A Shape Design to Avoid Interference based on Tensegric Modeling

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ABSTRACT

A new mesh modeling method for shape design to avoid interference between a subject shape and an obstacle shape is proposed. This method is based on the tensegric modeling, and the deformation result is achieved automatically according to few user inputs. Tensegric model is generated on a mesh model of subject shape and simulates pseudo-physical deformation for various constraints. In this method, a dis-interfered shape is calculated by two types of methods according to the obstacle representation; “potential based avoidance” and “mesh based avoidance”. The deformation result fits to the obstacle shape smoothly. So, this method is effective for cover shape design of complex shapes, e.g. mechanical product, and is applicable for soft tissue deformation, e.g. human body.

Keywords: interference avoidance, tensegric modeling, cover shape design.
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1 INTRODUCTION

In a design process by CAD, a product shape is constructed from various shapes, and interference between shapes occurs frequently during the operations. To design a shape avoiding the interference, a designer may apply Boolean set operations to one shape and another shape is preserved as unchanged. In our research, those shapes are represented by arbitrary triangular mesh models. We named the preserved shape “obstacle mesh”, another shape “interfered mesh”, and its deformed result “dis-interfered mesh”.

In general, a result shape by Boolean set operations has sharp edges, and some rounding operations as fillet processing are required. Then a designer checks all the interference, and executes those sequences manually. If the target shapes are very complex, designer’s operating cost is not negligible. Additionally, interferences occur at many places, so it is difficult to achieve global smoothness of the dis-interfered shape.

On the other hand, “pattern mapping” is popular as a surface feature modeling method, and the target shape is deformed by using the pattern shapes [1-5]. In these methods, the surface of the pattern shape is projected to the target shape in an arbitrary specified region, but a pattern shape is not allowed to have any overhangs, and a project direction is fixed basically. Therefore, the pattern mapping is not appropriate for interference avoidance between arbitrary shapes.
Recently, a new method for cover shape design by using triangular mesh is proposed [18]. A cover mesh is generated automatically as the convex hull of another mesh by this method. Although there is no interference between those meshes, the cover mesh is not represented as a concave shape in principle, and existing shape cannot be used as the design target.

Here, we propose a new mesh modeling method for shape design to avoid interference. The shape control algorithm of our method is based on “tensegric modeling” [7,8]. Fig.1 shows an example given by our method. An initial interfered mesh $M_A$ (plane; 3,200 polygons) and the obstacle mesh $M_B$ (octahedron) are shown in the top left, and the “avoidance region” (Sec.2.2.1) of $M_A$ is shown in the right. In the middle, the magnified images in dotted circle of the top left figure are given for $M_A$ and its corresponding tensegric model $T$. The tensegric model (Sec.2.1) is the deformation structure of the mesh, and $T$ is linked to $M_A$ via its vertices. Boundary conditions of $T$ are given by user inputs and the interference state between $M_A$ and $M_B$, and the geometry of $T$ is fed back directory to $M_A$. The dis-interfered mesh $M_A'$ is shown in the bottom left of Fig.1, and its side view with $M_B$ is shown in the right.

Our research purpose is to develop a modeling system for shape design to avoid interference between arbitrary meshes. The system provides simple operations for a novice user and smooth dis-interfered shapes. For example, various welfare products are used to put on human body, and need to cover them with appropriate smoothness and geometric relationship. In the past, a shape design for this purpose is limited to ready-made or handicraft, because the conventional systems can not satisfy those requirements. Our method is applicable to order-made system which satisfies the requirements. In this paper, we show the results given by prototype system, and discuss the effectiveness.

2 METHOD OVERVIEW

Here, we overview the calculation algorithm of the proposal method based on the tensegric modeling.

2.1 Tensegric Modeling

Tensegrity is a portmanteau word of “tensional integrity”, and a kind of architectural structure [6]. The elements of the tensegrity structure are tensile parts and prestress parts, and the statically determinate shape is given by the internal forces (pure tension and pure compression), the external force, and the geometrical constraints.

Fig. 1: An example given by the proposed method. Fig. 2: Calculation procedure.
In the past, we have developed a computational model of tensegrity (tensegric model) by combination of the unit structure, and extended the simulation to a mesh modeling method “tensegric modeling” and the applications [9-13]. A unit structure is placed on a triangle with an arbitrary geometry, and the vertices are controlled by the endpoints. In this method, the statically determinate shape is calculated stably with complex constraints, and the mesh has very smooth surface.

2.2 Algorithm

In the proposed method, a dis-interfered mesh between arbitrary 2-meshes is given automatically by pseudo physical simulation based on tensegric modeling. In this simulation, the deformation is executed by boundary conditions calculated for local interference avoidance at discrete time, and dis-interfered mesh is given by the iterative calculation. User inputs are subject meshes; $M_A$ and $M_B$, geometric constraints, properties of the tensegric model $T$ (Sec.2.2.1), “avoidance resolution” (Sec.3.2.1), and “avoidance parameter” $t$, and output is the dis-interfered mesh.

Here, $t$ $(0 \leq t \leq 1)$ represents interpolation between $M_A$ $(t = 0.0)$ and $M_A' (t = 1.0)$. In Boolean set operations, superiority about shape preservation is limited to 0/1, but the proposal method achieves interpolation of shape preservation which means bilateral avoidance by using $t$. If $t$ is not equal 1.0, the interpolated mesh $M_A''$ between $M_A$ and $M_A'$ is calculated, then the roles are replaced; $M_B$ is handled as an interfered mesh of $M_A''$, and $M_A''$ is handled as an obstacle mesh of $M_B$. Finally, the simulation is repeated one more time with $t = 1.0$, and the bilateral dis-interfered shape $M_A'''$ and $M_B'''$ are given as a result.

The calculation procedure is shown in Fig.2. Here, we describe the each step precisely.

2.2.1 User Inputs

Firstly, user inputs subject mesh $M_A$ and $M_B$, the geometrical constraints, the properties of the tensegric model $T$; unit height and part's length, avoidance resolution, and avoidance parameter. Then the system detects the interference of them; mesh vs. mesh or mesh vs. iso-potential surface. According to the result, region segmentation is executed automatically on the mesh, and is used for the simulation.

Here, the region means a face set has topological continuity. “Crossing region” $R_c$ is a region composed of the faces crossing each other between the meshes. “Inner region” $R_i$ is a disc region surrounded by $R_o$, and is in inside of another mesh. “Interfered region” $R_{itf} = R_{in} + R_o$ is an interfered region with another mesh, and means a minimal region for interference avoidance in theory. If region for interference avoidance is limited in $R_{itf}$, it is difficult to ensure smoothness of the surface at the boundary, because the vertices in $R_{itf}$ and others are close each other geometrically in general. So we implemented an additional region “buffer region” $R_{buf}$ which is an outside region of $R_{itf}$. $R_{buf}$ has a topological distance from the boundary given by user, and performs as a buffer about the deformation. “Avoidance region” $R = R_{itf} + R_{buf}$ is a target region of tensegric modeling and interference avoidance, and the other region is fixed geometrically.

In the prototype system, mesh data is represented by customized B-rep model, and these regions are extracted automatically by searching the entity topologically.

2.2.2 Tensegric Modeling

Tensegric model is generated on $R$, and the end points are linked to the correspondent vertices of $R$. According to the boundary conditions given by user inputs and the avoidance calculation, statically determinate shape is given automatically in the conventional way, and the geometry is copied to $R$.

2.2.3 Avoidance Calculation

If meshes are interfered, vertices of one are in inside of another. Avoidance calculation means a calculation of the vectors move the vertices (end points) of $R_o$, which are inside of the obstacle mesh, to the outside of it, and we call the vector “avoidance vector”. The vectors perform as boundary conditions; translation vectors or external force against the end points. The details are described in chapter 3.

This calculation is iterated automatically for the every equilibration calculation of $T$ until the interference is eliminated. Then, the final shape $M_A'''$ is given according to the user defined termination condition.
2.2.4 Morphing

If \( t \neq 1.0 \), the linear interpolated mesh \( M_{A''} \) between \( M_A \) and \( M_A' \) is calculated. A vertex of \( M_{A''} \) \( q'' \) is given as following formula;

\[
q'' = q + tv
\]  

(2.1)

Where \( q \) and \( q' \) are vertices of \( M_A \) and \( M_A' \), and \( v \) is the difference vector \((q' - q)\).

At the following procedure, \( M_{A''} \) is handled as new \( M_B' \), and \( M_B' \) is handled as new \( M_A \). Then the procedures (Sec.2.2.1-3) are repeated with \( t = 1.0 \). After that, \( M_B' \) is given as the dis-interfered mesh against \( M_{A''} \), and the bilateral avoidance is achieved as a result.

3 AVOIDANCE CALCULATION

Deformation of tensegric model is based on the translation of the end points, and can be occurred by giving the boundary conditions; geometrical constraints or pseudo external forces about it. We used the feature to the avoidance calculation. The translation is calculated by using 2-methods based on different obstacle representation “potential based avoidance” (Sec.3.1) and “mesh based avoidance” (Sec.3.2).

3.1 Potential based Avoidance

In our research, “potential” means pseudo energy in a 3D-field included subject meshes. The potential is infinity at the source, and decrease as the distance from it. Iso-potential surface \( E_0 \) represents a surface of the obstacle mesh \( M_B \) and the potential space, which is more than \( E_0 \) means inside of \( M_B \). In the potential based avoidance, avoidance vector is computed as an external force which moves an end point of \( T \) to the position whose potential is less than \( E_0 \). Here, \( E \) is a potential at the current position of an end point \( p \). If \( E \) is more than \( E_0 \), the end point is judged as in inside of \( M_B \), and the force is applied to the equilibration calculation of \( T \). The external force \( F \) at \( p \) is determined as following formulae;

\[
E = \sum_{i} \frac{W_i}{|O_i - p|}
\]  

(3.1)

\[
F = \frac{d}{|\nabla E|} \nabla E
\]  

(3.2)

\[
d = \frac{E - E_0}{|\nabla E|} \quad (E > E_0), \text{ otherwise } 0
\]  

(3.3)

where \( O_i (i=1,2, ..., n) \) is the position of each potential source, \( W_i \) is the weight factor of each potential, and \( d \) is the distance according to \( \nabla E \) from \( E \) to \( E_0 \). Fig.3 shows the geometrical relationship between the \( O_i \) and \( F \).

3.2 Mesh based Avoidance

The interference problem between arbitrary closed meshes is composed of the crossing problem between the faces and the convex hull problem between the vertices and mesh. However, the convex hull problem requires very high computational cost, and the interference is not occurred without the face crossing.

In this method, the interference is detected by using only the face crossing problem to reduce the cost. If a face is crossing another face, the vertices are judged in inside of another mesh. The avoidance vector \( V \) is calculated as translation vector moves the vertex (end point of \( T \), which is in inside of \( M_B \), to outside of it.

In this calculation, we handle the mesh interference problem locally (Sec.3.2.1) and globally (Sec.3.2.2). \( V \) is calculated in each case, and called “local avoidance vector” \( V_l \) and “global avoidance vector” \( V_g \). \( V \) is determined by the Eqn.3.6 (Sec.3.2.2) composed of these parameters.
3.2.1 I. Local Avoidance

The local interference means the crossing of face (=triangle) vs. face, edge (=line segment) vs. face. Generally, the crossing pattern is categorized into the next 2-types; (a) and (b), according to the combination of the mesh type. The translation vector against the vertex is calculated in each pattern, and we call it “crossing avoidance vector”. In this method, we approximate the subspace of obstacle mesh by the half-spaces defined by the faces and the normal vector. If a vertex of crossing face of $M_A$ is located in negative direction of normal vector of the partner face of $M_B$, the translation vector is calculated to move the vertex to the position whose direction is positive. Here, $E_A$ is an edge of $M_A$, $E_B$ is an edge of $M_B$, $S_A$ is a face of $M_A$, and $S_B$ is a face of $M_B$.

(a) $E_A$ vs. $S_B$

The parameter $c_a$ is determined as following formula, and is used to judge the crossing state between $E_A$ and $S_B$.

$$c_a = (q_a - p) \cdot N_a$$  \hspace{1cm} (3.4)

Where $q_a$ is a vertex of $S_B$, $N_a$ is the normal vector of $S_B$, and $p$ is an inner vertex of $E_A$ in the local obstacle space defined by $S_B$. If $d_a$ is more than 0.0, the system judges $E_A$ and $S_B$ are crossed each other. Here, $D_a$ is a crossing avoidance vector determined as following formula, and moves $p$ to $p'$ which is the projected position against $S_B$ according to $N_a$.

$$D_a = \frac{c_a}{|N_a|^2} N_a$$  \hspace{1cm} (3.5)

(b) $E_B$ vs. $S_A$

Here, $q_b$ is a vertex of $E_B$, and $N_b$ is an averaged normal vector of $E_B$ by neighboring faces. In this pattern, replace the alphabet “a” to “b” in the above formulae, and the same calculation is executed. Then, $D_b$ is given as a result.
Finally, the local avoidance vector $V_I$ is calculated from the crossing avoidance vector $D_{ai}$ ($i=1,2,\ldots,m$) and $D_{bj}$ ($j=1,2,\ldots,n$) as the following formula. Here, “avoidance resolution” $\alpha$ is the user specified constant, and $l$ is the average length of the edges in the $R$. Fig.4 shows the vectors at the each pattern.

$$V_I = \alpha l \left( \sum_i |D_{ai}| + \sum_j |D_{bj}| \right) \left( \sum_i |D_{ai}| + \sum_j |D_{bj}| \right) \left( \sum_i |D_{ai}| + \sum_j |D_{bj}| \right) (3.6)$$

### 3.2.2 II. Global Avoidance

The global interference means the crossing between the regions. Global avoidance vector $V_{II}$ is applied to the vertex $p$ which is judged as inner vertex in pattern I (Fig.5). Here, $R_{itf,B}$ is the interfered region of $M_B$, and $V_{II}$ means a weighted average vector of the normal vectors in $R_{itf,B}$. $V_{II}$ moves the inner vertices to outside, and have an effect for breaking the equilibrium state of the vertex translation by local avoidance vector, too. $V_{II}$ is determined as following formula;

$$V_{II} = \frac{\sum_k t_k N_k}{\sum_k t_k} \sum_k t_k (3.7)$$

Where $t_k$ ($k=1,2,\ldots,r$) is the area of an face included in $R_{itf,B}$, and $N_k$ is the normal vector of the face.

Finally, avoidance vector $V$ is calculated by the following formula, and the endpoint linked to $p$ is translated by it in the every iterative calculation.

$$V = \frac{|V'_{II}|V_I + |V'|_{II}V_{II}}{|V'|_{II}} (3.8)$$

### 4 EXAMPLES

We have developed the prototype system based on the proposal method, and we show the examples given by it in this chapter. The prototype system is programmed by C# programming language with OpenGL, OpenTK, and .NET libraries. And these examples stand on the note PC with Core 2 CPU 1.33GHz, 2.0GB memory, and Mobile Intel 945 Express 256MB.

Fig.6 demonstrates the result shapes based on the potential based avoidance. In the figures, square mesh (800-polygons) is the interfered mesh, and another shape (pink sphere) is iso-potential surface $E_0$ of the obstacle mesh. In this figure, the calculation is executed with some different boundary conditions. In the right, the potential source is placed at the geometrical center of the square mesh. As shown in the figures, the result shapes have smoothly deformed surfaces following the iso-potential surface $E_0$ but the avoidance is not fully achieved. We consider the reason is that the force by equilibration exceeds the force by the avoidance calculation. So, a new algorithm to guarantee the avoidance is needed to be implemented.

Fig.7-9 and Fig.11-12 demonstrates the result shapes based on the mesh based avoidance. In these figures, we evaluate the smoothness of the mesh surface according to the dot products $\dot{v}$ between the normal vectors and a basis vector of the world coordinates. The results are rendered without shading by the colors; red, green, and blue, according to $\dot{v}$. The square mesh (3,200-polygons) in these figures is the interfered mesh, and another mesh (pink) is the obstacle mesh. As shown in these figures, the result shapes have smoothly deformed surfaces following the obstacle mesh with complex geometry, and the interference avoidance is guaranteed. Although it seems to be discontinuously at part of these, we suggest that the buffer regions are too small against the deformation volumes.

Fig.10 shows the rapid prototype models made by using the meshes in Fig.9. These models are outputted by the machines; Spectrum Z510 ((C) Z Co.) and EDEN 350 ((C) OBJET Co.). As shown in this figure, the dis-interfered models covered the complex obstacle model with a certain degree of distance, and it suggests that this system is also useful for a cover shape design.

Fig.11 and Fig.12 demonstrate the examples of a bilateral avoidance. As shown in these figures, the result meshes avoided interference each other with a certain degree of distance, and it shows that the shape preservation can be controlled by avoidance parameter $t$.  

Fig. 6: The results with different conditions: Top: $E_0 = 0.5$ (left) and $E_0 = 0.25$ (right), Bottom: Fixed the open edges (left), 4-corners (middle) and free (right).

Fig. 7: Hand: 758-polygons, $h=0.5$, $pl=1.0$, $res=3.0$, $bl=5$, Computation time = 123.8[sec].

Fig. 8: Alphabet ‘G’: 738-polygons, $h=0.5$, $pl=1.0$, $res=3.0$, $bl=2$, Computation time = 336.1[sec].

Fig. 9: Complex shape: 13,768-polygons, $h=0.5$, $pl=1.0$, $res=3.0$, $bl=3$, Computation time = 4609.4[sec].
Fig. 10: Rapid prototype models; The obstacle model covered by the dis-interfered models (left) and open state (right).

Fig. 11: Initial state (left) and interpolated shapes (right, $t=0.0-1.0$) in a bilateral avoidance: Interfered mesh: cube (4,800-polygons), Obstacle mesh: sphere (1,520-polygons).

Fig. 12: Dis-interfered meshes based on $t$ in a bilateral avoidance.
5 CONCLUSIONS

In this paper, we proposed a new mesh modeling method based on the tensegric modeling for the shape design to avoid interference. The result shapes have smooth surface following the obstacle mesh, and the avoidance is guaranteed in the mesh based avoidance under the complex condition despite few user inputs. Additionally, bilateral avoidance is a unique result by the proposal method. In the future, possible and desirable extensions about this method are shown as follows.

- Improvement of the surface quality;
  The smoothness of the deformed mesh seems to be not up to satisfiable level, and the geometric relationship between the meshes has not be estimated quantitatively yet. So we should propose new solutions for these problems.
- Implementation of t-FFD [14];
  t-FFD is a free-form deformation method for the arbitrary mesh, and a dense mesh is deformed effectively and smoothly by a sparse mesh. The detailed feature shape of the interfered mesh can be preserved by implementing t-FFD, and drastic progress of the computational speed can be expected.
- Implementation of efficient mesh interference detection;
  In the proposal method, the mesh interference is detected in a round-robin, so the computational speed can be progressed by using conventional efficient methods [15-17].
- Implementation of the sophisticated user interface of the system

REFERENCES

