Computer-Aided Design AND APPLICATIONS

Exploring the Integration of MathCAD-Assisted Mathematics Experiments into Higher Mathematics Teaching

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Abstract. This paper analyzes and designs the teaching of higher mathematics through Mathcad-assisted mathematics experiments. This study uses MATLAB software to design teaching cases with the support of constructivist learning theory through a combination of theoretical and practical research methods, and implements the teaching with senior students, and analyzes the implementation effect. The results and shortcomings of this study are reflected. The use of MATLAB to assist the teaching of mathematical probability and statistics can improve students' interest in mathematics, better grasp and understand the knowledge of probability and statistics, and cultivate students' ability to explore the laws and use mathematical knowledge to solve practical problems, which is more advantageous than the traditional teaching model. To verify the practicality of this teaching model, teaching cases and experiments in line with students' thinking development were designed on this basis, and qualitative and quantitative analyses of the results before and after the experiments were made. The study showed that this model of teaching mathematical concepts can promote students' meaningful learning, provide teachers with some reference for designing the teaching process, and improve the actual teaching efficiency. It also enables students to develop a multi-perspective, multi-level, and multi-faceted understanding of concepts, to deeply grasp the essence of concepts, and to improve their mathematical thinking skills at the same time.

Keywords: MathCAD-assisted mathematics; experimental integration; advanced mathematics; teaching exploration
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1 INTRODUCTION

In the context of education reform, what should teachers do to promote students' image thinking and improve their literacy in daily teaching? Increased educators realize that presenting abstract mathematical problems visually through information technology can enhance students' interest in learning, stimulate their enthusiasm for learning, and then actively accept and actively acquire knowledge. The choice of visualization technology is particularly important in real-world teaching and learning and directly affects whether it is truly and effectively useful for teaching mathematics, rather than the technology itself. Most mathematics teachers present textbook contents and exercises in the form of slides through PPT [1]. The number of empirical studies on mathematics education is small, and the socio-cultural context in the classroom has received little attention from scholars. The emphasis on the connection between college mathematics and the real-life world aims to make learners active, caring, and reflective citizens through mathematics [2]. This study takes the culture of silent learning in the university mathematics education field as the context and explores the situation of classroom culture generation in higher mathematics classrooms in engineering institutions during modeling activities through collaborative action research in light of the strengths and characteristics of the mathematical community of inquiry model.

In this way, the rebuilding of mathematics classroom culture must balance the silent tradition of learning with the reform trend of encouraging students' active participation, taking into account both the nature of the mathematics discipline and the tradition of education: it requires students to think deeply in a dialogical context rather than the one-way communication of the teacher; it requires students to investigate, explain, argue and reason in a collective learning process; it requires students to make a proper connection between mathematical knowledge and professional learning. It requires students to make the necessary connections between their mathematical knowledge and their professional learning, and for teachers and students to work together to create a distinctive mathematics classroom culture.

2 RELATED STUDIES

Dotsenko et al. [3] have conducted an in-depth study on the problems of probability learning among secondary school students. They selected students of different age groups in different mathematical contexts, and by setting up problems to understand different students' different opinions about the same concepts, they found the phenomenon that based on their life experiences, students are more likely to acknowledge the fact that the occurrence of events is random, but they are not clear about the objective laws reflected behind these uncertain event. Vygovskaya [4] discussed the necessity, feasibility, and significance of experimental teaching in probability statistics course with the characteristics of a probability statistics course. Cuadri et al. [5] introduced MATLAB software into the teaching process to simulate some probabilities and statistics to deepen students' understanding and proficiency in relevant concepts in probability statistics. Andrade-Arenas et al. [6] conducted a field study on the status of high school mathematics function teaching and proposed two different types of classes with the help of MATLAB to assist high school function teaching based on the constructivist learning theory and the characteristics of function courses.

Abdullaeva and Sultonova based on the four stages of the APOS theory, divided students' conceptual understanding of logarithmic and exponential functions into four levels of practical instruction: at the first level, students can perform simple power operations; at the second level, students compress the power operation activity into a procedure through internalization [7]. Abdullaeva and Sultonova used the APOS theory to test the trainee teachers' understanding of the problem of conversion of place value and decimal point. Cuadria et al. [8] of Haifa University in

Israel also uses APOS theory when observing students' assimilation of fractions, and APOS theory is widely used.

Variation teaching is a way for teachers to transform the form of problem representation, change the extension of concepts, change conditions or conclusions in the actual classroom so that students can see the essence of the problem and explore the laws of mathematical objects in constant variations. Variation teaching about mathematical concepts is to change the irrelevant non-essential characteristics of mathematical concepts in mathematics class, to seek the same in the change so that they can gain the knowledge of abstraction in the change and improve the abstraction ability of students. Mathematical concepts are inherently abstract and not easily understood. Students cannot directly accept many mathematical concepts, and for such abstract concepts, students can draw on their perceptual experience. For example, the concept of a heterogeneous straight line for the second-year students is relatively abstract, especially the poor sense of three-dimensional is not easy to understand. The concept of a heterogeneous straight line in three-dimensional space is difficult to feel intuitively with two-dimensional plane graphics, so teachers can use a variety of teaching. Mathematical concepts have boundaries, and the properties outside the boundaries are called the extents of the concept. According to the essential connotation of the concept and the nature of a specific thing to determine whether it belongs to the concept extents or extents. Variation in the space of extensions is an effective way to optimize the mathematics variation classroom.

3 ANALYSIS OF MATHCAD-ASSISTED MATHEMATICS EXPERIMENTS INTO HIGHER MATHEMATICS TEACHING MODE

3.1 MathCAD-Assisted Mathematical Model Analysis

Through continuous technical development, the scope of the visualization concept has been gradually expanded and its content has been enriched by Kozlov et al. [9]. Mathematical visualization belongs to the category of knowledge visualization, except that the object of study is more specific, targeting the mathematical subject level to visualize and, in turn, applying visualization techniques to mathematical content or mathematical teaching aspects to achieve educational purposes. Mathematics itself has different characteristics from other disciplines, for example, it can be understood through various forms of expression, including textual language, symbolic language, and graphic language. For different mathematical contents, the choice of different expressions is beneficial for people to understand abstract mathematics more efficiently. The visual presentation is superior to these three forms of language expressions, and it can express abstract mathematical content with concrete diagrams, as shown in Figure 1.

The transformation between numbers and shapes in mathematics and the abstract position relationship in geometry is difficult to explain clearly in traditional teaching. In the world of visualization teaching, the dynamic demonstration process can stimulate students' imagination, search for patterns, make induction, and establish the dynamic connection between numbers and shapes; meanwhile, it can inspire students to pay attention to the connection between the local and the whole, and the transformation from the special to the general. When the information is input into the brain, the dynamic presentation of the situation can reveal the essence of knowledge and dynamically "see" the "back" of the abstract problem, which is conducive to the expansion of thinking. For example, the origin of conic curves, many students only know that ellipse, hyperbola, and parabola are collectively called conic curves, but they do not understand why these curves are called "conic curves" and how they are related to the three-dimensional figure of a cone. In the visualization based on GGB, we can use the dynamic plane to intercept the cone, and we can observe the shape of the intercepted section as circle, ellipse, hyperbola, and parabola, and we can see the different shapes of the section due to the different angles between the dynamic plane and the cone axis. Therefore, visualization teaching should fully exploit the advantages brought by dynamic demonstration and follow the principle of dynamic demonstration to make complex problems simpler.

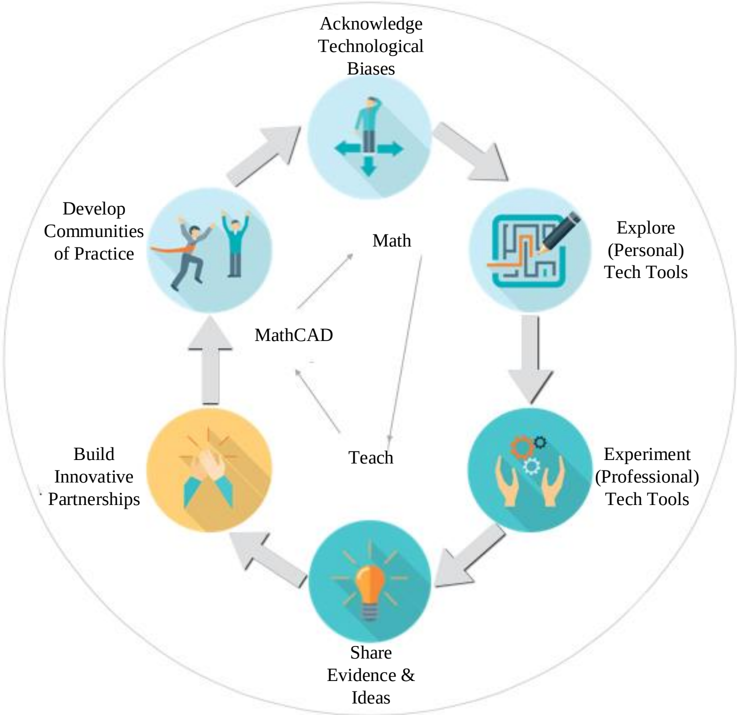

Figure 1: Mathcad-assisted mathematical model framework.

Reflection on the acquisition of the meaning of the question, i.e., the examination and re-recognition of the acquisition and processing of the information on the question, involves the equivalence of the transformation of the conditions and the representation of the question. The examination of the correctness reflected the meaning of the form conclusion. The main point is to recognize the problem-solving strategies and principles adopted in the problem-solving activities, mainly reflecting on the information combination strategies adopted, i.e., why certain topic information is arranged and combined in a certain way, to recognize the purpose and effect of their operation. If the information combination strategy is generalized, the solution strategy behind the operation can be explored; if the solution strategy and principles are further abstracted, the mathematical ideas and methods behind the topic can be refined.

3.2 Teaching Experiment Design

To test the effectiveness of the above theoretical study, this study applies the instructional design to actual teaching to verify the theoretical design through practice, and for this purpose, the practical part of this study will use a combination of instructional experiment and measurement analysis. Specifically, the teaching experiment of this study is based on the theoretical study mentioned above, and the experimental model of a single-factor equal group is used to operate the teaching environment reasonably according to the purpose of this study. Since the essence of the experiment is to explore whether there is a causal relationship between different objects, the cause of this experiment is the teaching of multiple representations of mathematics versus conventionalized teaching, and the effect is the effect of different teaching.

To understand the effects of differentiated instruction, we use measurement studies, which are based on some theory or law to quantify the behavior or results of students. To do this, we need to develop appropriate scales, which are distributed to students to obtain the effects of differentiated instruction. Based on the above orientation of student learning assessment, we will use Peas' self-assessment scale, which is widely used, to measure the cognitive load of students in the learning process; for the measurement of students' mathematical representation and transformation ability, we write test questions related to the basic theorem of plane vectors based on the evaluation and measurement theory of mathematical representation ability, and we use the total score as a measure of students [10]. We also use the total score as the result of the measurement of students' learning effectiveness. The data obtained from the scale tests will be analyzed descriptively and tested for independence with the help of SPSS 20.0.

This study focuses on teaching experiments and measurement analysis in terms of empirical research, with the expectation that such empirical research will verify the feasibility and practical effects of theory application. After the empirical study, we also reflected and discussed the results to reveal the relationship between the empirical results and this research question.

In summary, the design idea of this study can be summarized as follows: First, the research on multiple representations in mathematics is sorted out to fully understand the theories related to learning and teaching of multiple representations in mathematics, and the research on plane vector fundamental theorems is sorted out to further clarify the research questions. Secondly, we analyze the multiple representations of the fundamental theorem of plane vectors and their transformations from the perspective of multiple representations in mathematics and construct teaching strategies for the fundamental theorem of plane vectors based on the perspective of multiple representations. Besides, the teaching process of multiple representations of the fundamental theorem of plane vectors and the specific teaching design is further developed and experiments are conducted accordingly. Then, the results of the experiments are quantified and analyzed by using the test survey method to obtain experimental results. Finally, the experimental results are reflected and discussed, and the whole study is summarized and prospected, as shown in Table 1.

	Scenario	Spoken	Graphics	Operation	Symbols	Language	Numbers
Scenario	-	△	△	▲	△	△	△
Spoken	△	-	△	△	▲	△	▲
Graphics	▲	△	-	▲	△	▲	△
Operation	△	▲	△	-	△	△	△
Symbols	▲	△	△	△	-	△	▲
Language	△	△	▲	△	△	-	△
Numbers	▲	△	▲	△	▲	▲	-

Table 1: Conversion mechanisms between several representations.

The analysis of the conversion mechanism between multiple representations is the basis for the complementarity and penetration of multiple representations, and also plays an important role in helping students to explore the essence of theorems, for example, when learning theorems by abstracting the physical situation mathematically, although the situation has changed, the substance has not changed, so the vectors in mathematics can also be decomposed by using the parallelogram rule.

Understand the various representations of the theorem, and understand the essence and richness of the theorem from multiple perspectives; be able to use the theorem to reasonably select the base and represent other vectors in the plane; flexibly select and convert the representations of the theorem to solve mathematical problems. To experience the process of mathematizing physical situations and finding and refining mathematical problems from them, and developing the ability to find problems mathematically and ask questions. By using the research idea of moving from the particular and concrete to the general and abstract, experiencing the development of the theorem through observation, conjecture, geometric manipulation, induction, and other activities, gradually comprehending and enhancing the basic mode and ability of mathematical research, and developing the ability to solve problems mathematically. Through the teaching of multiple representations gradually develop a world view of understanding things from multiple perspectives, as shown in Figure 2.

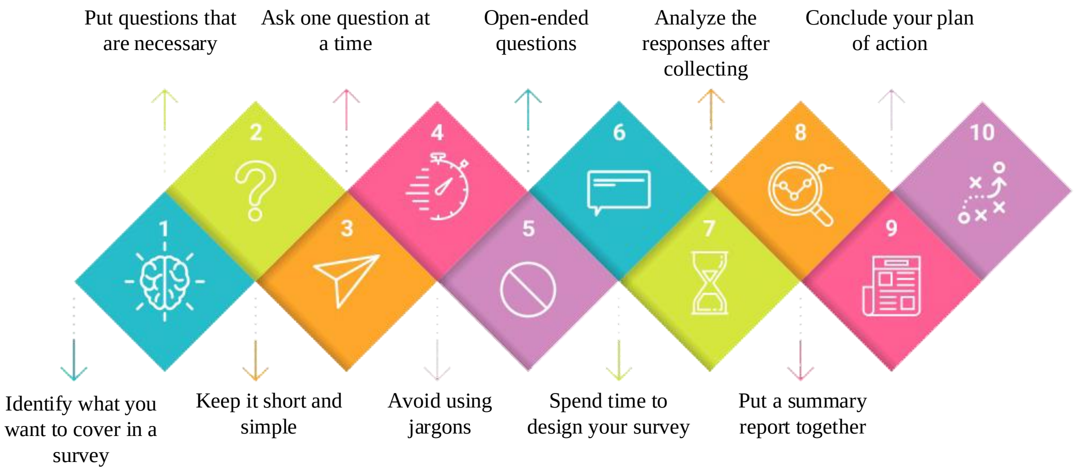

Figure 2: Design of teaching experiment steps.

Students have a preliminary understanding of the meaning and multiple representations of the fundamental plane vector theorem, but the application and variation of the theorem are needed to promote students' deeper understanding of the theorem and the flexible translation and integration between multiple representations. Therefore, teachers should set up examples and organize variation training for students to consolidate and deepen their overall knowledge of the theorems and to promote the generation of integration codes for the theorems. In setting the application and variation of theorems, teachers should carefully design the difficulty gradient of the problems according to the teaching objectives and ensure the effectiveness of the exercises to promote the integration of students' representations.

4 ANALYSIS OF RESULTS

4.1 Results of Pre-Test Data

The output of SPSS 20.0 was organized into the table shown below by conducting independent sample t-tests on the pretest scores of the two classes. From Figure 3, the mean math scores of the control class were slightly better than those of the experimental class, and the overall difference was not significant. Also, the significance level of the t-test was 0.545, indicating that there was no significant difference in math levels between the experimental and control classes, so this study illustrated from the data that the two subject classes were parallel classes.

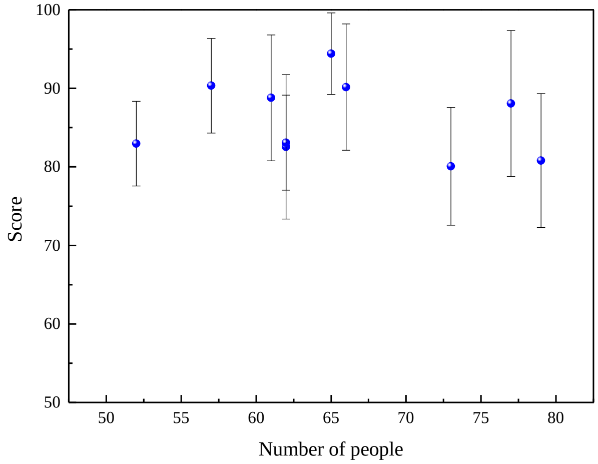

Figure 3: Results of independent sample t-test for pretest scores.

By conducting independent samples t-tests on the cognitive load measurements of the students in the experimental and control classes during the learning process, the SPSS output was compiled into the results shown in Figure 4. From Figure 4, it can be seen that the mean cognitive load of the experimental class is lower than that of the control class, and it can also be seen that the significance level of the cognitive load of the students in both classes is 0.026, which indicates that the difference between the cognitive load of the students in the experimental and control classes is significant, thus indirectly indicating that the teaching model based on the theory of multiple representations of mathematics is better at reducing the cognitive load of the students compared to the conventionalized teaching model.

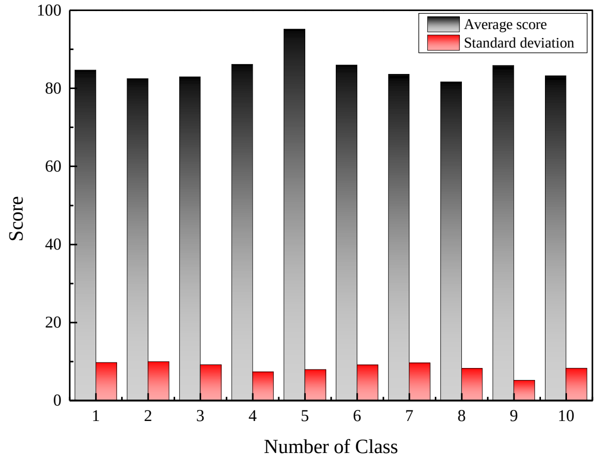

Figure 4: Results of independent sample t-test for cognitive load.

To test the significance of the difference between the two classes, we conducted an analysis of covariance (ANOVA) on the pre-test and post-test data with the help of SPSS, and the "between-subjects results" in the output showed that the sig value of the "class pre-test score" column was less than indicates. The slopes of the regression lines between the pre-test scores and the post-test scores of the two classes are not equal, which means that the data cannot be analyzed for covariance. Therefore, we conducted independent samples t-tests on the posttest scores of the students in the two classes and obtained the results shown in Figure 5. This indicates that the teaching under the theory of multiple representations in mathematics helps to improve students' academic performance to a certain extent.

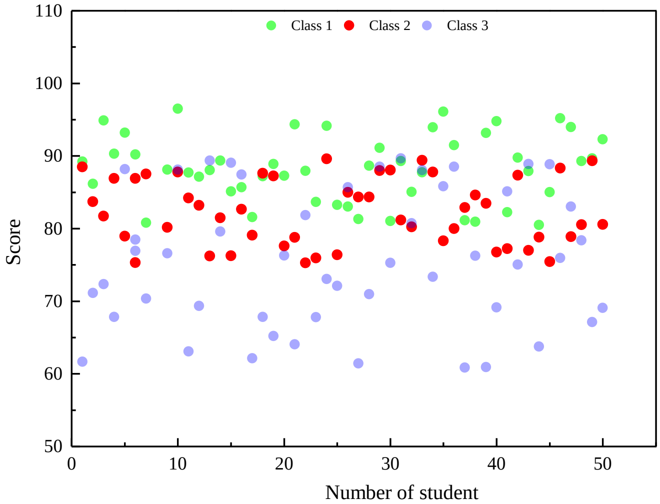

Figure 5: Results of independent sample t-test for posttest scores.

By summarizing the student performance by each score band, we obtain the results shown in Figure 5. From the figure, it can be concluded that the number of students in the first three low score bands is more in the control class than in the experimental class, while the number of students in the last three middle and high score bands is more in the experimental class than in the control class, which shows that the good rate in the control class is lower than in the experimental class.

4.2 Analysis of Experimental Results

As can be seen from Figure 6, the significance level of the chi-square test 0.075 indicates that the variance is chi-square and there is no significant difference. And 0.002 indicates a significant difference between the experimental and control groups' posttest scores within the 95% confidence interval. Originally, the two classes were parallel classes with similar scores selected randomly by computer based on their midterm scores, and the experimental group was taught with the aid of MATLAB software, while the control group was taught in the classroom traditionally and ordinarily, but after three weeks of implementing different teaching modes, the experimental group's scores improved, and the experimental group performed better compared to the control group. This also proves that teaching probability and statistics in high school mathematics with the aid of MATLAB is more advantageous than the traditional teaching model.

The experimental class and the control class of the high school where I practiced are both parallel classes, and the students' enthusiasm and interest in learning mathematics are not strong enough. We counted the monthly examination results of the two classes and used Excel to make a bar chart to show the results of the two classes more visually. The students in the experimental and control classes were tested through the last monthly test, which lasted 120 minutes and scored 150 points, with the same scoring criteria for both classes. There were 42 students in the experimental class and 40 students in the control class, and the number of students in both classes was greater than 30, which was a large sample. As shown in Figure 7, the results of this test paper were collected and then SPSS 22.0 statistical analysis software was applied to analyze and study the effectiveness of the experimental class, and the data with 150 points were used as the pre-experimental data of the two classes, after which independent samples t-test was conducted to test whether there was a significant difference in the level of conceptual understanding of exponential functions between the two classes.

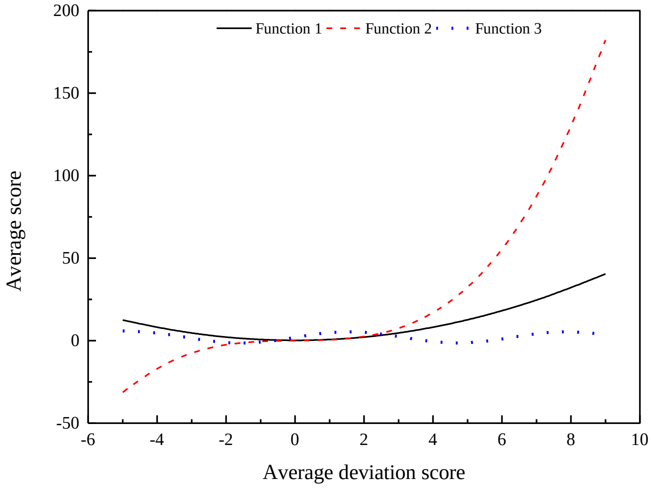

Figure 6: Image of the terms of an equivariant series.

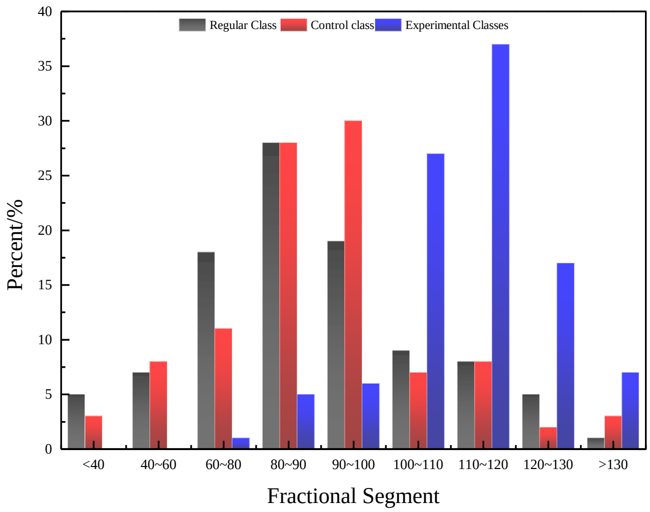

Figure 7: Bar chart comparing the pre-test scores of the experimental class and the control class

In the new lesson on arbitrary trigonometric functions, the experimental class was guided by the APOS theory and the theory of variational teaching, while the control class was taught by the conventional teaching method. After the teaching, the author developed a test paper on the concept of arbitrary trigonometric functions for the experimental class and the control class, to understand the students' mastery of the concept of arbitrary trigonometric functions and the teaching effect of the experimental class, and tested the students of both classes for 90 minutes, with 150 points. There were 42 students in the experimental class and 40 students in the control class. The number of students in both classes was greater than 30, which was a large sample.

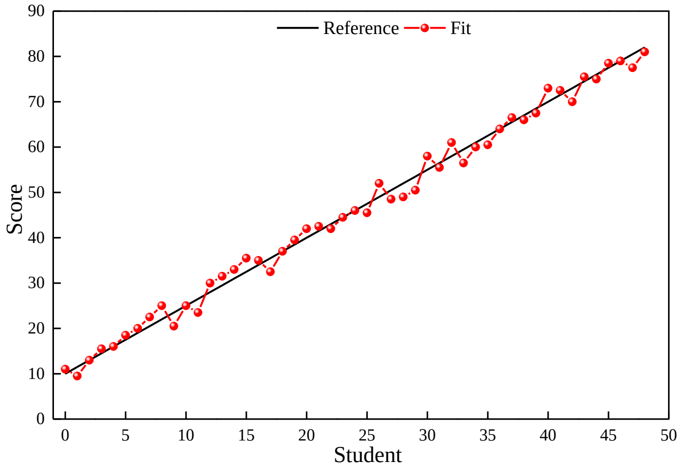

Figure 8: Comparison of post-test scores between the experimental class and the control class.

In the post-test of the concept of arbitrary angle trigonometry, the mean scores of the control and experimental groups were 85.63 and 77.95 respectively after the independent sample test, and the significant level of the chi-square test was found to be 0.561>0.05, indicating that there is no particularly significant difference in the chi-square, and the result of taking the first line of the independent sample test assuming equal variances 0.005<0.05. As shown in Figure 8, there is a 95% confidence level, indicating that there is a significant difference between the post-test scores of the control and experimental classes. The variation of teaching about the concept of arbitrary angle trigonometry based on the APOS theory affects students' performance.

Judging from the classroom students' classroom status, most of the students in the experimental class was more active and willing to actively participate in the four stages of teaching. This mode of teaching is more likely to stimulate students' enthusiasm for learning, and strong motivation helps learners construct concepts on their own. By comparing the scores of the pre and post-tests, the average scores of the students in the experimental class were relatively high, especially the problems of deeper conceptual understanding were better solved. The mathematics concept class under the combination of APOS theory and variable teaching theory was effective in improving the scores of the upper and middle students and had little effect on the scores of the lower students. The results of this analysis were not simply obtained through single experimental teaching, but this teaching model was used in the whole semester of concept teaching.

5 CONCLUSION

This study first provides an overview of the theories related to Mathcad mathematical multiple representations and related pedagogical studies to clarify the objectives and ideas of this study and then analyzes the profound connotations, rich representations, and their transformation methods contained in the fundamental theorem of plane vectors. The four teaching principles of the clear mainline, information packing, proximity presentation, and dual-channel are proposed for teaching the basic theorem of plane vectors in high school mathematics. Finally, the teaching of the fundamental theorem of plane vectors was redesigned according to the previous objective analysis and teaching principles, and a practical study was conducted. There exists a certain facilitation effect of teaching probability and statistics in high school mathematics with the aid of MathCAD software, which can help students to improve their motivation and academic performance. The change in the traditional authoritative role of the teacher, rather than ignoring its existence, reinforces the image of the mathematics teacher's professional role. The unique mathematical dialogue of collective argumentation is a joint effort by teachers and students to build mathematical claims that involve consensus reached by multiple people.

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REFERENCES

[1] Mazurenko, E.-V.: Aspects of using computer programs in teaching higher mathematics at university, Vestnik of Samara State Technical University, Psychological and Pedagogical Sciences, 8(1), 2017, 48-56. https://journals.eco-vector.com/1991-8569/article/view/52250
[2] Etcuban, J.-O.; Campanilla, B.-S.; Horteza, A.-D.: The use of Mathcad in the achievement of education students in teaching College Algebra in a university, International Electronic Journal of Mathematics Education, 14(2), 2019, 341-351. https://doi.org/10.29333/iejme/5718
[3] Dotsenko, O.-A.; Zhukov, A.-A.; Kochetkova, T.-D.; Leontyeva, E.-G.: The Problem-Oriented Approach in the Basic Mathematical Courses for Engineering Education, Handbook of Research on Engineering Education in a Global Context, 5(10), 2018, 176-176. https://doi.org/10.4018/978-1-5225-3395-5.ch016
[4] Vygovskaya, L.-S.: Solving the Inverse Problem of Analytical Geometry in Mathcad System Using the Tools of the R-Function Class, Prepodavatel XXI vek, 2020(3), 2020, 115-124. https://doi.org/10.31862/2073-9613-2020-3-115-124
[5] Cuadri, A.-A.; Martín-Alfonso, J. -E.; Urbano, J.: A teaching methodology based on Mathcad for improving the calculation of pumping power, Education for Chemical Engineers, 28(6), 2019, 68-78. https://doi.org/10.1016/j.ece.2018.11.007
[6] Andrade-Arenas, L.; Cabana-Caceres, M.; Perez, W. -R.; Sandoval, J. -V.: The Impact of Using Software to Learn Mathematics in Courses of Peruvian Engineering Programs, In 2020 IEEE World Conference on Engineering Education (EDUNINE), 16(4), 2020, 1-4. https://doi.org/10.1109/EDUNINE48860.2020.9149478
[7] Abdullaeva, S.-M.; Sultonova, U.-N.: Use of mathcad software in the preparation of students majoring in engineering, International Journal of Scientific and Research Publications (IJSRP), 10(12), 2020, 650-653. https://doi.org/10.29322/IJSRP.10.12.2020.p10877
[8] Cuadria, A.-A.; Martín-Alfonsoa J.-E.; Urbanob, J.: A teaching methodology based on mathcad for improving the calculation of pumping power. Education for Chemical Engineers, 28, 2019, 68-78. https://doi.org/10.1016/j.ece.2018.11.007
[9] Kozlov, A.; Lapteva, S.; Shevnina, T.: The Development of Student Professional Competence in the Field of Modeling Technological Processes in the Oil and Gas Industry Based on Computer Mathematics, Journal of Computational and Theoretical Nanoscience, 17(9-10), 2018, 4755-4761. https://doi.org/10.1166/jctn.2020.9374
[10] Abdurahmonov, B.-A.: Opportunities to use mathematical packages of information technology in teaching mathematics, Economics and Innovative Technologies, 2018(1), 2018, 78-86. https://uzjournals.edu.uz/iqtisodiyot/vol2018/iss1/78/