Re-designing Heterogeneous Objects by Attribute Discretization

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ABSTRACT

For rapid prototyping of a heterogeneous attributed computer-aided design object, the continuous attributed object model has to be discretized by replacement with homogeneous attributes over spatial domain during the build path generation with the existing layered manufacturing technology. With this motivation, research is undertaken on re-designing heterogeneous continuous-attributed objects. The varying attributes of these objects are mapped into homogeneous attributes on finite spatial enumeration grids for attribute discretization. The methodology and mathematical study on heuristic searching of optimal grid configuration are presented in this paper. Essentially, this research addresses a core heterogeneous object modeling issue in CAD regarding the relationship between geometric spatial enumeration models and attribute field.

Keywords: Heterogeneous Object Modeling, Re-design, Discretization, Meshing, Rapid Prototyping

1. INTRODUCTION

For Rapid Prototyping (RP) of a heterogeneous attributed Computer-Aided Design (CAD) object, the continuous attributed object model has to be discretized by replacement with homogeneous attributes over spatial domain during the build path generation with the existing layered manufacturing technology. As a result, an attribute approximation error exists in the fabricated prototypes. Since current RP research on CAD objects only focus on homogeneous solid models for the reduction in surface roughness, the research problem discussed here could be the essential complement to any work that addresses the need to preserve the attribute information in RP.

2. REVIEWS ON RELATED WORK

2.1 Heterogeneous Object Modeling

Traditional CAD systems can only represent the geometry and topology of an object. No material information is available within the representation. Obviously, this poses a great limitation for the downstream applications of the representation. A modern day CAD system should therefore be able to model material information inside an object. Studies on Heterogeneous Object Modeling (HOM) have been a hotspot in recent years [1-7]. In contrast to traditional solid modeling which assumes the material inside a solid is homogeneous, HOM allows material definition and variation inside the solids. Heterogeneous objects are generally classified as objects with clear material domains and also those with continuous material variations. It has been widely accepted that heterogeneous components have some key advantages over homogeneous objects: anisotropic properties can be obtained; different combinations of various materials can be achieved; and traditional limitations due to material incompatibility (e.g. stress concentration, non-uniform thermal expansion) can be solved by gradual material variations.

Future CAD systems should then have to accommodate both material and structure data, including multiple materials within a given part and functionally gradient properties. Designers will then be able to design the geometry, material composition variations (e.g. Functionally Graded Material, FGM) and structure variation (e.g. laminates). Optimization may then be carried out not only on geometry design but also on material composition and micro-structure to achieve the desired part properties. This requirement is one of the primary drivers for developing the fabrication of functional prototype in Layered Manufacturing (LM).

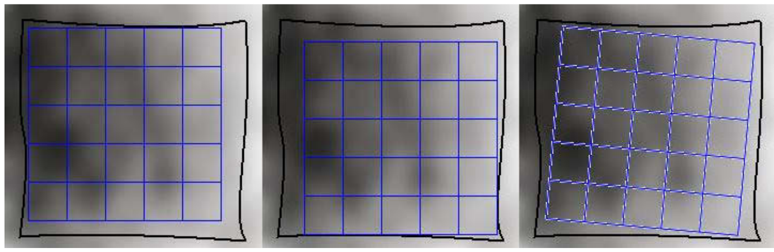

Fig. 1. The 3-DOF configurations of voxel array locate at a 2-D attributed object: translation (left & center) and rotation (right).

2.2 Preservation of Surface Profile in Rapid Prototyping

RP systems are usually based on LM technology, which fabricate 3-D parts by stacking 2-D layered contours. The Stereolithography Apparatus (SLA) is one of the most popular prototyping machines, which fabricates parts out of photo-sensitive polymeric resins directly from a CAD model without intermediate tooling by stacking thin layers of the parts' cross-section on top of each other. Stair-stepping effect then occurs when an inclined surface is fabricated. Such effect is one of the major factors that worsen surface quality of parts fabricated in many RP systems such as SLA. Due to the poor surface quality, most parts so fabricated are not suitable for direct use in functionality testing and tool making. These parts require post-processing to improve surface roughness. Hence, there is a strong need to develop algorithms which reduce the post-processing times and costs. In most cases, the layer stacking direction is selected according to the machine operator's experience and discretion.

Some studies have been carried out either to find appropriate objective functions for optimization or to develop expert systems that can determine the optimal fabrication direction [8-10]. The simplest idea for reducing surface roughness is to reduce the layer thickness of a slice, which leads to better-finished parts but increases the build time. Several methods have been developed to improve the fabrication process such as meniscus smoothing, diagonal irradiation, and the adaptive layer thickness system [10].

3 PROBLEM FORMULATION AND THE PROPOSED APPROACH

This paper presents the concept of re-designing heterogeneous continuous-attributed objects with attribute discretization, by mapping the varying attributes of these objects into homogeneous attributes on finite geometric partitions. Essentially, this research addresses a core heterogeneous object modeling issue in CAD regarding the relationship between geometric spatial enumeration models and attribute field.

For practical applications, the output heterogeneous (multiple-material) objects [3] have to be functionally evaluated accounting for the adjustment in discretization. In the case when this requirement is not fulfilled, the re-design of CAD model's shape and/or original continuous attribute are required such that the output multiple-material objects can be functionally "closely" equivalent to the CAD model.

The spatial enumeration grid representations (e.g. pixel & voxel) are adopted in this study as the geometric partitions. For a specific shape/size of the meshing grid, an optimal meshing configuration(s) must exist among the bounded (i.e. available configurations within the object boundary) 6 Degree-of-Freedom (DOF) configuration space (3 for translation & 3 for rotation in 3-D geometric space) for the minimization of discretization/approximation error with or without other cost function constraint(s). For the case of a 2-D attributed object, the configuration space is in 3-DOF (Figure 1). Since it is computationally infeasible to check for too many configurations evenly over the entire configuration space for the solution(s) of optimal grid configuration, a heuristic search is adopted in this study. By calculating the first derivatives of the grid array's total approximation error with respect to the array's geometric coordinates at each candidate configuration as illustrated in the next section, the lowest value among the first derivatives of the voxel array in x- and y- translations and rotation is adopted for shifting [11] the array rigidly over the 6-DOF configuration space with finite small searching step size, and can iteratively shift to configurations with lower and lower total approximation error. Hence, the grid array will heuristically converge to the global or local optimization recursively, starting from any initial configuration(s).

4. MATHEMATICAL STUDY ON GRID ARRAY

4.1 Overall Approximation Error on Grid Array

Let $A_{(x)}$, $A_{(x,\ y)}$ and $A_{(x,\ y,\ z)}$ be the explicit known values of scalar attribute in 1-D, 2-D and 3-D geometric space respectively. Also, let M be the averaged attribute value of a specific 1-D, 2-D or 3-D grid defined as:

$$M = \int A_{(x)}\, dx / G \tag{1a}$$
$$\text{(OR)}\ \iint A_{(x,\ y)}\, dx\, dy / G^2 \tag{1b}$$
$$\text{(OR)}\ \iiint A_{(x,\ y,\ z)}\, dx\, dy\, dz / G^3 \tag{1c}$$

Then, the overall approximation error (ERROR) in discretizing a 3-D continuous-attributed object into N voxels (for i = 1, 2, …, N) is defined as:

$$\text{ERROR} = \sum_i \iiint \|A_{(x,\ y,\ z)} - M_i\|\, dx\, dy\, dz \tag{2}$$

4.2 Shifting an Interval on 1-D Attribute

With a graphic interpretation (Figure 2), we have the following findings in Eqs. (3) & (4) on the first and second derivatives of M with respect to the geometric coordinate x, expressed in terms of the attribute values $A_{(P+G)}$ & $A_{(P)}$ and their first derivatives with respect to the geometric coordinate x.

$$dM/dx = (A_{(P+G)} - A_{(P)}) / G \tag{3}$$

$$dM^2/dx^2 = (dA_{(P+G)}/dx - dA_{(P)}/dx) / G \tag{4}$$

Also, with a graphic interpretation (Figure 3), the first derivative of M with respect to the geometric coordinate x can also be expressed in Eq. (5) below. The $\text{Length}_{\text{below}}$ and $\text{Length}_{\text{above}}$ in Eqs. (5), (7) & (8) refer to the total lengths of the intervals which have the attribute values below and above the averaged value (M) respectively. The approximation error (D) of a single 1-D grid/interval is defined in Eq. (6) below, similar to the expression in Eq. (2). Hence, the first derivative of D with respect to the geometric coordinate x and/or M can be expressed in Eqs. (7) & (8).

By Eq. (3),

$$dM = dx^*(A_{(P+G)} - A_{(P)}) / (\text{Length}_{\text{below}} + \text{Length}_{\text{above}}) \tag{5}$$

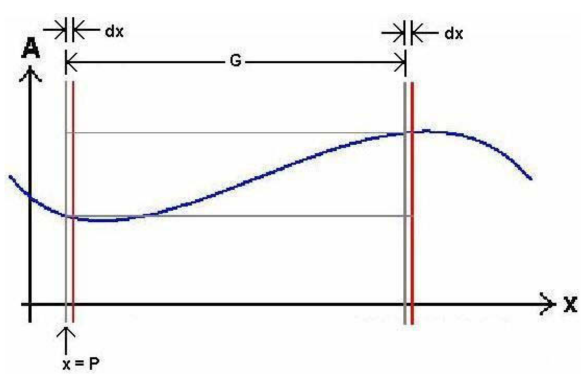

Fig. 2. A graphic interpretation on shifting the region of a single meshing interval [P, P+G] on 1-D attribute

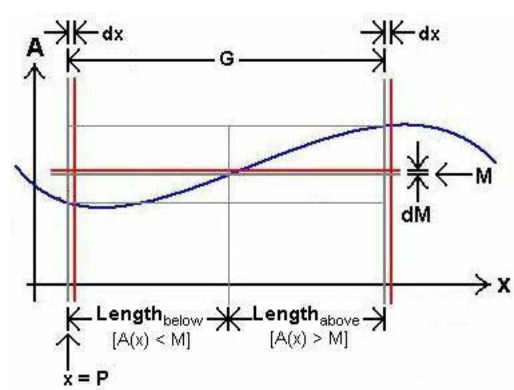

Fig. 3. A graphic interpretation on differentiating the approximation. error with respect to the region of a single meshing interval [P, P+G] on 1-D attribute

$$\text{For } D = \int \|A_{(x)} - M\|\, dx \tag{6}$$

$$dD = (\|A_{(P+G)} - M\| - \|A_{(P)} - M\|)^*dx + (\text{Length}_{\text{below}} - \text{Length}_{\text{above}})^*dM \tag{7}$$

By Eq. (5),

$$dD/dx = 2^*(\text{Length}_{below}^* || A_{(P+G)} - M || - \text{Length}_{above}^* || A_{(P)} - M ||) / (\text{Length}_{below} + \text{Length}_{above}) \quad (8)$$

4.3 Shifting a Pixel Grid on 2-D Attribute

4.3.1 Translational Shifting

With a graphic interpretation illustrating the x-directional case (Figure 4), the first derivative of M with respect to the geometric coordinate x can be expressed in Eq. (9) below. The approximation error (D) of a single 2-D grid (pixel) is defined in Eq. (10) below, similar to the expression in Eqs. (2) & (6). Also, the Area_{below} and Area_{above} in Eqs. (11) & (12) refer to the total areas of the regions which have the attribute values below and above the averaged value (M) respectively. Hence, the first derivative of D with respect to the geometric coordinate x and/or M can be expressed in Eqs. (11) & (12).
Integrating over [0, G], we have

$$dM = dx^*\{\int A_{(G,\, y)}\, dy - \int A_{(0,\, y)}\, dy\} / G^2 \quad (9)$$

$$\text{For } D = \iint || A - M ||\, dx\, dy\,, \quad (10)$$

and by integrating over [0, G], we have

$$dD = dx^*\{\int || A_{(G,\, y)} - M ||\, dy - \int || A_{(0,\, y)} - M ||\, dy\} + (\text{Area}_{below} - \text{Area}_{above})^* dM \quad (11)$$

By Eq. (9),

$$dD/dx = \{\int || A_{(G,\, y)} - M ||\, dy - \int || A_{(0,\, y)} - M ||\, dy\} + (\text{Area}_{below} - \text{Area}_{above})^* \{\int A_{(G,\, y)}\, dy - \int A_{(0,\, y)}\, dy\} / G^2 \quad (12)$$

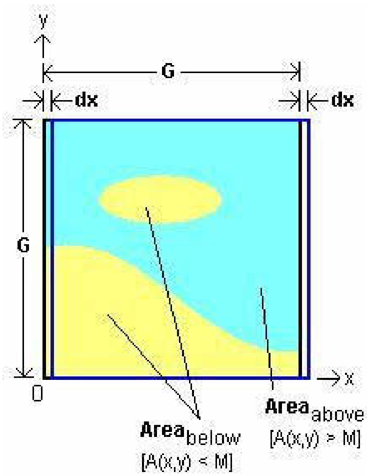

Fig. 4. A graphic interpretation on translational shifting of a single pixel grid on 2-D attribute in positive x-direction.

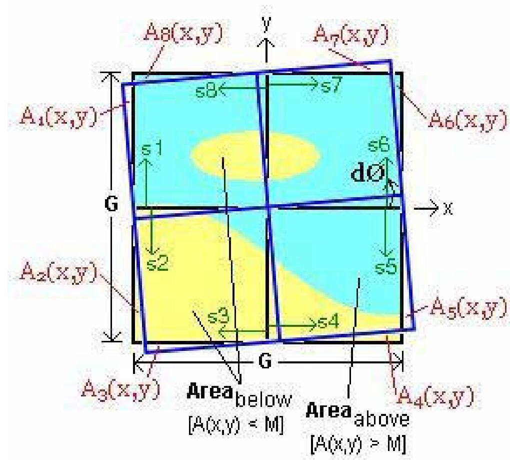

Fig. 5. A graphic interpretation on rotational shifting of a single pixel grid on 2-D attribute in counter-clockwise.

4.3.2 Rotational Shifting

With a graphic interpretation (Figure 5), the first derivative of M with respect to the geometric coordinate Ø can be expressed in Eq. (13) below. $A_{1(x,\, y)}$, $A_{2(x,\, y)}$, …, $A_{8(x,\, y)}$ in the following equations below refer to the specific attribute values along the segments of pixel boundary illustrated in Figure 5. Also, s_1, s_2, …, s_8 in the following equations below refer to the integral variables corresponding to the $A_{1(x,\, y)}$, $A_{2(x,\, y)}$, …, $A_{8(x,\, y)}$ respectively. The approximation error (D) of a single 2-D grid (pixel) is defined in Eq. (10) previously. Moreover, the $Area_{below}$ and $Area_{above}$ in Eqs. (14) & (15) refer to the total areas of the regions which have the attribute values below and above the averaged value (M) respectively, having the same definitions in the Subsection 4.3.1. Hence, the first derivative of D with respect to the geometric coordinate Ø and/or M can be expressed in Eqs. (14) & (15).

Integrating over [0, G/2], we have

$$\begin{aligned} dM = \{&[\int(A_1*s_1*dØ)ds_1 + \int(A_3*s_3*dØ)ds_3 + \int(A_5*s_5*dØ)ds_5 + \int(A_7*s_7*dØ)ds_7] \\ &- [\int(A_2*s_2*dØ)ds_2 + \int(A_4*s_4*dØ)ds_4 + \int(A_6*s_6*dØ)ds_6 + \int(A_8*s_8*dØ)ds_8]\} / G^2 \end{aligned} \tag{13}$$

With Eq. (10), and by integrating over [0, G/2], we have

$$\begin{aligned} dD = dØ*\{&[\int(||A_1 - M||*s_1)ds_1 + \int(||A_3 - M||*s_3)ds_3 + \int(||A_5 - M||*s_5)ds_5 + \int(||A_7 - M||*s_7)ds_7] \\ &- [\int(||A_2 - M||*s_2)ds_2 + \int(||A_4 - M||*s_4)ds_4 + \int(||A_6 - M||*s_6)ds_6 + \int(||A_8 - M||*s_8)ds_8]\} \\ &+ (Area_{below} - Area_{above})*dM \end{aligned} \tag{14}$$

By Eq. (13),

$$\begin{aligned} dD/dØ = \{&[\int(||A_1 - M||*s_1)ds_1 + \int(||A_3 - M||*s_3)ds_3 + \int(||A_5 - M||*s_5)ds_5 + \int(||A_7 - M||*s_7)ds_7] \\ &- [\int(||A_2 - M||*s_2)ds_2 + \int(||A_4 - M||*s_4)ds_4 + \int(||A_6 - M||*s_6)ds_6 + \int(||A_8 - M||*s_8)ds_8]\} \\ &+ (Area_{below} - Area_{above})* \{[\int(A_1*s_1)ds_1 + \int(A_3*s_3)ds_3 + \int(A_5*s_5)ds_5 + \int(A_7*s_7)ds_7] \\ &- [\int(A_2*s_2)ds_2 + \int(A_4*s_4)ds_4 + \int(A_6*s_6)ds_6 + \int(A_8*s_8)ds_8]\} /G^2 \end{aligned} \tag{15}$$

The above established findings Eqs. (12) & (15) can be used for heuristic searching in the solution configuration space for the optimal/sub-optimal grid configuration(s) on 2-D attributed objects. Similar expressions can be derived for the shifting of voxel grid on 3-D attribute.

5. DISCUSSIONS AND FUTURE WORK

Currently, an array of voxels is adopted for attribute meshing for RP. For more generic manufacturing conditions, the study may be generalized as the discretization of continuous heterogeneous-attributed CAD object into (i) a collection of shells with uniform or non-uniform thickness, each shell is homogeneous with geometry to be calculated and optimized, and (ii) arbitrary volumetric parts, each of which is homogeneous with geometry to be calculated and optimized. When the available number of homogeneous attributes is limited with quantization, the optimization criteria have to be adjusted correspondingly. Besides, adaptive meshing techniques might be adopted for faster RP fabrication.

For a more general setting of the heuristic search, a complete 6-DOF configuration space can be searched with the optimization cost function involving (i) constraint on the object's surface smoothness, and (ii) with or without locally weighting on approximation error over the attribute domain. Moreover, a set of independent attributes can also be considered as an extension of the current scalar attribute. In addition, more candidate optimization criteria might be considered such as (i) reduction in center-of-gravity offset, and (ii) reduction in the change in object's Poisson's ratio.

6. CONCLUSION

This paper presents a research applicable to practical manufacturing systems, such as the RP fabrication. The research problem approached here could be the essential complement to any work that addresses the need to preserve the attribute information in RP. The work is undertaken on re-designing heterogeneous continuous-attributed objects. The methodology and mathematical study on heuristically searching optimal grid configuration are presented in this paper. Essentially, this research addresses a core heterogeneous object modeling issue in CAD regarding the relationship between geometric spatial enumeration models and attribute field.

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