When 3D data is put into circulation on the Internet, data size reduction for trimmed surfaces is an important issue in digital engineering. In this paper, we propose a data compression method based on surface fitting method. Our method enables efficient surface data compression by using the surface fitting method. In our method, the surface fitting method is applied to a closed region surrounded by the boundary edges of a 3D shape model. With the approximation method that uses boundary curves and sample points based on a tangent plane at the boundary edges, the surface fitting method can also be applied to shapes with holes or concave shapes. In order to maintain G^1 continuity with adjacent surfaces, the cross boundary derivatives obtained by the surface interpolation method are applied to the connection section with adjacent surfaces. When a surface is generated in good accuracy, the original surface element can be deleted and data size can be drastically reduced. The deleted surface element can be reconstructed from the boundary edges using our method.

Keywords: trimmed surface, data compression, G^1 continuity, notch shape.
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1 INTRODUCTION

3D CAD systems are used in a lot of fields such as prototyping, analysis, evaluation and product brochures. As 3D CAD systems become popular, accurate precision is required for 3D shape models, so that the data will be complicated and data size will increase. Gigantic data size is a big issue for data exchange of 3D shape models on the Internet. As the data size increases, it takes longer to transfer the data, and this will disturb circulation of 3D shape models. Due to this, data compression of 3D shape models is an important issue.

A lot of researches have been done for compressing 3D shape models, while there have been few researches for surface compression and most of the old compression methods are based on compression for polygon meshes. One of the compression methods uses subdivision surfaces [4]. In this method, division of a polyhedron is repeated regularly to generate a smooth surface, and the continuity between the surfaces of the generated 3D shape model is guaranteed. In addition, if the number of subdivision is controlled, the model is also used as LOD (Level of Detail). If, however, the accuracy required in CAD systems is satisfied, the number of polygons will become huge and so the data size will be huge. Therefore, it is necessary to consider the compression method for surface data.

Wakita et al. have developed a compression method for surface data[12]. They proposed XVL (eXtended VRML with Lattice) that enables transmission of lightweight 3D data with good quality by changing surface data into an original surface expression called lattice structure. The lattice structure is built with surface data and outlined polygon data, and surface data is expressed by Gregory patches[3]. In XVL, it is possible to generate a free-form surface from outlined polygon data, so that free-form surface data is not necessary upon data transfer. Compared with the case where a polygon mesh with the same quality as surface data is transmitted, the transmission time becomes extremely shorter.

The surface fitting method that Wakita et al. have adopted is the surface interpolation method [11] using Gregory patches. The surface interpolation method applies two or more surface patches to a closed region surrounded by boundary edges. In an arbitrary N-sided region (N Í 4), internal curves are generated based on Catmull-Clark subdivision method [2] to divide the region into N four-sided regions. Each of the four-sided regions is interpolated with a Gregory patch. The surface interpolation method fits surfaces with considering the continuity with adjacent surfaces. Therefore, the surfaces on the four-sided regions will be connected with G^1 continuity. Catmull-Clark subdivision method, however, divides a closed region depending on the form of the boundary edges. Due to this, for concave shapes shown with blue lines in Fig. 1 (b) and (c), the internal curves will be generated to go outside the regions as shown with the red lines. Moreover, expected surfaces cannot be applied to a shape with a hole since it is difficult to divide the region. CAD data generated by 3D CAD systems is often expressed with trimmed surfaces [5] and it includes a lot of notches such as holes or concave sections. In other words, it is difficult to compress trimmed surfaces by using the method of Wakita et al. that uses surface interpolation method. Then, another surface fitting method is required for trimmed surfaces with holes or concave shapes.

In this paper, first, we propose a surface fitting method that can be applied to trimmed surfaces with considering the continuity with adjacent surfaces. After that, the proposed surface fitting method is used to examine compression of 3D surface shape data. The surface fitting method used in our compression method can generate free-form surfaces from boundary edge information for the shapes with holes or concave shapes. Therefore, when 3D shape data is transmitted, the surface information is not sent but only the corresponding prediction function is sent, and the 3D shape will be reconstructed according to the received prediction function. This enables data compression that guarantees the accuracy of original shapes.

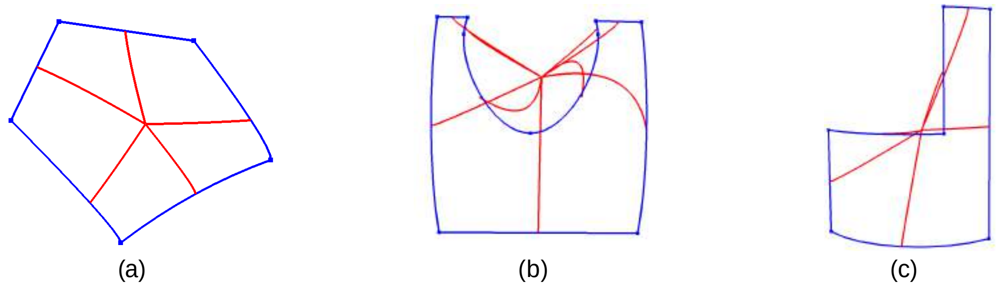


Fig. 1: Division of regions by Catmull-Clark subdivision method: Division curves overflow to the outside of closed regions of concave shapes as shown in (b) and (c). As a result, the generated surfaces will also be protruded outside the regions.

2 FRAMEWORK OF DATA COMPESSION METHOD

The method proposed in this paper compresses the data of free-form surface models and its framework is composed of data compression part and data extension part, as shown in Fig. 2. In data compression part, surface elements are alternatively deleted from 3D shape data to reduce data size. When surface elements are deleted, the prediction function for the deleted surface is saved as attribute information. First, the surface fitting method proposed in this paper is applied to a closed region representing each surface of the 3D shape data. After that, errors are evaluated for the generated surface. When it is judged that the surface can be fitted with good accuracy, the corresponding surface element will be deleted. The details of the surface fitting method and error evaluation method are described in section 4. In data extension part, the received data is used to extend the surface data. A surface is fitted by using boundary edges and the prediction function of the attribute information, and the 3D shape model will be reconstructed. A surface will be extended within the accuracy of preset tolerance and connected with adjacent surfaces with G^1 continuity.

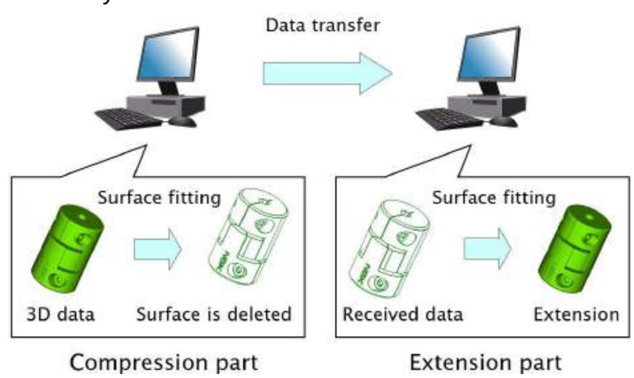


Fig. 2: Concept of surface compression method.

3 DATA COMPRESSION AND EXTENSION

This section describes compression part and extension part of our data compression method. Fig.3 shows the flow of surface data compression part. The surface fitting method used in this paper cannot be applied to a case where a rectangle is connected with all the surrounding four surfaces with G^1

continuity. On the other hand, the surface interpolation method can generate a surface, without being influenced by the connection relation with an adjacent surface if it is a convex shape that does not include any holes. Due to this, it is more desirable to apply the surface interpolation method in as many cases as possible, if the surface fitting method is considered from the viewpoint of the continuity between surfaces. Under this proposition, the surface interpolation method is applied to the closed region first, and the error between the generated surface and the original one is measured. If a surface can be generated with sufficient accuracy, the surface elements are deleted. If a surface with a large error is generated, the surface fitting method described in section 4 is applied. After that, the error between the generated surface and the original one is measured, and if a surface can be applied with sufficient accuracy, the surface elements are deleted. The method of error evaluation is described in section 4.4. With the algorithm mentioned above, the data size of 3D shapes is reduced. When surface elements are deleted, the prediction function for the deleted surface is saved as attribute information. This relates a closed region with the information of the method that should be applied, surface interpolation method or surface fitting method. Processing of Fig. 3 is performed to all the surfaces of a 3D shape model.

In data extension part, a surface element is extended using the boundary edges and attribute information received upon data transfer. The flow of surface element extension is shown in Fig. 4. The surface element is extended by applying the prediction function saved as an attribute to the boundary edges. In our method, a trimmed surface can be generated so that the boundary edges lie on the surface within the tolerance. The surface element deleted in compression part is imported from the attribute information and the prediction function is applied to the element for restoring the 3D shape model.

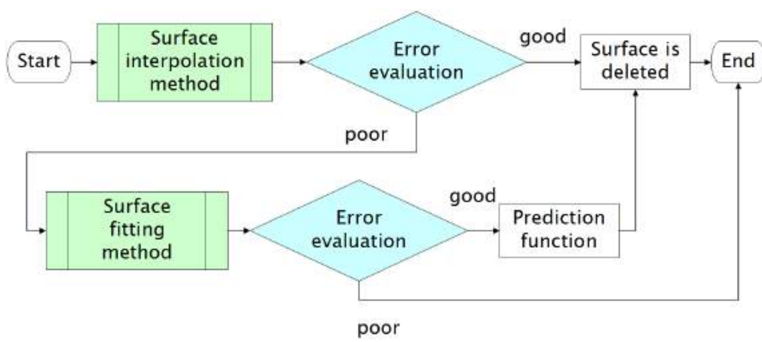


Fig. 3: Flow chart of surface fitting in compression part.

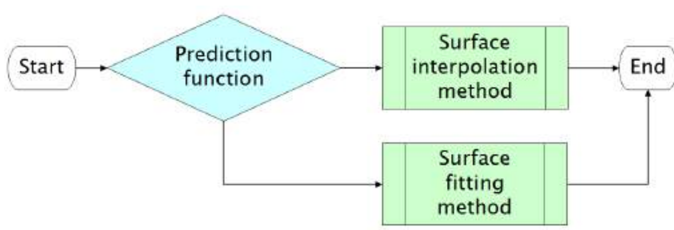


Fig. 4: Flow chart of surface generation in extension part.

4 SURFACE FITTING METHOD

In our method, the surface fitting method with input of boundary edges is used for prediction coding upon compression. If a surface can be restored in sufficient accuracy, the surface elements are deleted. We use the method [11] based on N-side filling [10] for surface fitting.

As described in section 1, the surface interpolation method can generate G^1 continuous surfaces with adjacent ones. This method, however, is difficult to apply to shapes containing holes or concave shapes. Since CAD data contains a lot of trimmed surfaces with holes or concave shapes, this difficulty becomes a big issue.

In contrast, the method based on N-side filling that the authors proposed in [8] is applicable to trimmed surfaces that contain shapes with holes or concave shapes. The method [8] generates sample points on the boundary curves and a point cloud based on a tangent plane outside a closed region and a B-spline surface is generated from the sample points using the least-squares method. The method [8], however, applies a surface to each closed region and the connection with adjacent surfaces is not considered, so two surfaces are connected with C^0-continuity. Due to this, if the method [8] is included in the data compression method, each surface will be extended in good accuracy, although two adjacent surfaces are discontinuous on their boundary in 3D shape model. Discontinuity with adjacent surfaces extremely reduces the quality of 3D shape data. The authors extended the method [8] and proposed the surface fitting method in which G^1 continuity with adjacent surfaces was considered [9].

CAD data contains a lot of shapes with holes or concave shapes. As shown with red circles in Fig. 5, shapes with holes existing on the boundaries of two surfaces connecting with G^1 continuity appear frequently. Since the method [9] is inapplicable to such shapes, it is insufficient to raise the compression ratio. So, in this paper, we extend the method [9] and aim at improvement in the compression ratio to enable application to the shapes shown in Fig. 5.

The surface fitting method proposed in this paper unites the advantages of the surface interpolation method [11] and the N-side filling method [10]. The concept of our method is shown in Fig. 6. This figure shows surface fitting when surface F has adjacent surfaces F_1 and F_2 on its right and left sides connecting with G^1 continuity and there is no surface on its upper and lower sides, shown in (a). Our method generates a four-sided surface that includes a closed region. Each point of Fig. 6 (b) expresses a surface control point and the gray region expresses the generated B-spline surface. As the red markers in Fig. 6 (b) show, for a section where surfaces are required to be connected with G^1 continuity, the cross vectors computed with the surface interpolation method are applied to the surface. A cross vector is a tangent vector that crosses a boundary as shown in the red arrows in Fig. 7. For the other sections, as shown with the blue markers in Fig. 6 (b), internal control points generated by the N-side filling method are applied to the surface. In addition, the boundary curves of the B-spline surface are generated by approximation of point sequences, as shown with the yellow markers in Fig. 6 (b), in order to apply to the shapes in Fig. 5. This enables generation of a surface connected with G^1 continuity with adjacent surfaces, even if the surface contains holes or concave shapes.

Our method is roughly composed of the following three steps:

- STEP1. Four B-spline curves are generated in the outside of a closed region. The generated curves will be the boundary curves of a B-spline surface. With these curves, a four-sided region is constructed.
- STEP2. With using the surface interpolation method, the cross vectors are computed for connecting adjacent surfaces with G^1 continuity.
- STEP3. In the inside of the boundary curves generated in STEP1, sample points are generated being based on tangent planes. From the generated sample points and the boundary

curves, a B-spline surface is generated. After that, two surfaces are connected with G^1 continuity by restraining the connection section with the cross vectors computed in STEP2.

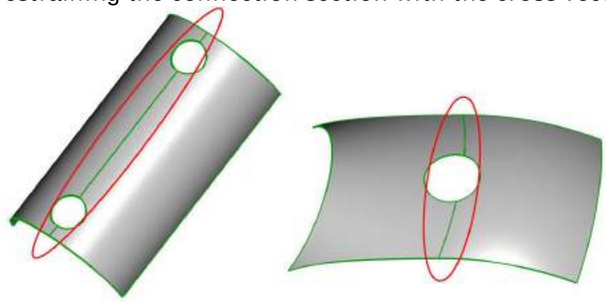
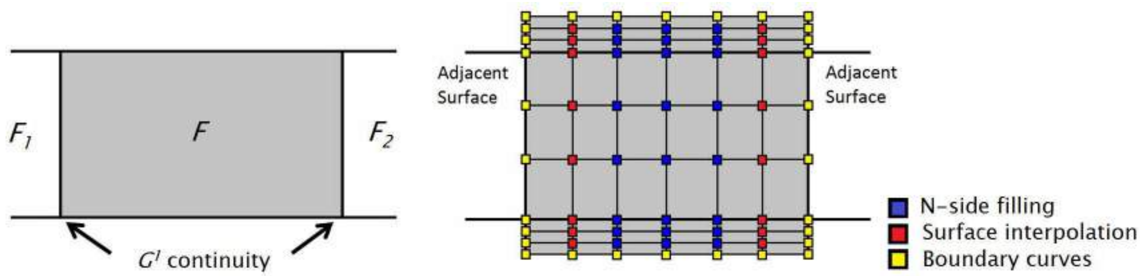


Fig. 5: Shapes with holes existing on the boundaries of two surfaces.



(a) (b)

Fig. 6: Concept of surface fitting method: (b) concept of surface fitting method to the gray region in (a).

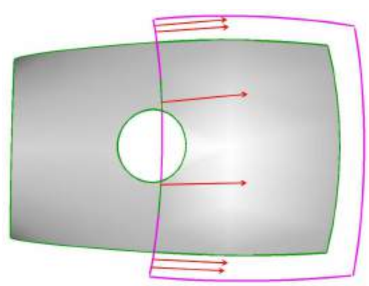


Fig. 7: Generation of cross vectors.

4.1 Boundary Curve Generation

This section describes the method of generating the boundary curves of a B-spline surface. Boundary curves are generated by the following procedures:

- Generation of the bounding box which covers a closed region.
- Calculation of intersection points between cross boundary derivatives and the bounding box.
- Approximation of intersection points with a B-spline curve.
- Connection of B-spline curves.

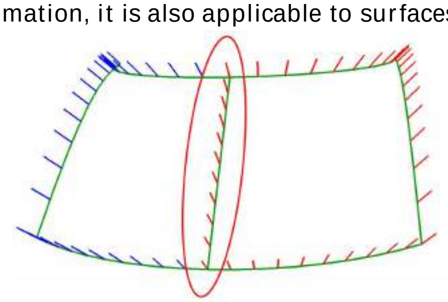
Before generating a surface, the sections where two surfaces are connected are analyzed to judge whether the boundary should be G^1 continuous. To be more concrete, as shown in Fig. 8 (a), the normal vectors are calculated for the boundary edge of the two adjacent closed regions (surrounded by the red circle in Fig. 8 (a)). If the normal vectors on the boundary edge where two surfaces share coincide each other, the two surfaces are judged to be connected with G^1 continuity. Based on the connection information between the two surfaces judged as above, a four-sided region that includes a closed region is generated.

First, as shown in Fig. 8 (b), four reference planes (shown with red lines) that include a closed region are generated. A reference plane is defined in the local coordinate system generated based on the sequence of the boundary edges representing a closed region[8]. Therefore, it is possible to generate a reference plane uniquely, without being dependent on the posture in 3D space, which means that the reference planes are obtained by using affine invariables.

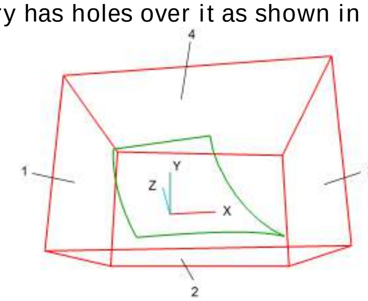
Next, the boundary curves that include a closed region are generated. As shown in Fig. 5, CAD data often contains the boundary of two surfaces over which holes exist. In order to treat such shapes, our method generates the boundary curves of the B-spline surface by approximating point sequences. First, as shown with the blue lines in Fig. 9 (a), cross boundary derivatives are generated from all the boundary edges of a closed region and a set of intersection points with reference planes is generated[8]. A boundary curve is generated from a set of intersection points belonging to a reference plane. When the boundary edge is connected with the adjacent surface with G^1 continuity, sample points are generated on the boundary edge instead of the intersection points and they are approximated by a B-spline curve. When the boundary edge is not connected with the adjacent surface with G^1 continuity, a set of the generated intersection points is approximated by a B-spline curve. Our method uses the least-squares method for surface approximation. In the least-squares method, no restriction is set for errors and approximation is repeated until the number of control points reaches sample points, or until the residual sum of squares between the sample points and the approximated curves or surfaces become smaller than 1.0. As shown in Fig. 9 (b), four purple boundary curves are generated for the region enclosed with green curves.

Finally, after generating the four boundary curves, corner points of the boundary curves are connected to generate a closed four-sided region. To be more concrete, the corner points of the boundary curves where the connection section is G^1 continuous are extended in the tangent vector direction and the intersection with a reference plane is obtained. The average of the obtained intersection points and the corner points of the boundary curves is calculated and the obtained points are added to the sample points. After that, four boundary curves are generated by re-fitting the sample points. The generated four-sided closed region is shown in Fig. 9 (c).

Since our method generates the boundary curve that connects two surfaces with G^1 continuity by approximation, it is also applicable to surfaces whose boundary has holes over it as shown in Fig. 10.

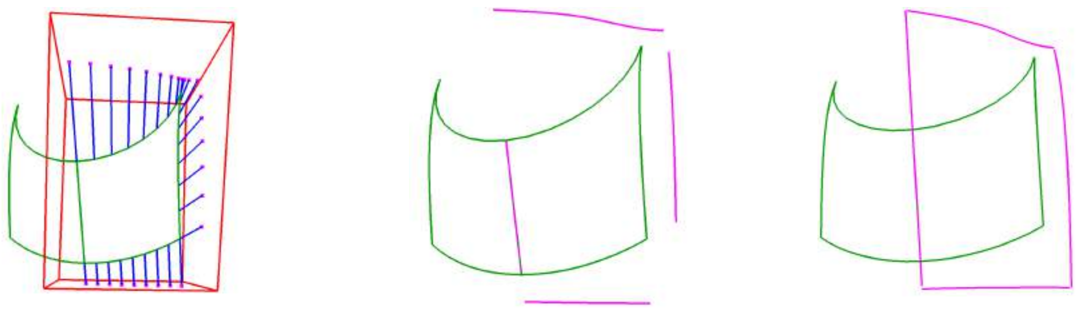


(a)



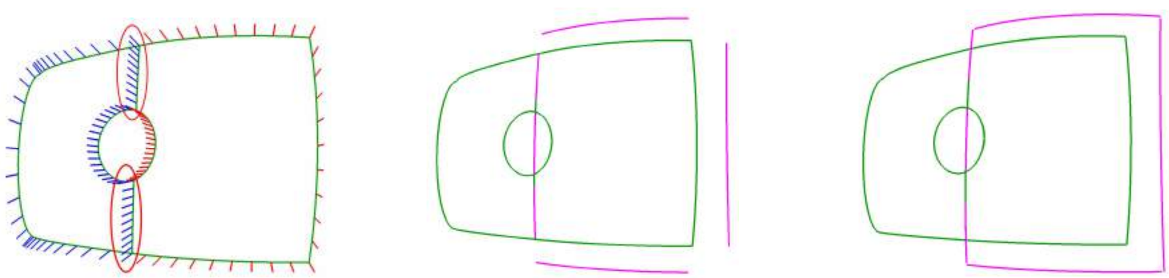
(b)

Fig. 8: Analysis of a connection section and generation of reference planes: (a) The section where two surfaces are connected is analyzed by using the normal vectors on the boundary edge. (b) Reference planes are generated in the local coordinate system obtained from the boundary edges.



(a) (b) (c)

Fig. 9: Generation of boundary curves: (a) A set of intersection points of cross boundary derivatives and reference planes is generated. (b) A B-spline curve is generated by approximating a sequence of points. The corner points of the boundary curves generated in (b) are connected, and the closed four-sided region (c) is generated by approximating the points again.



(a) (b) (c)

Fig. 10: Case where a hole exists over a G^1 continuous boundary edge: (a) There is a hole on the boundary edges, shown within red circles, connecting the adjacent surfaces with G^1 continuity. (b) A sequence of points obtained from boundary edges is approximated to generate a B-spline curve. In the same manner, the corner points of the boundary curves generated in (b) are connected and a four-sided closed region is generated by approximating the points again.

4.2 Restricted Control Point Generation

This section describes how to connect surfaces to be G^1 continuous. By restraining the control points shown with the red markers in Fig. 6 (b), a B-spline surface will be G^1 continuous with the adjacent surface. To be more concrete, the cross vectors of the boundary edge of the two surfaces are calculated based on the surface interpolation method[6][7]. A knot is inserted to the parameter corresponding to an end point of the boundary edge where two surfaces are G^1 continuous, and the boundary curve generated by the method described in section 4.1 is divided. After that, as shown in Fig. 11 (a), the cross vectors connected to each of the divided boundary curves are calculated based on the basis patch method[6][7], and restrained control points are obtained.

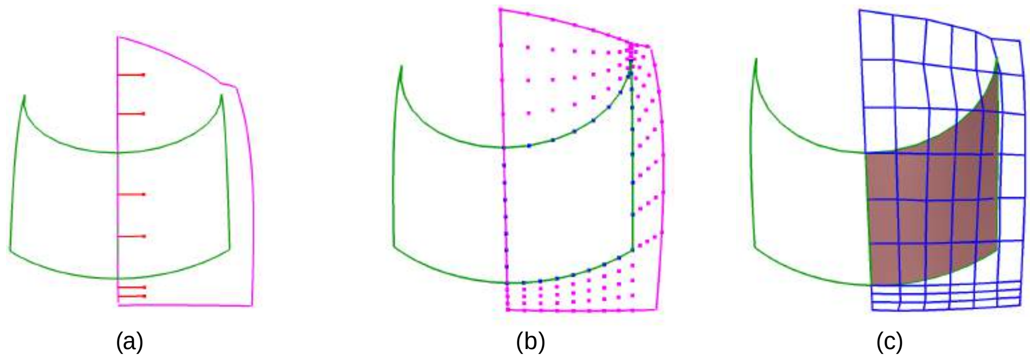


Fig.11: (a) Connection of two surfaces using Gregory control points, (b) sample points and (c) the generated surface with restraining the connection section.

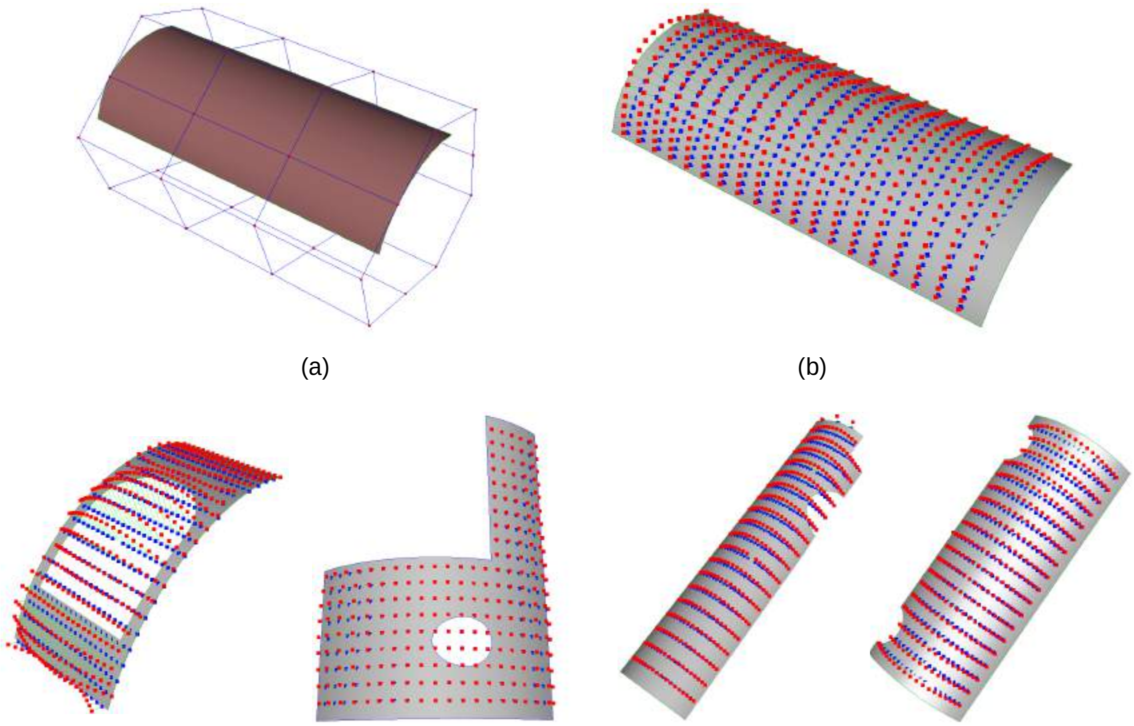
4.3 Sample Points and Surface Generation

This section describes how to generate sample points and a B-spline surface. First, sample points are generated inside the boundary curves of the B-spline surface generated in section 4.1 and outside the closed region, according to the cross boundary derivatives[7]. The purple markers in Fig. 11 (b) are the sample points on the cross boundary tangent lines. Next, sample points are generated on the boundary edges as shown in the blue markers in Fig. 11 (b). The generated point cloud lies on the tangent plane of the sample points of the boundary edge (the blue markers in Fig. 11 (b)). The internal control points of a B-spline surface are calculated by using the boundary curve described in section 4.1 and the sample points generated in this section with the least-squares method[8]. When a surface is generated, the connection section is restrained by using the control points generated in section 4.2, and a B-spline surface that connects adjacent surfaces with G^1 continuity is generated. The generated surface and surface control points are shown in Fig. 11 (c).

4.4 Error Evaluation of the Generated Surface

This section describes the error evaluation method of the B-spline surface generated in section 4.3. To verify the accuracy, the distance between the source surface that the trimmed surface retains and the generated surface is measured. The source surface is divided equally in both u and v directions into twenty sections so that a square grid is generated. The generated grid points lying inside the outer loop of the trimmed surface are extracted. The extracted point cloud is projected to the generated B-spline surface and the shortest distance is measured. After that, it is verified whether the shortest distance is below the tolerance. In addition, it is verified whether the source boundary edges lie on the generated B-spline surface within the tolerance or not.

An example of shape evaluation is shown in Fig. 12 and Tab. 1. The blue markers in Fig. 12 (b) to (f) represent the points on the generated surface to which the grid points of the source surface are projected. The red markers are obtained by extending the blue markers in each normal direction of the corresponding tangent plane. The lengths between the blue markers and the red ones show the distances between the source surface and the generated surface multiplied by twenty five. Tab. 1 shows the result of applying the error evaluation to the surfaces shown in Fig. 12 (b) to (f). In Tab. 1, three kinds of values are shown: *Avg.* indicates the average error margin value obtained by averaging the distances between the generated surface and the source one, *Max* indicates the maximum error margin value representing the maximum distance between the generated surface and the source one, and *Ratio* indicates the ratio of the bounding box size and the maximum distance. *Avg.* and *Max* indicate absolute errors and *Ratio* indicates relative errors. In this paper, when *Ratio* is smaller than 1%, it is assumed that a shape is approximated in good accuracy.



(a) (b)

(c) (d) (e) (f)

Fig.12: Error evaluation of generated surfaces: (a) shows the control points of a trimmed surface. (b) to (f) show the distances between the generated surfaces and the source ones.

Object	*Evaluation Object*	*Avg.*	*Max*	*Ratio*
(b)	*Trimmed surface*	0.115352	0.201063	0.213553 %
	Boundary edges	0.043451	0.179122	0.190249 %
(c)	*Trimmed surface*	0.039736	0.121211	0.359014 %
	Boundary edges	0.029863	0.057359	0.169890 %
(d)	*Trimmed surface*	0.006278	0.017272	0.080947 %
	Boundary edges	0.003254	0.010960	0.051366 %
(e)	*Trimmed surface*	0.006956	0.012310	0.102466 %
	Boundary edges	0.004188	0.012333	0.102662 %
(f)	*Trimmed surface*	0.119726	0.192948	0.204945 %
	Boundary edges	0.027265	0.179203	0.190344 %

Tab. 1: Error evaluation: The distances between the generated surfaces and the source trimmed ones are calculated and *Ratio* values are obtained.

5 EXPERIMENTAL RESULT

Our method was applied to CAD data in the IGES format obtained from web2CAD[1], and the practicality was verified. Five CAD data used in the experiment are shown in Fig. 13. The result of compression of 3D shape data for which our method was applied is shown in Tab. 2. Three kinds of data, IGES, XVL and ours, are compared for verification. We can find that IGES and XVL data are compressed at the maximum of 99.3% and 64.7% respectively by applying our method.

The result of surface data extension by using the boundary edges and prediction function is shown in Fig. 14. We can find that the surfaces are generated in good accuracy even if there are holes or concave shapes. In order to verify the continuity with adjacent surfaces, the normal vectors on the boundary edges of the generated surface are calculated and shown in Fig. 15. Since the normal vectors of the generated surfaces coincide with those of the adjacent surfaces on their boundary edges, we can find that two surfaces are connected with G^1 continuity.

The CAD data used in this experiment are contained many quadric surfaces. In order to show the validity of our method, we applied our method to the concave shape which has adjacent surfaces with G^1 continuity as shown in Fig. 16. We can find that the surface which maintained G^1 continuity with adjacent surfaces was generated.

DATA	*IGES*	*XVL*	*Ours*	*Compression ratio*	
				IGES	*XVL*
(a)	403,614 byte	5,462 byte	2,762 byte	99.3%	49.4%
(b)	188,354 byte	7,503 byte	3,711 byte	98.0%	50.5%
(c)	534,394 byte	18,216 byte	6,435 byte	98.8%	64.7%
(d)	1,508,718 byte	30,906 byte	12,347 byte	99.2%	60.0%
(e)	1,461,896 byte	39,885 byte	15,670 byte	98.9%	60.7%

Tab. 2: Comparison of data size.

6 CONCLUSION AND FUTURE WORKS

In this paper, we proposed the data compression method of the trimmed surface based on the surface fitting method. The surface fitting method used in our method can connect adjacent surfaces with G^1 continuity. To be more concrete, first, the boundary curves of a closed region and sample points are generated by using the tangent planes on the boundary edges. After that, the connection section is restrained using the cross vectors obtained by the surface interpolation method, and a B-spline surface is generated by a least-squares method. Since the surface fitting method used in our method is also applicable to the shapes with holes or concave shapes, it is possible to compress trimmed surfaces. In order to verify the practicality, our method was applied to CAD data. As a result, 3D shape data was able to be compressed efficiently. Moreover, we found that trimmed surfaces can be extended with satisfying the accuracy of 3D shape data.

In our method, in order to connect adjacent surfaces with G^1 continuity, knots are inserted to increase the control points. At that time, some sample points may lack and surface generation may fail. Fig. 17 shows the result of applying our method to a closed region surrounded by the surfaces in all directions connecting with G^1 continuity. The control points will be unstable if the number of sample points decreases for the number of control points. As this example shows, there are some shapes to which the algorithm shown in Fig. 3 cannot be applied, such as a shape with holes or concave shapes that connect all adjacent surfaces with G^1 continuity. Such shapes are not popular in CAD data, and they might not be so important from the viewpoint of data compression. However, we consider that broadening the range of application of our surface fitting method is quite meaningful. One of our future works is to improve the surface fitting method so as to be applied to all types of shapes including the ones shown in Fig. 17. In addition, four boundary curves may not be generated by our method in certain cases[9]. For example, in periodic shapes like cylindrical faces, the reference plane and cross boundary derivatives lie in parallel and no intersection point can be calculated, as shown in Fig. 9 (a). In our method, such periodic shapes are divided into some regions manually and a surface is generated to each of the regions. Therefore, to apply our method to such periodic shapes is one of our future works.

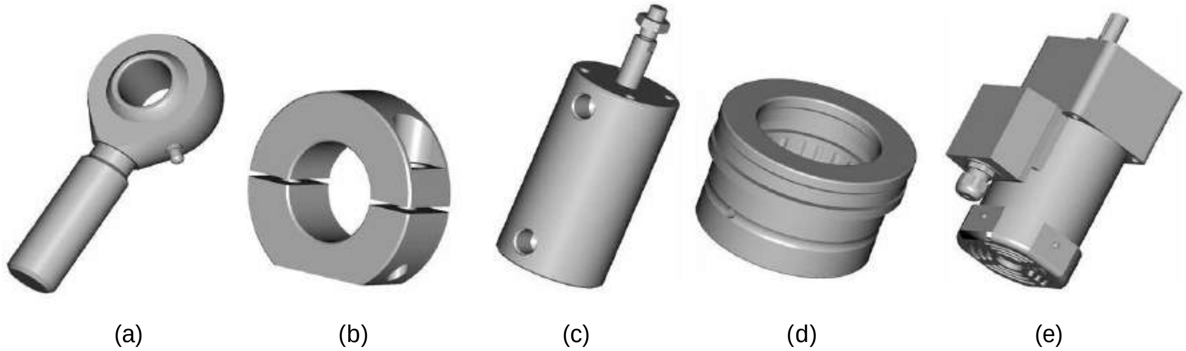


Fig. 13: CAD data used in the experiment: Five practical IGES data obtained by web2CAD.

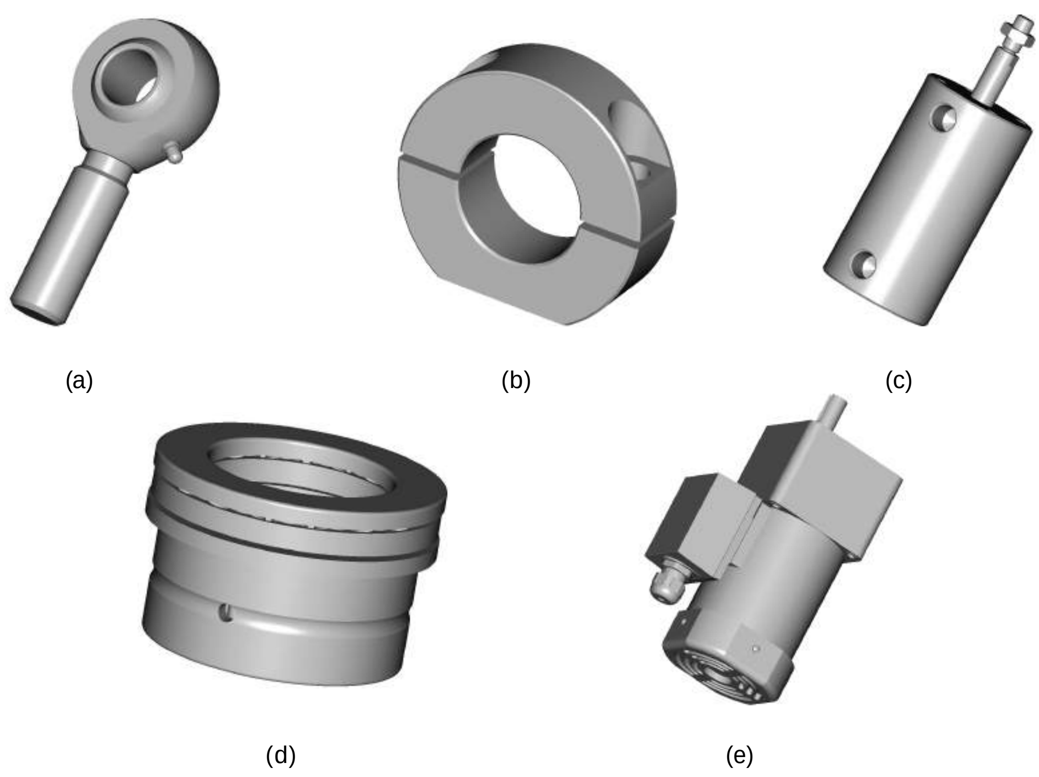


Fig. 14: Result of surface data extension: Our method is applied to the CAD data shown in Fig. 13 and the surface data is deleted and compressed. After that, the surface data is extracted by using the boundary edges and the prediction function.

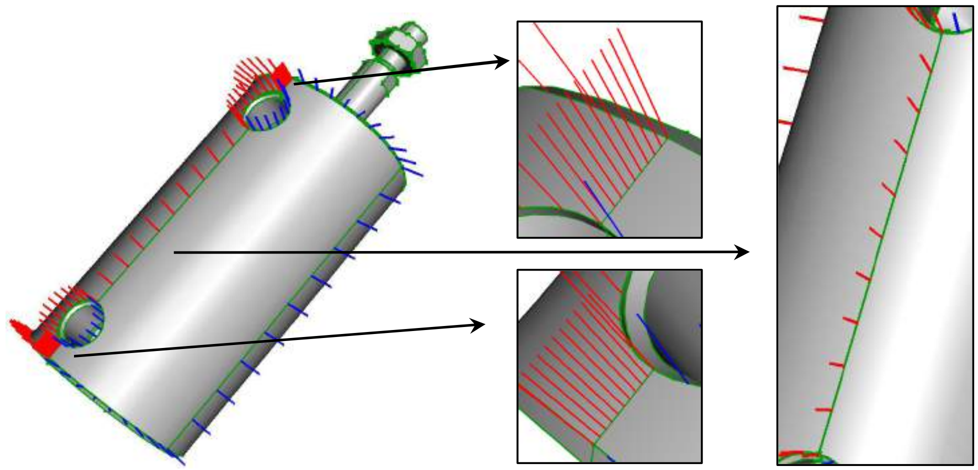
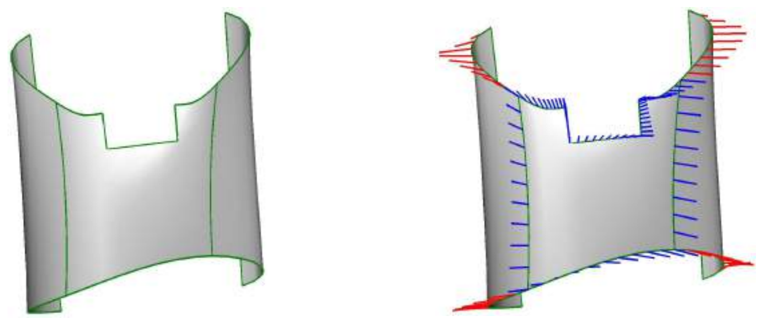
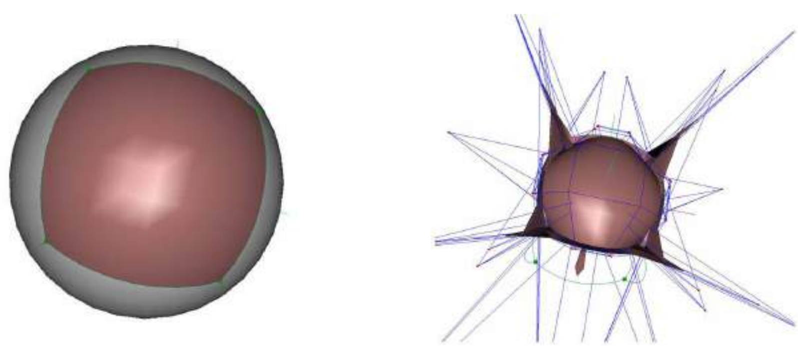


Fig. 15: Result of normal vector presentation: To verify the continuity between two surfaces, the normal vectors on the boundary edges were calculated. We can find that the normal vectors of the generated surfaces coincide with those of the adjacent surfaces on their boundary edges.



(a) (b)

Fig. 16: The result of applying our method to the concave shape which has adjacent surfaces with G^1 continuity: (a) is a result of shading presentation, and (b) is a result of normal vector presentation.



(a) (b)

Fig. 17: Example of the shape to which the surface fitting method proposed in section 4 cannot be applied: (a) A closed region surrounded by surfaces in all directions with G^1 continuity. (b) Our method was applied to the closed region of (a), and the control points of the generated surfaces are shown.

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