



Reference Based Geometric Modeling for Heterogeneous Objects

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ABSTRACT

Heterogeneous Object (HO) is a blend of two or more materials with different characteristics with or without any specific boundary interface. Material properties of such objects are controlled according to the defined functional requirement. Some of the rapid prototyping processes can fabricate such objects but require both geometric and material distribution information in a related CAD model. Developed 'Gradient Reference' based approach has capability to represent such information for geometric modeling and fabrication of HO. The approach introduces the concept of gradient references and boundary enclosures to distribute the material in the heterogeneous object domain. A system structure integrating different modules is described to produce required CAD data for rapid prototyping of HO. The proposed model is extended to implement a user defined material distribution function in an object and properties at different locations in an HO are defined accordingly. In the current work, a sub-division algorithm is proposed to incorporate the material information in conjunction with geometric model for fabrication of HO. The algorithm incorporates distance and blending functions for smooth material distribution in the heterogeneous object domain. Finally, some heterogeneous objects are modeled using gradient reference approach.

Keywords: heterogeneous objects, gradient reference, material distribution function.

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1 INTRODUCTION

A heterogeneous object (HO) is made of multiple materials, with sub-objects embedded and discrete or continuous material distributions to accomplish desired functional requirements. Since heterogeneous objects overcome homogeneous ones in terms of combined mechanical, thermal and electrical properties of the preferred materials, thus, functionally efficient and cost reducing designs can be achieved within a single solid. Emerging technologies i.e. biomedical, geophysical and nano-scale modeling involve objects with multiple materials that results in required anisotropic properties across the object domain [11]. Rapid prototyping (RP) techniques have the capability to produce HO using 3D CAD models by varying material composition region-wise, layer-wise, or point-wise in the object domain. The required 3D CAD model should have not only the geometric information but also the information of material property at each point inside an object. The existing computer aided models

lack powerful, generic and uniform representations for heterogeneous objects. Effective computer aided HO modeling tools for integration with analysis and rapid prototyping set up are limited. System level modeling work has also not yet reported in the literature to the complete extent. The detailed system structure framework, visualizations techniques, data structures and user interactions required for heterogeneous modeling environments have also not captured their due attention.

Keeping in view the above problems, a gradient reference approach is developed for CAD modeling of heterogeneous objects. The resulted CAD model not only contains the geometrical and topological information of the objects, but also the material information of the objects. The gradient reference approach is uniform and more generalized and assures the local control in the heterogeneous region. The developed model is extended to implement proposed system structure for a user defined material distribution function. A sub-division algorithm for material distribution is proposed such that color information of an object can be outputted to any RP machine for painting/printing the prototype.

The paper is organized as follows: in Section 2, the previous work is reviewed. Section 3 describes a system structure for CAD data flow to represent, analyze and fabricate HO while in Section 4, the mathematical model for heterogeneous object is extended to distribute material for a user defined function. Section 5 of the paper represents briefly the developed gradient reference CAD approach while the Section 6 elaborates the sub-division algorithm; Section 7 presents some examples of the heterogeneous objects and the conclusions along with the future scope of the work comprises the last section.

2 REVIEW OF PREVIOUS RESEARCH

CAD based HO modeling have been extensively studied by computer and manufacturing community. An approach to model multi-material objects based on R-m sets and R-m classes is proposed primarily for application in layered manufacturing [12-13]. Boolean operators were defined to facilitate the modeling process. Jackson [5] and Liu [14-15] has defined a local composition control (LCC) approach to represent heterogeneous object in which a mesh model is divided into tetrahedrons and different material compositions are evaluated on the nodes of the tetrahedrons by using Bernstein polynomials. A method based on material tree structure is developed by Chiu and Tan [2] to store different compositions of an object. The material tree was then added to a data file to construct a modified format being suitable for RP manufacturing. A scheme named 'source-based' method is proposed to distribute material primitives, which can vary any material with an object [21]. Qian and Dutta [17-18] proposed a feature-based modeling scheme which was extended to heterogeneous object representation through boundary conditions of a virtual diffusion problem in the solid, and then designers could use it to control the material distribution. The work is further extended by taking parameterized functions in terms of distance(s) and functions using Laplace equation to smoothly blend various boundary conditions, through which designers could edit geometry and composition simultaneously [14-15]. A hierarchical representation for designing and optimizing objects composed of multiple regions with continuously varying material properties is suggested by Kou and Tan [11]. This approach uses B-rep to represent geometry and a heterogeneous feature tree to express the material distributions. A level-set based variational scheme is proposed by Wang [26] which has adapted a variational model as the objective function to locate any point in the material region of a well-defined gradient or on the boundary edges and surfaces of discontinuities. The set of discontinuities is represented implicitly, using a multiphase level set model. A mesh-free approach is presented which is based on the generalized Taylor series expansion of a distance field to model and analyze a heterogeneous object satisfying the prescribed material conditions on a finite collection of material features and global constraints [25]. However, almost all such researches are mainly focused on the computer representation of heterogeneous object, rather than developing a procedure for rapid prototyping and fabrication of heterogeneous object. Liu [14] and Qian [17] verified some of the approaches in commercial software packages, such as SolidWorks and Unigraphics. A system independent of commercial CAD package is developed by Marson [16] and Qian [18] to deal with the HO modeling, but that does not take care of the slicing procedure for RP manufacturing. A new file format called Additive Manufacturing File (AMF) format, easily convertible to STL file format is proposed by Jonathan [9-10].

“The existing representations have their own strengths but require improvements to overcome certain limitations. Smooth and flexible material variations, local control, modeling generalized HO and system structure for integration with RP set up are a few residual issues that require more focus [1], [12], [17-18], [21]. To accomplish such goals, a gradient reference approach is developed, which can be used to model generalized heterogeneous objects.

In this paper, we address the CAD gradient reference model with systematic methodologies for representation, formulation and manipulation of HO. The proposed ‘gradient reference’ approach is generalized for one dimension grading in comparison to feature based and source based approaches [3-4]. An algorithm is developed and presented for smooth and flexible material variation in the object domain. The proposed approach assures better local control for precise material distribution in grading region. A system structure for modeling, visualization and CAD data representation for rapid manufacturing of HO is presented. A detailed description of each module is not presented in this manuscript due to the constraints of paper length.

3 SYSTEM STRUCTURE FOR CAD MODELING OF HO

A system structure is developed by interlinking different modules required for modeling, analysis and fabrication of HO. The basic structure of CAD modeling system (Fig. 1) contains six main modules; namely (1) Geometric modeling; (2) Material modeling; (3) Integrated geometry and material slice generation module; (4) Analysis integration module; (5) RP integration module; and (6) Visualization and display module.

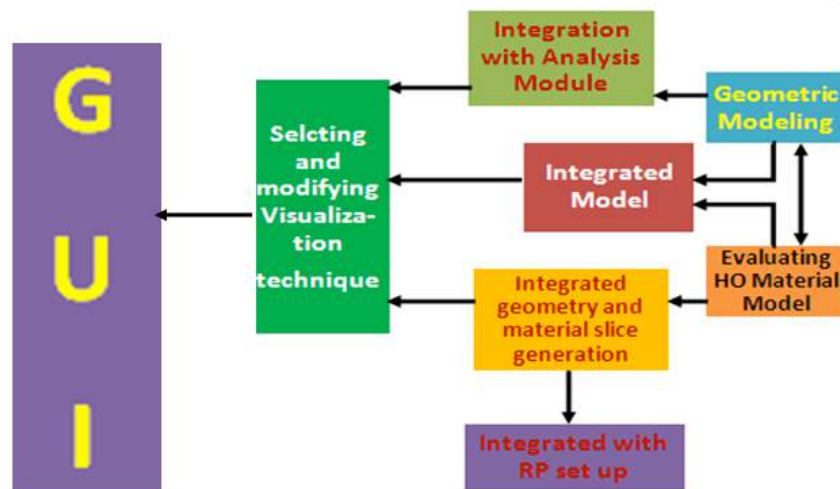


Fig. 1: Basic system structure for CAD data flow to represent HO.

The geometric modeling module mainly deals with the data structure set up for geometric model, mesh generation and sub-division of surfaces for improving the smoothness of meshes. The material modeling module evaluates material information of a heterogeneous region within a CAD model according to the specifications set by the users. In our system, we exploit the geometric model to describe the shape information. The material model describes material composition in terms of material space. The accuracy and quality of the final part fabricated by rapid prototyping depends on the 2D geometric slices/layers of a model. The third module gives the slice generating information by integrating the material information in a layer. In the previous work of the authors [4], a mesh model slicing algorithm is used to display the internal structures and material distributions in each region of the HO, as displayed in Fig. 2.

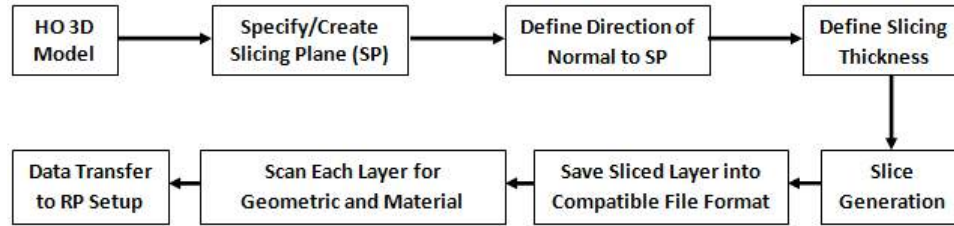


Fig. 2: Slicing algorithm for HO model.

A recursive triangle sub-division algorithm is employed for better local control in a heterogeneous slice [4]. Each triangular facet in a mesh model is sub-divided into number of sub-facets for accurate and precise material distribution in the chosen region, as described in Fig. 3. The sliced layers containing the information of materials information, lines, arcs and circles are saved in ASCII/DXF/CLI files. This data can be easily extracted and used as control data for RP machines.

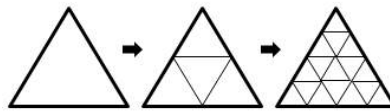


Fig. 3: Recursive triangle subdivision.

Fourth module imports and exports the data for the finite element analysis of HO using neutral file format (e.g. DXF, ACIS or IGES format). The data communication between proposed analyses integration module and analysis software (ANSYS) supports ACIS file format for processing of HO. The material information exchanges are carried out through data files using Visual C++ programming language. Rapid prototyping technique offers a possibility to manufacture heterogeneous object. Fifth module integrates the CAD modeling system with RP set up for efficient data exchange to fabricate HO. The proposed system structure is employed for modeling analysis and rapid manufacturing of heterogeneous objects. The system structure is implemented using OpenGL libraries and AMF formats for geometry and material models respectively [9-10]. The STL file format, used for homogeneous objects, can be easily converted into AMF format and further edited to include material information for rapid manufacturing of HOs. Sixth module mainly provides visualization techniques for HO and displays complete information on user friendly display. The proposed system structure provides an efficient method for data representation and transfer; and thus can be used for downstream technological applications for design, analysis and manufacturing of heterogeneous objects.

4 MATHEMATICAL REPRESENTATION OF HO FOR GRADIENT REFERENCE APPROACH

The developed mathematical model represents intricate geometries as well as material variation simultaneously. It also assures smooth material variations throughout the complex object and local material alterations [3-4]. A noteworthy point to be emphasized here is that in the developed model, the topological information is utilized to ensure smooth material variations throughout the complex geometry. The heterogeneous object domain is defined by merging the geometric and material information of defined number of cells in the HO space. Specific material composition for pre-defined number of primary materials in each cell can be found out using material distribution functions and material composition arrays [4]. The properties of the HO are manipulated by controlling and discretizing each cell into number of sub volumes at various identified locations that can be defined at any location [4].

The material distribution for a user defined material distribution function $f(x)$ in a unit length HO is described in Fig. 4. The defined HO is made up of a mixture of metal and ceramic, having modulus of elasticity 460×10^3 MPa and 150×10^3 MPa respectively and Poisson ratio of 0.30 and 0.42 respectively.

The materials in this HO are distributed as a function of distance x along the grading direction. The material distributed function $f(x)$ for four sub-regions in the unit length HO is defined in Eqn. 4.1. First region is purely a ceramic region, second region describes a linear distribution, and third one is made of metal and fourth represents a parabolic distribution of materials.

$$f(x) = \begin{cases} 1, & 0 < x \leq 0.25 \\ (1-x), & 0.25 < x < 0.5 \\ 0, & 0.5 \leq x \leq 0.75 \\ x^2, & 0.75 < x < 1 \end{cases} \quad (4.1)$$

Percentage volume fraction of metal and ceramic along a specific direction in the heterogeneous object domain is shown in Fig. 4(b).

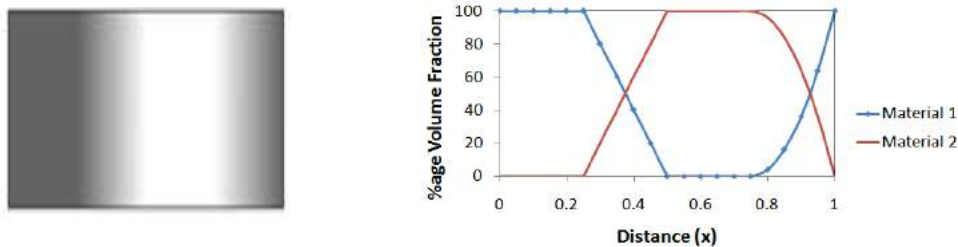


Fig. 4: Material distribution in HO: (a) Gradient region, and (b) Graphical presentation of percentage change in volume fraction of two materials.

The properties of heterogeneous unit volume vary with respect to percentage volume fraction of constituent materials and are determined using Voigt's rule [3-4]. For the smooth material transition properties, one of the available blending functions can be used. Effective properties i.e. elastic limit and Poisson ratio for the HO domain are displayed in Fig. 5(a) and 5(b) respectively.

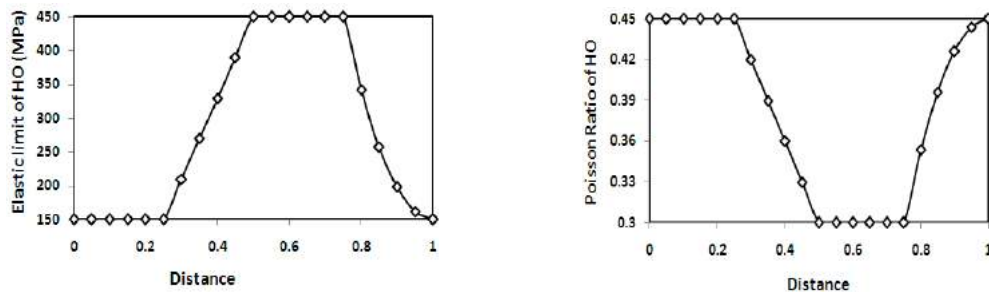


Fig. 5: Properties variation in HO: (a) Effective elastic limit, and (b) Effective Poisson ratio.

5 GRADIENT REFERENCE CAD MODEL FOR HO

The 'gradient reference' approach reduces various limitations, hence, is more comprehensive to create heterogeneous objects with one dimension grading in contrast to methods already reported [17-18], [21]. The grading references approach requires addressing the following two aspects for CAD modeling for any FGM/HO, as shown in Fig. 6.

- Gradient reference (GR): Point, Line, Plane
- Boundary enclosures (BE): Volume definition

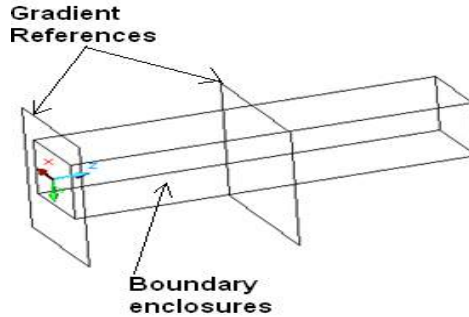


Fig. 6: Gradient references and boundary enclosures.

The gradient references are the basic entities i.e. point, line and plane in the object which originates the gradient region and limit the gradient region with defined boundary enclosures in an object. Such boundaries can be defined either by planes/surfaces of object or may be created exclusively by the user, thus creating a closed space defined as closure of gradient space. By controlling and manipulating the gradient references and boundary enclosures, complex heterogeneous objects can be realized. In this approach, the basic entities used in defining the geometry of the object can be used as gradient references/boundary enclosures. However, a user can create his / her own references independent of geometry of the object for local control in HO.

5.1 Gradient Reference (GR)

The gradient references includes user defined point, line and plane. It may be any 3D object entity i.e. vertex, axis, edge, or a surface of the object. The material distributions for these gradient references are discussed below and are shown with the help of Fig. 7.

- Point with spherical closure (function of sphere radius)
- Line and curve with cylindrical closure
- Gradient region between planes, bounded by enclosing surfaces of object

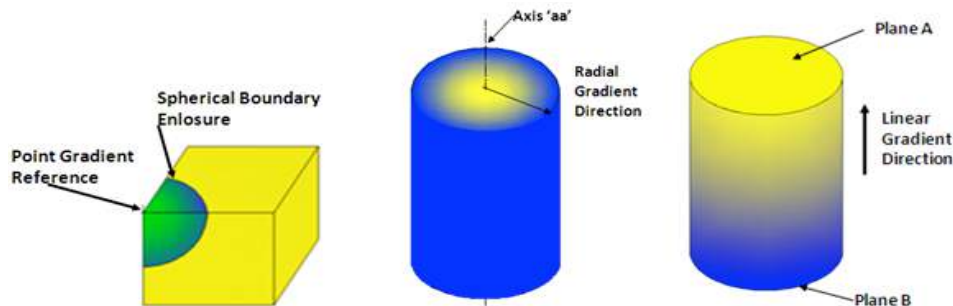


Fig. 7: Gradient references: (a) Point gradient reference bounded by user defined radius, (b) Line gradient references as axis 'aa' bounded by user defined radius and planes, and (c) Plane gradient references bounded by surface enclosures.

5.2 Boundary Enclosures (BE)

Apart from dependency on gradient references, material distribution in HO depends on boundary enclosure/shape of the object. BE/shape dependence is defined by relation between the object boundary enclosures and gradient reference.

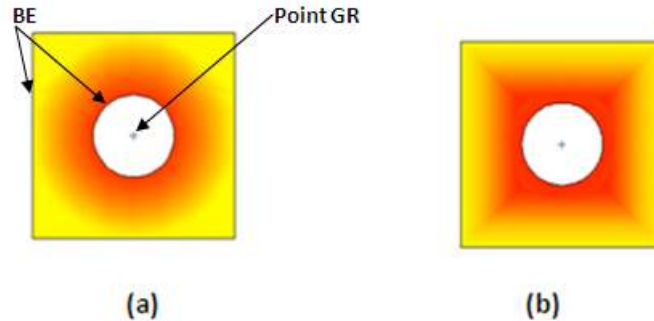


Fig. 8: Shape dependency concept for material distribution using point gradient reference: (a) Material distribution independent and non-associating with shape of the object, and (b) Material distribution, associated and adapted to the shape of the object.

If the relation between BE and GR is non-associative in nature, then the material distribution function is independent of shape/BE and the gradient region is only restricted by BE. Otherwise, for the associative relation, the material distribution function depends on and is also bounded by the shape/BE of the object in the HO domain.

An example of material distribution in a 2D object is displayed in Fig. 8 to describe the shape dependency concept. A point gradient reference with radial distribution is located at the center of the circle for the material distribution in the 2D object. Material distribution does not adapt the shape but is restricted to the end boundaries of the object i.e. square and circle, thus having non-associative relation between GR and BE. In this case, material composition is same at equal radial distance according to virtual spherical enclosure (Fig. 8(a)). Material distribution for associated relation between GR and BE, adapts the shape of end boundaries, thus have different composition at equal radial distances, as illustrated in Fig. 8(b).

5.2 Local Control

The gradient reference model assures local control over the gradient region and is independent of universal co-ordinate system. With this, it is possible to distribute the material in complex shape objects. Material distribution in the object domain can be controlled by manipulating; (a) gradient references, (b) boundary enclosures, (c) material distribution function, (d) blending/smoothing function, and (e) type of relation i.e. associative /non-associative.

The effect of grading on the properties of heterogeneous objects can be easily modified by controlling the respective gradient references [4]. The effect of changing the gradient references is shown with Fig. 9. The material distribution has remained linear for non-associative relation as in Fig. 9(a), while adapt the shape of changed gradient reference for associative relation as shown in Fig. 9(b). Examples illustrated in Fig. 9(a) and 9(b) displays the different gradients to support this concept.

6 SUB-DIVISION ALGORITHM FOR MATERIAL DISTRIBUTION IN HO DOMAIN

As discussed in Section 5, the grading region in the heterogeneous object domain is bounded by boundary enclosures. The pattern of material distribution in this region may be associated with boundary enclosures and thus depends upon the geometry of the object as described in Section 5.2. In such cases, the boundary enclosures may be of similar or dissimilar shape.

Similar shape boundary enclosures have two same topology entities, parallel or offset to each other i.e. two parallel planes defined by opposite faces of a cube, two cylindrical surfaces of a hollow cylinder, etc. Dissimilar boundary enclosure grading region are defined by two different topology entities in the model i.e. a square plate with a circular hole.

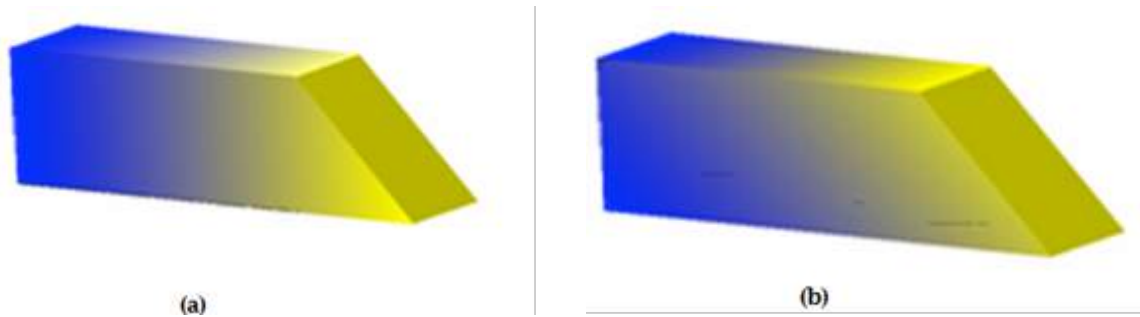


Fig. 9: Local control: (a) Linear material distribution between parallel gradient reference planes i.e. linear material distribution and (b) Sub-division of gradient region to adapt changed gradient reference i.e. between non-parallel gradient reference planes.

Material distribution function for such objects will adapt the shape and topology of boundary enclosures. Material distribution for similar/dissimilar shape BE is performed using the algorithm presented with the help of Fig. 10.

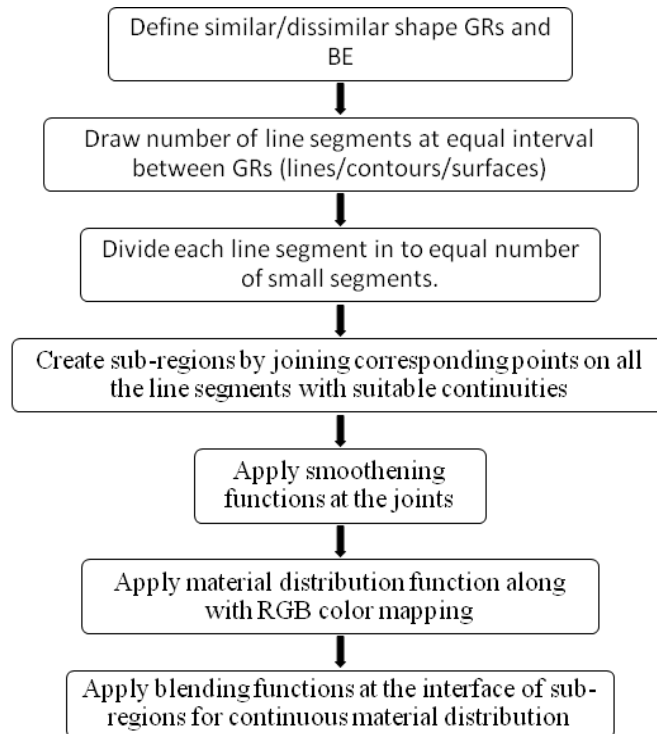


Fig. 10: Sub-division algorithm.

Blending functions with different continuities, depending upon curvature of references, are used for smooth joining of curve segments. These functions allow the construction of constant radius blends for any type of surfaces as long as their offset surfaces are smooth, without singularities and self intersections. Edge blends are created by sweeping rational quadratic curves. Corner blends are created by a convex combination of Taylor interpolants.

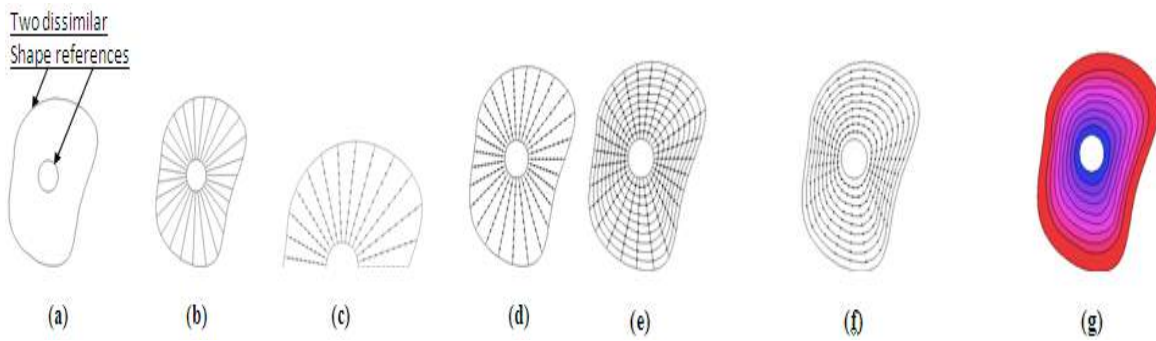


Fig. 11: Material distribution for associative BEs: (a) Dissimilar shape BE, (b) Sub-division of grading region in equal angle intervals, (c) Dividing each line in equal number of divisions, (d) Full view of sub-divided object, (e) Joining of all respective points using blending function and different continuities at ends, (f) Removal of division lines, and (g) Material distribution.

An example describing the algorithm for material distribution in heterogeneous objects having associative BEs is illustrated in Fig. 11.

7 EXAMPLES

The proposed gradient reference approach has been successfully implemented for heterogeneous object design. Some examples of heterogeneous objects are presented in Fig. 12 to show the validity of the proposed approach.

8 CONCLUSION AND FUTURE SCOPE

This work presents a gradient reference approach for modeling heterogeneous objects with complex geometry and simple one dimensional material variation. The basic structure listing sequence of CAD data flow is elaborated to visualize HO on a graphical user interface, analyze HO for property variation and fabricate HO using rapid prototyping technology. The distribution of material is obtained by using different material information functions and sub-division algorithm. The gradient reference approach is implemented for a user defined material distribution function to define the percentage volume fraction and study the change of properties in the HO domain. It ensures smooth material variations throughout the object; imposes independent material alterations on the cells so that their original properties can be properly retained in the resultant object; offers local control on material distribution and consistent data representation. The approach is also computationally robust and efficient.

The present work is limited to material distribution for one dimension grading only. The work can be extended to two and three dimension grading systems. The present approach may be extended and implemented to multiple, complex and irregular material distributions. Certain rapid manufacturing issues for the fabrication of HO are required to be addressed for the current approach. The present work can be extended to fabricate HO in real rapid manufacturing environment. Further the proposed approach can be used for object modeling i.e. solid modeling with other physical attributes such as mechanical properties, material distribution. etc. The work may be advanced to dynamic heterogeneous object modeling (DHOM) which is a new class of heterogeneous objects and has applications in life science domain, biomedical applications, dynamic process simulation, bio-CAD etc.

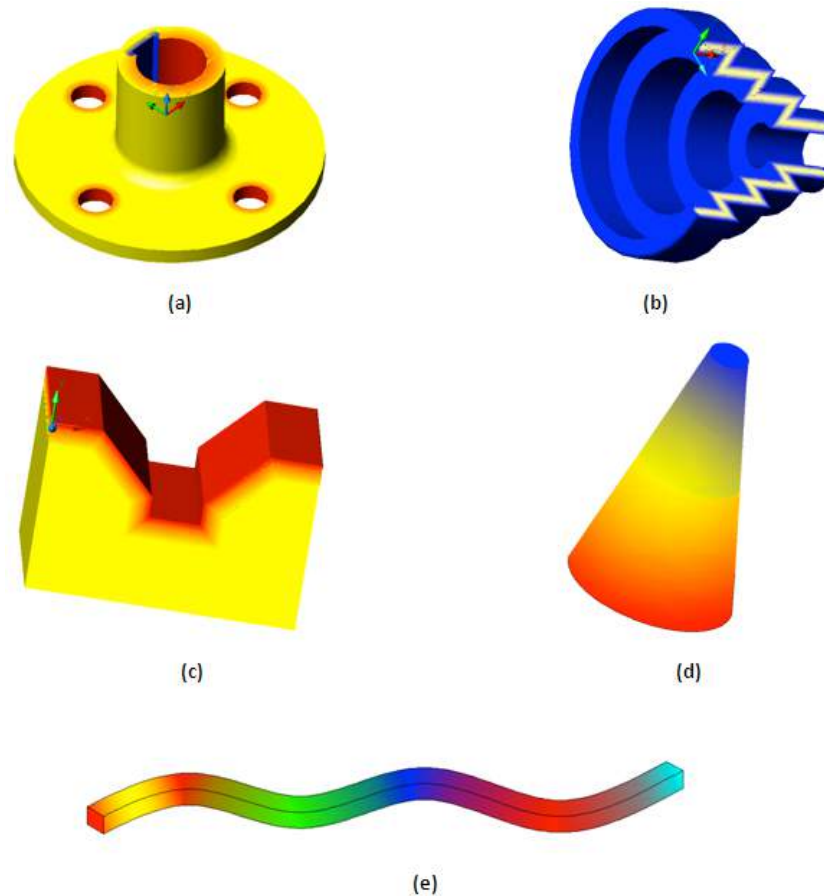


Fig. 12: Examples of heterogeneous object designs: (a) Flange coupling, (b) Stepped cone pulley, (c) Machine block, (d) Solid truncated cone, and (e) Multi-contour rectangular cross-section rod.

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