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Thoughts on Hierarchical Modeling Methods for Complex Structures

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ABSTRACT

Hierarchical modeling refers to the modeling of parts with solid geometry, material composition, and possibly distributions of properties at multiple scales. Typical solid and heterogeneous models represent geometry at one scale but do not contain geometric information about features at scales that are orders of magnitude smaller. In this paper, the related topics of heterogeneous and hybrid modeling, subdivision methods, and level of detail approaches are surveyed. The capabilities of these technologies are compared against the requirements for modeling cellular materials and other structures with complex geometric and material constructions. A specific hierarchical modeling method is proposed. Three 2D examples are used to illustrate the application of the proposed modeling method.

Keywords: hierarchical modeling, heterogeneous modeling, solid modeling.
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1. INTRODUCTION

Manufactured parts typically have a complex distribution of microstructure, mechanical properties, and sometimes materials. Commercial CAD systems are not able to represent such distributions, but some heterogeneous or multi-material research systems are able to represent at least some of this information. In this work, we are interested in modeling methods that enable the capture of geometry as well as material and microstructure distributions at multiple length scales. This requirement for multiple length scales leads to the term “hierarchical modeling” and is meant to include geometry, material, and microstructure distributions. That is, hierarchical modeling implies heterogeneous modeling, at least in this work.

The specific engineering domain of interest is that of cellular materials and other parts fabricated using additive manufacturing (AM) (i.e., “rapid prototyping”) processes. However, the domain of application of hierarchical modeling is much broader. The concept of cellular materials is motivated by the desire to put material only where it is needed for a specific application. Achieving high stiffness or strength and minimal weight are typical objectives [13, 23]. Multifunctional requirements are also common, such as structural strength and vibration absorption. We hypothesize that designed mesostructures will enable structures and mechanisms to be designed that perform better than parts with bulk or non-designed mesostructures, particularly for multifunctional applications. Testing this hypothesis requires the ability to bridge multiple length scales (micro to macro, or even nano to macro). Fig. 1 shows several images of cellular structures and a cross-section of a metal part fabricated using a laser cladding process (LENS), which illustrates some aspects of typical microstructures that result from laser AM processes.

To achieve hierarchical modeling, it is first helpful to identify specific requirements on heterogeneous and multi-scale methods. The requirements that we propose include the capability to:

- Represent and design with hundreds of thousands of shape elements, enabling large complex design problems as well as designed material mesostructures.
- Represent complex material compositions and microstructures and ensure that they are physically meaningful.
- Determine mechanical properties from material compositions and mesostructures across length scales.
- Ensure that specified shapes, material structures, and properties are manufacturable.
- Ensure that as-manufactured designs achieve requirements.

Of these requirements, the first two are addressed directly in this paper. The other three highlight important properties of the modeling method for supporting other design and manufacturing objectives.

a) extruded shapes b) 3D lattice structure c) microstructure from LENS process

Fig. 1: Example cellular structures and additive manufacturing microstructure.

A Design for Additive Manufacturing (DFAM) framework was introduced in some of our earlier work [23] and is illustrates our desire to achieve design requirements by manipulating both the geometry of a part and its material composition. This framework is based upon the process-structure-property relationships from the materials science domain [19], where analysis of a material consists of examining the microstructure of the material after processing it, and determining its mechanical properties from the microstructure. In materials design, material developers seek to reverse the process by specifying desired behavior, deriving target mechanical properties, designing desired microstructures, and developing manufacturing process conditions to achieve those microstructures.

In the design of cellular materials, we introduce an additional hierarchy into this framework that captures the geometric complexity of the microstructure or material composition. As a result, part behaviors can be controlled by adjusting cellular structure geometry in addition to material microstructure. The concept is illustrated in Fig. 2. Let a solid model with material composition and other material-related attributes be denoted as $\mathcal{S} = (\mathcal{G}, \mathcal{M})$, where $\mathcal{G}$ is the solid geometric model and $\mathcal{M}$ is the attribute space, with some attributes being applied only to regions of $\mathcal{G}$. $\mathcal{S}$ denotes the structure of the design. The manufacturing process space, $\mathcal{P}$, consists of process plans with sequences of operations and values of process variables. Property space $\mathcal{T}$ contains information about part properties that are derivable from S using physical principles; e.g., mechanical, thermal, and electrical properties. Finally, we will add behavior space, $\mathcal{B}$, which contains information about a part's actual and desired behavior given some loading and boundary conditions.

The overall DFAM method consists of a traversal of the framework in Fig. 2 from Behavior to Process, then back again to Behavior. The traversal from Behavior to Process can be called design, where functional requirements are mapped to properties and geometry that satisfy those requirements to structures and through process planning to arrive at a potential manufacturing process. Going in the reverse direction, one can simulate the designed device and its manufacturing process to determine how well it satisfies the original requirements.

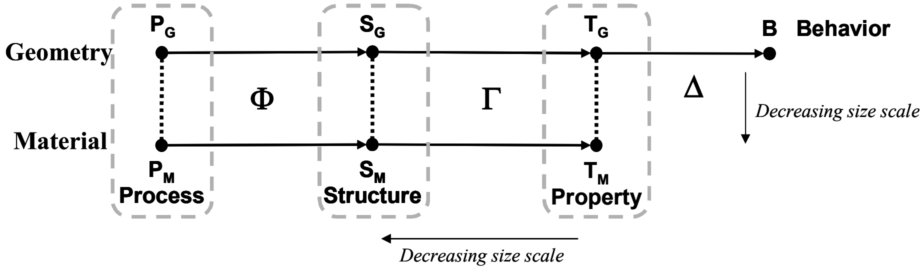


Fig. 2: DFAM framework with geometry and material layers.

In this paper, the area of heterogeneous modeling is surveyed briefly, then a more specific description of multiresolution modeling method is offered in order to identify methods and models from the geometric modeling literature that may contribute to hierarchical modeling. From these surveys, a new hierarchical modeling method is proposed in Section 4. Three simple 2-dimensional examples are presented in Section 5, one that illustrates multi-scale geometric modeling, one that demonstrates multi-scale material modeling, and one that demonstrates randomized fiber orientations that vary with variations in geometry. Conclusions are offered in the final section.

2. HETEROGENEOUS MODELING

Broadly speaking, the objective of heterogeneous modeling is to model a part's geometry and the distribution of materials and other properties throughout the geometry. Many heterogeneous modeling methods have been developed by different research groups. Most of this work can be classified broadly into two categories: spatial discretization methods, and implicit and other non-discretization methods. Research on discretization methods started with the observation that material compositions could be assigned to specific volumes within a part. Finite element modeling approaches assigned material compositions to individual elements or nodes; material compositions within elements were interpolated in a manner similar to stress interpolation using shape functions [14, 15.]. Rather than rely on finite element decompositions, other researchers applied voxel-based representations that utilized spatial occupancy enumeration of part geometry. Again, material composition information was applied to either individual voxels or interpolated over sets of voxels using a part's bounding surface [25, 29]. General cellular decompositions have also received considerable attention. Dutta and coworkers [3, 15, 16] have developed a series of heterogeneous solid modeling approaches over several years. Material compositions are assigned to primitive geometric entities and a set of modeling operations was defined that operate on these entities.

In the area of non-discretized approaches, some researchers have separated the representation of material compositions and properties from the underlying part geometry [27]. Others have utilized implicit modeling approaches, which has advantages in that a common mathematical model is used for both geometry and material composition [12, 24]. A method based on hypertextures was proposed by Park et al. [20] and extended [21] that provides more intuitive user controls, according to the developers.

Probably the most sophisticated heterogeneous object modeling approach, due to Kumar et al. [15], is based on fiber bundles. A fiber bundle model includes not only a geometric model of the part, but also a mapping from points located within the part to additional properties such as material composition. Thus geometry is considered to be the "base attribute," and each point in the object is considered to correspond to a point in the base (Euclidean) space. The object geometry is represented as a description of the overall geometry together with a finite set of disjoint decompositions which will be used to map into the attribute space.

A typical model of geometry and material composition will be presented. A volume fraction-based *n*-phase, graded material model will be used here. The material composition at a point is represented by volume fractions of n_m phases: (v_1, v_2, ..., v_{nm}). Then, the space of material compositions is given by Eqn. 1:

$$\mathcal{M} = \left\{ \mathbf{m} = \left(v_1, \ldots, v_{n_m} \right) \in R^{n_m} \mid \sum_{i=1}^{n_m} v_i = 1 \right\} \tag{1}$$

The condition ensures that material combinations at all points are feasible (volume fractions sum to 1). For example, a combination of three materials could be given as (0.4, 0.5, 0.1), which indicates that the geometric domain is composed of 40% material 1, 50% material 2, and 10% material 3.

For convenience, we will introduce a set of pure materials, denoted by the standard basis in *n*-space; i.e., $m_1 = (1, 0, 0)$, $m_2 = (0, 1, 0)$, $m_3 = (0, 0, 1)$ if three materials are being modeled ($n = 3$). We will denote this standard basis of materials by $\mathbf{M}^n$. Then, a particular material combination will be given as $\{v_1 \cdot m_1, v_2 \cdot m_2, v_3 \cdot m_3\}$, where again the v_i represent volume fractions. The space of material compositions can be rewritten as

$$\mathcal{M} = \mathbf{V} \times \mathbf{M}^n = \left\{ \mathbf{v} \bullet \mathbf{m}, \mathbf{m} \in \mathbf{M}^n, \mathbf{v} = \left(v_1, \ldots, v_{n_m} \right) \in R^{n_m} \mid \sum_{i=1}^{n_m} v_i = 1 \right\} \tag{2}$$

Heterogeneous representations are often represented by the combination of geometry models, from a space of geometric entities $\mathcal{G}$, and material models. As such, the space of heterogeneous models is given by:

$$\mathcal{S} = \mathcal{G} \times \mathcal{M} \tag{3}$$

3. MULTI-RESOLUTION MODELING

Multi-resolution modeling methods are used typically for model simplification or for level-of-detail control for visualization or virtual reality applications. Multi-resolution modeling originated from the desire for computational efficiency in computer graphics from the mid-1970's, when James Clark [10] introduced hierarchically structured object representations, with progressively more detailed representations for viewing objects that are increasingly closer to the viewer. An example used to motivate this work is the replacement of a complex model with a coarse tessellation (e.g., replace a sphere with a dodecahedron) if the viewer is far from the sphere, which is an example of level-of-detail control. The first three steps of Catmull-Clark subdivision of a cube is shown in Fig. 3. In the limit, the subdivision converges to a sphere. This concept was extended to objects modeled with parametric surfaces [6], which gave rise to the subdivision surface and volume methods. Broadly speaking, literature in this area can be categorized based on subdivision or discrete geometric modeling approaches.

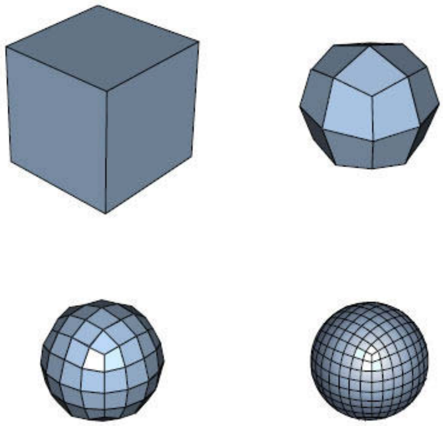


Fig. 3: First three steps of Catmull–Clark subdivision of a cube [28].

3.1 Subdivision Approaches

Subdivision approaches to multiresolution modeling are appealing in that they enable the addition of shape details or smoothing by subdividing an appropriate mesh. Some of the simplest subdivision approaches work with a control polygon (curve) or polyhedron (surface) as the mesh is subdivided. Different approaches worked with triangular meshes or quadrilateral meshes [6]. Other approaches use simplicial or other complexes for their mesh. From a design perspective, an interesting application of subdivision methods is to add shape detail at a particular subdivision step by manipulating the mesh or control vertices. If smaller shape details are desired, one or more subdivision steps can be performed, then the user can manipulate the mesh at the resulting level.

As stated, the mesh can represent a control polygon or polyhedron. Tensor-based Bezier, B-spline, or NURBS parametric curve/surface formulations are often used [2]. That is, each face of a subdivision (see Fig. 3) is modeled by a parametric surface; vertices of the mesh are corner points of the surface's control polyhedron. Others have taken a non-tensor-based approach, utilizing simplicial meshes and multivariate splines [7], arguing that non-tensor-based methods are needed for non-structured domains and they can achieve comparable smoothness with relatively low polynomial degree.

Subdivision approaches have been extended to deal with volumes, enabling multiresolution modeling of 3D solids, not just their bounding curves and surfaces.

A line of research has explored the integration of non-manifold and manifold regions [11, 30]. Chang and Qin [8] presented a unified approach to non-manifold subdivision modeling, using a simplicial complex for their domain and box splines for interpolating surfaces. Subdivision rules are modified by spatial averaging to achieve smoothness conditions. They developed special subdivision rules for handling regions between manifold and non-manifold topologies.

The usage of subdivision approaches for modeling cellular structure is of questionable value since the main objective is to generate smooth surfaces and/or shape detail. However, the decomposition of simplicial complexes can model finer and finer partitions of part geometry, which may be useful in modeling composite materials and possibly lattice structures. The interpolation of shapes using splines may be extended to modeling distributions of material compositions, particles or fibers for composite materials, and grains in metals.

3.2 Discrete Geometric Models

Typically, discrete geometry refers to tessellated surfaces, enumeration methods for decomposing solids (e.g., octrees), and cellular decompositions (e.g., simplicial complexes). Part models consist of discrete vertices, edges, faces, tetrahedra, etc. A class of methods known as level-of-detail (LOD) techniques have been developed for visualization and virtual reality applications [5, 31]. Multiple part representations are used, at different levels of detail, to minimize rendering times when parts are far away from the viewer, while enabling accurate rendering when the viewer is close. The subdivision methods are used often for generating these representations, an example of which is shown in Fig. 3.

Recently, other discrete representations have been developed with the objective of providing efficient methods of constructing solid models, visualizing large models, or generating models for other applications such as analysis or manufacturing. In a variation of the octree representation, a discrete representation can be generated by classifying, against a part model, a set of sample points distributed on the nodes of a regular, axis-aligned grid [5]. Nodes lying inside the part model are colored black (for instance), while nodes outside the model are colored white. The part boundary can be represented as a collection of sticks, which are grid edges that connect black and white nodes.

Chen and Wang [9] introduced the layered depth-normal images (LDNI) which can be used with graphics hardware to very quickly compute Boolean operations between solid models. The LDNI representation consists of three images of the part, one image along each coordinate axis. A ray from each pixel is projected into the part space to intersect with the part model at point p_{ij}, for the i,j pixel. Associated with each pixel of each image is the depth of the surface from the viewing plane and the surface normal at p_{ij}. Others have presented similar work, based on sampled rays, including ray-rep [17] and Layered Depth Images (LDI) [26], without surface normal information. Some applications included solid model construction, collision detection, and visualization.

One issue associated with the use of sampled discrete models is the need to convert from boundary representations to the discrete representation, and then back again for visualization (and other modeling) purposes. Isosurface extraction algorithms are needed to reconstruct part surfaces from a discrete representation [5]. Given a set of sampled points and/or depths, many ambiguities arise when trying to reconstruct surfaces, such as locations of surface boundaries, surface continuities, sharp corners, and the like. Marching cubes algorithms and their variants [18] are often used to compute surfaces, and various heuristics are often applied to resolve ambiguities [1].

It is not clear if the types of discrete modeling approaches described here will benefit hierarchical modeling of cellular structures. These approaches may be helpful to increase efficiency for various computations, but probably will not serve as the primary representation for composite materials or lattice structures.

3.3 Hybrid Approaches

A variety of other approaches have been investigated. One approach integrated precise solid modeling into a virtual reality (VR) environment [31]. The representation consisted of six levels of information: assemblies, parts, features, feature elements, geometric and topological relationships, and polygons. These six levels were organized into three layers, including a high layer constraint-based model for precise object definition, a mid-layer constructive solid geometry/boundary representation hybrid solid model for supporting the hierarchically geometry abstractions and object creation, and a low

layer polygon model for real-time visualization and interaction in the VR environment. The constraint system was used to support object creation and also to support VR interactions.
Another approach sought to integrate NURBS-based design, analysis, and optimization using a hierarchical partition of unity construction [22]. This approach shares some similarities with the subdivision approaches. It also integrates consideration of material distributions within part models, so is a heterogeneous representation. In their method, NURBS are used to discretize the geometry, material, and behavioral fields in their representation. Behavioral fields are associated with the geometric primitives and their values are determined by a global analysis problem. The authors showed that this NURBS construction leads to recursive partition of unit constructions on the knot spaces due to the partition of unity properties of NURBS basis functions. Examples demonstrated applications to simultaneous topology and material distribution optimization, shape and size optimization, and optimal design to mitigate the effects of cracks.

4. HIERARCHICAL MODEL

A hierarchical modeling approach is proposed that enables the specification of geometric and material composition models at multiple size scales. The model is based on the concepts of subdivision and heterogeneous modeling methods introduced earlier. The approach is also based on concepts of textured surfaces and solids.

4.1 Hierarchical Modeling Concepts

The proposed hierarchical modeling approach is to support geometric and material models at multiple size scales. A part model can be given as a 3-D geometric model and a material composition at the macro scale. In addition, the designer can specify a more precise geometry and material composition as smaller size scales. For example, a part may have a shape appropriate for a product housing and is to consist of nylon with 10% by volume of a nano-clay for stiffening. At the macro scale, the nano-clay filler (say, with mean diameter 500 nm) is too small to represent exactly in the geometric model. However, the nano-clay geometry becomes important when considering the part microstructure. In this case, the micro-scale geometry becomes important and the material composition cannot be assumed to be uniformly distributed.

In heterogeneous modeling, a part model contains a geometric model and a material model. Typically, the part modeling space, $\mathcal{S}$, is given by the product of a space of 3-D geometric models, $\mathcal{G}$, and a material composition space, $\mathcal{M}$ (recall Eqn. 3). In the proposed hierarchical modeling approach, part models will be composed of a macro scale part model, $\mathcal{S}_M$, one or more smaller scale models, denoted $\mathcal{S}_\mu$ for micro-scale or $\mathcal{S}_n$ for nano-scale, and a set of rules ($Rules_{M\mu}$, $Rules_{mn}$) that relate the different size scales, as given in Eqn. 1:

$$\mathcal{S} = \mathcal{S}_{\mathrm{M}} \times \mathrm{Rules}_{\mathrm{M}\mu} \times \mathcal{S}_{\mu} \times \mathrm{Rules}_{\mu\mathrm{n}} \times \mathcal{S}_{\mathrm{n}} \tag{4}$$

4.2 Hierarchical Geometric Models

We will borrow methods from the subdivision literature (e.g., [2, 6, 8]) in order to develop rules for relating size scales. Curves, surface, and solids can be subdivided using different methods. In general, any subdivision method can be used to divide part geometry into smaller regions. When geometric subdivisions become appropriately small, the geometry for the smaller size scale is inserted into each subdivision. The material composition models are handled similarly. Fig. 4 shows an example with a hollow diamond microstructure (unit size) that is mapped into a subdivided region in a larger 2D shape. In the microstructure, the solid region is all of a given material, while the white regions are voids. The macro-scale geometric domain is composed of the same material as the microstructure. In this case, $g_M \in \mathcal{G}_M$ is the macro-scale geometry, $\{g_{\mu_s}, g_{\mu_v}\}$ is the microstructure geometry ($g_{\mu_s}, g_{\mu_v} \in \mathcal{G}_\mu$), where g_{μ_s} represents solid and g_{μ_v} represents void, the two materials are $m_1 = (1, 0)$, $m_2 = (0, 1)$ (representing solid and void, respectively), $m_M \in \mathcal{M}_M$ is m_1, and $m_\mu \in \mathcal{M}_\mu$ is $\{m_1, m_2\}$. As a result, $s_M = (g_M, m_M)$, $s_M \in \mathcal{S}_M$, $s_\mu = \{(g_{\mu_s}, m_1), (g_{\mu_v}, m_2)\}$, $s_\mu \in \mathcal{S}_\mu$, $s_n = \varnothing$, and $rules_{M\mu}$ = {subdivision by mapping, relate size scales via linear interpolation}.

In this paper, we will use a simple mapped mesh subdivision approach, partitioning surface patches into smaller patches (quadrilateral subdivision) and partitioning solid volumes into smaller hexahedral volumes. Geometry from the micro-scale model is then mapped into each subdivided patch or volume with the appropriate material composition.

We can go a step further and generate intermediate models between given size scales. The approach taken in this paper is to map geometry from the smaller scale into regions generated at each subdivision step. Material compositions are interpolated from those given at the larger and smaller scales. B-spline models are used to interpolate material compositions across the subdivision steps. This enables the use of linear or quadratic interpolation across many subdivisions. Resulting intermediate models will share some of the shape and material composition characteristics of the larger and smaller scale models and may be suitable for visualization or analysis at intermediate size scales.

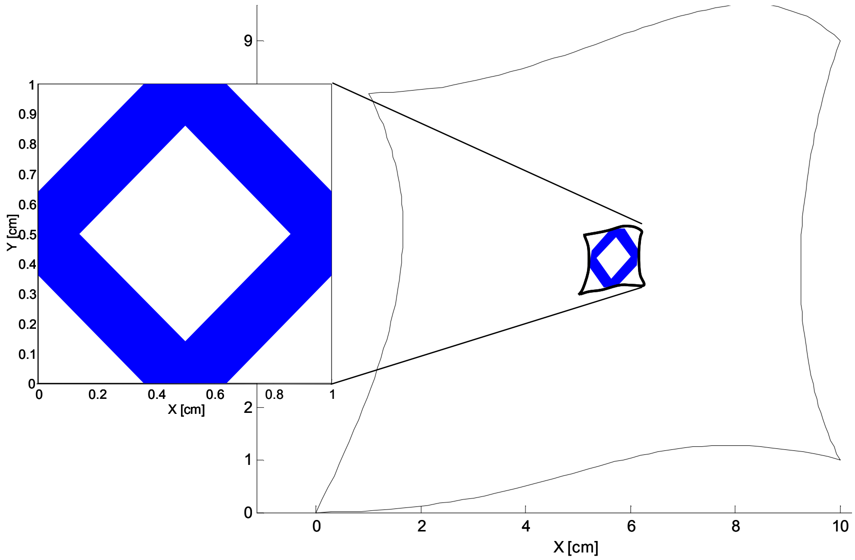


Fig. 4: Mapping a unit geometric primitive into subdivided region.

4.3 Hierarchical Material Models

In a manner similar to that of hierarchical geometric models, hierarchical material models will be constructed with a material distribution specified at the macro-scale and a different material model specified at a smaller scale. For cases where different materials are represented as separate phases at micro or nano-scales, we can continue the example from Section 4.1. The nylon material with 10 percent nano-filler would be represented as $m_M = \{0.9\ m_1, 0.1\ m_2\}$, where $m_1 = (1, 0)$ representing nylon and $m_2 = (0. 1)$ representing nano-filler at the macro-scale. That is, at the macro-scale, the material is viewed as a continuum with a combination of materials 1 and 2. At the micro-scale, the different material phases are represented separately. The matrix material, m_1 occupies the geometry of the part, or the geometry of a subdivision, while the nano-filler has a rectangular geometry that is embedded in the matrix.

As an approximation, the hierarchical material model can represent material compositions in the part by interpolating between the macro-scale and micro-scale, or nano-scale, material models. One simplest approach is to progressively increase the content of each material phase in different subdivisions as the part model is progressively subdivided. The choice of which material phase to increase in each subdivision should be made on a proportional basis. For the example of 90 percent nylon and 10 percent nano-filler, 9 out of 10 subdivision should favor nylon, while the 10th subdivision has an increased percentage of nano-filler. Such intermediate models will have material distributions that approximate the desired distribution in the macro-scale and micro or nano-scale models.

5. EXAMPLES

Three examples will be used to illustrate the proposed hierarchical modeling method, one that uses simple geometry and material compositions, one that uses more complex micro-scale geometry, and one that includes a randomized fiber orientation to mimic that caused by injection molding.

5.1 Rectangular Plate with Particulate Filler

The first example has a simple rectangular shape 10x8 cm in size, with a particulate filler that is uniformly distributed throughout the part at a volume fraction of 0.1 (10% by volume). For simplicity, we will assume that the particles are roughly rectangular in shape about 2.5 mm in size. At each subdivision step, the geometry will be decomposed such that rectangles are divided into 4 equally sized rectangles. As such, 5 subdivision steps are needed to generate the "micro-scale" from the macro-scale. Linear interpolation of material compositions will be used. For visualization purposes, red will be used to indicate the polymer material that comprises the part, and blue will be used for the particles. The initial material composition (90% "red" and 10% "blue") corresponds to an initial color that is a reddish purple.

Using the notation from earlier, the macro-scale geometry, $g_M \in \mathcal{G}_M$, is a rectangle, the micro-scale geometric models $\{g_{\mu_1}, g_{\mu_2}\}$ ($g_{\mu_1}, g_{\mu_2} \in \mathcal{G}_\mu$) are also rectangles, where g_{μ_1} represents the matrix material m_1 and g_{μ_2} represents the nano-filler m_2, the two materials are $m_1 = (1, 0)$, $m_2 = (0, 1)$, $m_M \in \mathcal{M}_M$ is ($0.9m_1$, $0.1\ m_2$) and $m_\mu \in \mathcal{M}_\mu$ is $\{m_1, m_2\}$. As a result, $s_M = (g_M, m_M)$, $s_M \in \mathcal{S}_M$, $s_\mu = \{(g_{\mu_1}, m_1), (g_{\mu_2}, m_2)\}$, $s_\mu \in \mathcal{S}_\mu$, $s_n = \varnothing$, and $rules_{M\mu}$ = {subdivision by mapping, relate size scales via linear interpolation}.

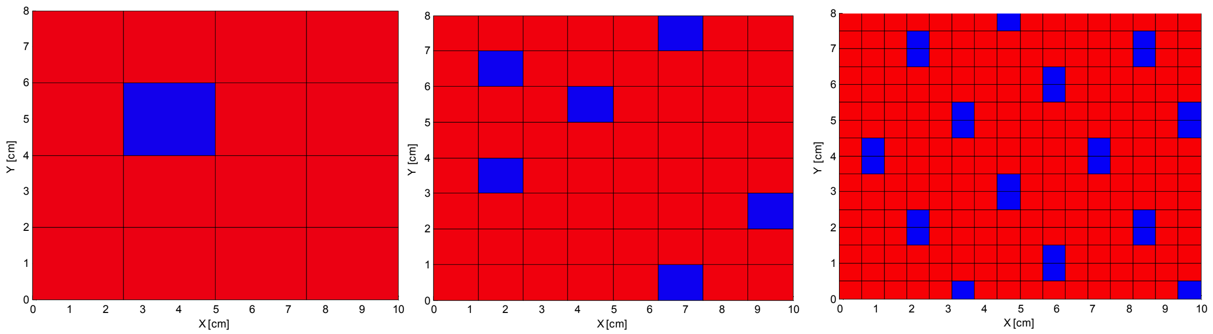
Fig. 5 shows the results of the second, third, fourth, and fifth subdivision steps. Note that the red and blue colors become more pure until step 5 when the colors are pure red and pure blue. Every tenth cell in the model has its concentration of material 2 (blue) increased, while the other nine cells increase in material 1. At the fifth step, every tenth cell will be 100 percent material 2, which is consistent with the 10 percent nano-filler (material 2) specification. The material compositions in the reddish and bluish cells are given in Table 1 for the macro-scale part and for each subdivision step. Note that in subdivision step 1, only the geometry is divided. The material composition remains the same, since only 4 cells were created, far less than the 10 cells necessary to start introducing the particles into the material composition.

5.2 Curved Plate with Cellular Structure

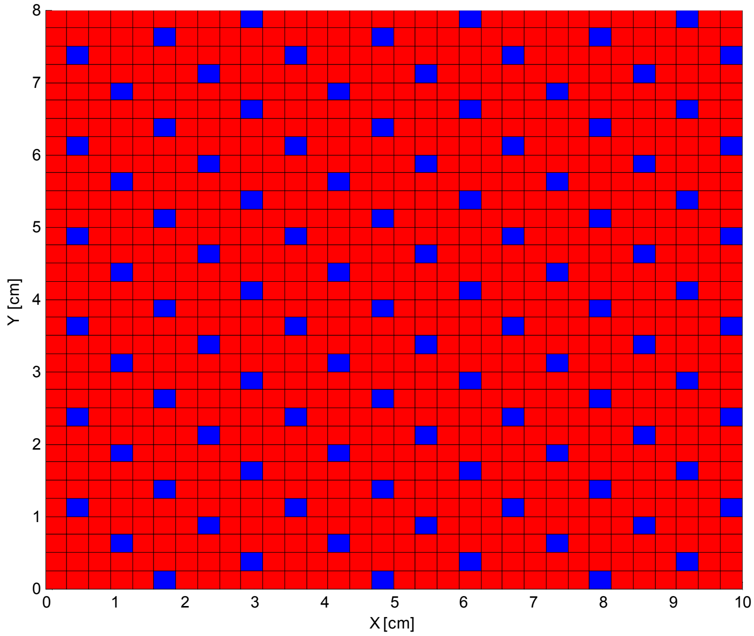
The second example illustrates the capability of dividing curved geometry, as well as the specification of different geometries at different size scales. Again, the plate is about 10x8 cm in size, but with curved boundaries. Also, the plate is planar and is modeled as a bicubic Bezier surface patch. The micro-scale geometry is the hollow diamond shown in Fig. 4. As with the previous example, the micro-scale geometry is enlarged for purposes of illustration, so the diamond-shaped cells are 2.5 mm on a side, which results in the need for 5 subdivision steps. A mapped mesh approach is taken for subdivision. At each subdivision step, each quadrilateral cell is divided into four smaller cells as in the previous example. The material composition at the macro-scale is all one material, represented by a blue color. At the micro-scale, the hollow diamonds are all of the same material. The void regions are represented as a white color (no material).

Subdivision Step	Matrix Cells		Nano-Filler Cells	
	Proportion of Material 1	Proportion of Material 2	Proportion of Material 1	Proportion of Material 2
0	0.8	0.2	-	-
1	0.8	0.2	-	-
2	0.85	0.15	0.15	0.85
3	0.9	0.1	0.1	0.9
4	0.95	0.05	0.05	0.95
5	1	0	0	1

Tab. 1: Material compositions for each subdivision step.



a) 2nd subdivision step b) 3rd subdivision step c) 4th subdivision step



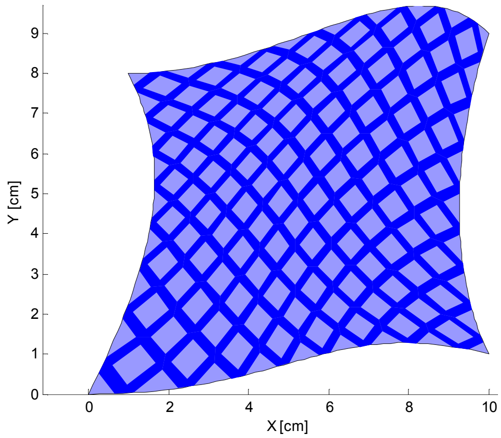
c) 5th subdivision step.

Fig 5: Hierarchical material example.

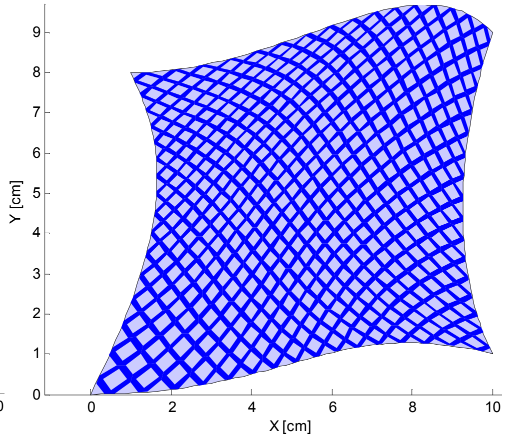
At intermediate size scales, the material composition will be interpolated linearly between the material compositions at the macro-scale and the micro-scale. As a result, only the void regions (micro-scale) vary as the size scale is reduced.
The third, fourth, and fifth subdivision steps are shown in Fig. 6a,b,c, respectively. Note that the “void” region becomes lighter as the size scale decreases, becoming white in the fifth step.

5.3 Randomized Fiber Orientation
The third example illustrates how microstructure can be varied as part geometry varies. In this case, the orientation and density of fiber filler will vary to mimic variations observed in injection molded parts. In particular, fiber distributions become denser and aligned with the flow as polymer fills the mold cavity. For this example, the microstructure geometry is a quadrilateral region with a 6 mm long fiber that has a random orientation and position within the quadrilateral. As the polymer flows into narrowed regions, the fibers become oriented in the flow direction. As shown in Fig. 7, the polymer flow is from left to right; one can observe that fibers are randomly oriented in the wide region on the left, but become progressively aligned horizontally as the narrower right side is reached. The fiber

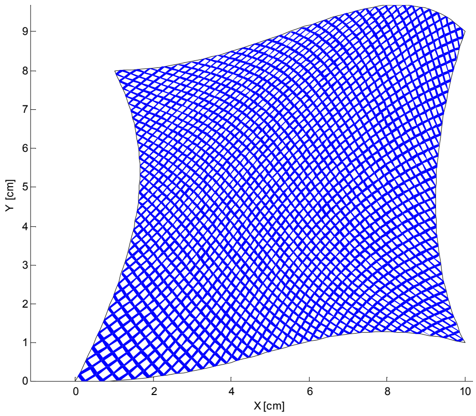
density is nominally about 10 percent, but becomes denser as one moves to the right. Fig. 7 illustrates the 5th subdivision step.



a) 3rd subdivision step



b) 4th subdivision step



c) 5th subdivision step.

Fig. 6: Hierarchical geometry example.

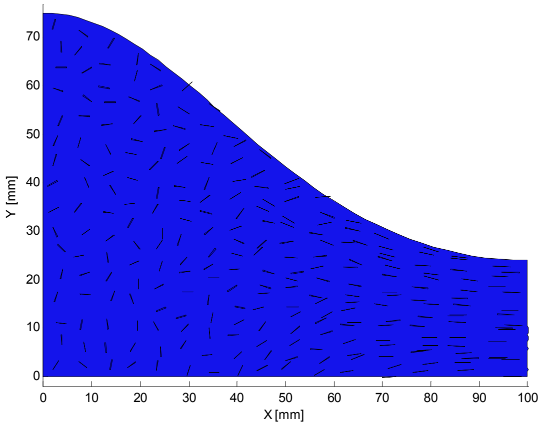


Fig. 7: Randomized fiber orientation example; 5th subdivision.

6. CLOSURE

Hierarchical modeling in this paper refers to the modeling of parts with solid geometry, material composition, and possibly distributions of properties, all at multiple length scales. A brief survey was performed on topics related to hierarchical modeling, specifically heterogeneous and hybrid modeling, subdivision methods, and level of detail approaches. The capabilities of these technologies were compared against the requirements for modeling cellular materials and other structures with complex geometric and material constructions. A specific hierarchical modeling method was proposed. Three 2D examples illustrated the application of the proposed modeling method. From this preliminary work, the following conjectures can be made:

- new types of models are needed to represent objects with geometry, material, and property distributions at multiple length scales, since existing approaches to heterogeneous and hybrid modeling are insufficient;
- the proposed approach that combines subdivision with material and/or property interpolation over the subdivided regions is effective in capturing the desired distributions at multiple length scales;
- two complementary approaches seem promising: the proposed subdivision approach presented here and the implicit modeling approach, particularly those that are based on hypertextures;
- since the examples are simple and only 2D, considerably more work is needed to more fully develop the proposed hierarchical modeling methods and explore its application to more complex part designs.

7. ACKNOWLEDGEMENTS

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