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3D Facial Action Units and Expression Recognition using a Crisp Logic

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ABSTRACT

This work proposes a method for recognizing the main 13 Facial Action Units and the 6 basic emotions. The methodologies rely on Differential Geometry to extract relevant discriminant features from the query faces, and on some linear quantities used as measures: Euclidean, geodesic, and angles between 17 automatically extracted soft-tissue landmarks. A thresholding system which evaluates local properties of connected regions, selected through tailored geometrical descriptors, supports the identification of the AUs. Then, a technique based on crisp logic allows the identification of the global expression. The three-dimensional context has been preferred due to its invariance to different lightening/make-up/camouflage conditions.

Keywords: Face expression recognition, Emotional design, Intelligent drive.
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1 INTRODUCTION

Face expression recognition (FER) has registered a slow but growing interest among the scientific community from the Seventies. In the last decade, works published on this topic are more than 500 per year, with a double amount reached in 2011-2014. They address the issue of automatically identifying from a facial image its emotion-based expression. This research branch was fostered by the psychological studies undertaken by Paul Ekman in 1970 [1] [2], who formulated the "theory of basic emotions", which are six: anger, disgust, fear, joy, sadness, surprise. Later in 1978, he presented his Facial Action Coding System (FACS) composed by Action Units (AUs), which are catalogued relying on relaxation or contractions of one or more facial muscles [3] [4]. More AUs define a facial expression. These have been often used as a basis for facial expression recognition studies. Nonetheless, other researchers dealt with six or four facial expressions without analysing AUs.

Among the varied contributions, some authors adopted approaches relying on geometrical descriptors such as curvatures and shape index. Berretti et al. used mean and Gaussian curvatures for detecting fiducial points (landmarks) in a 3D face expression recognition framework based on Scale Invariant Feature Transform (SIFT) descriptors applied to the detected points. They gained a global recognition rate (RR) of 78.43% [5]. Similarly, Bolkart and Wuhrer predicted landmark locations for entire facial motion sequences by adopting mean curvature, Gaussian curvature, and Shape Index, to automatically recognize dynamic face expressions. Final classification rates were 90.71% to recognize the expressions anger, happiness, surprise and 90.60% to recognize happiness, sadness, surprise [6]. Broadbent et al. introduced the "area weighted histogram of shape index", curvedness, mean curvature, and Gaussian curvature as facial features to automatically distinguish between the six main expressions. Tests undertaken on the Binghamton University 3D Facial Expression Database (BU4DFE) database gave 95% accuracy [7].

Fang et al. adopted shape index and spin images, "a representation of the geometric neighborhood around a specific point of a three-dimensional object". Spin images encode point coordinates on the surface with respect to a local basis, i.e. oriented point. Average FER classification rates on BU4DFE database were 74.63% when all six Ekman's expressions are involved, 95.75% when only joy, surprise, and sad ones are taken into consideration [8]. Huang et al. mapped shape index on 3D facial surfaces to build a novel facial representation model. To test it, the authors conducted FER for static facial models of sixty subjects selected from the BU-3DFE, reaching a 83% RR with six expressions on static data [9]. Li et al. estimated point-by-point values of shape index and principal curvatures on facial depth maps for 3D FER with classification rates ranging from 76.4 to 82%, depending on the composed feature/descriptor adopted (histograms), on six expressions of BU-3DFE [10]. The same dataset was adopted by Powar et al. for recognizing smiling (95% RR), surprised (92%), and sad (70%) expressions. Mean, Gaussian, and principal curvatures were used as features to describe facial surfaces and support comparison process [11]. Mean and Gaussian curvatures, and shape index, together with Curvedness, were also investigated by Savran et al. for automatically detecting facial 25 AUs obtaining different rates depending on the adopted method. 96.3% is reached when the three-dimensional modality is used. Experimentations were carried out on Bosphorus and Cohn-Kanade databases [12] [13]. Shape index was evaluated by Zhen et al. for developing a muscle movement-based automatic 3D FER algorithm. The six expressions of the BU-3DFE dataset were recognized with an average performance of 83.2% [14].

Other geometrical descriptors were also adopted. Daoudi et al. introduced Scalar Vector Field (SVF) defined on radial curves of 3D faces for automatic 4D (3D video) expression recognition. The SVF grounds on Riemannian shape analysis and captures deformations occurring between three dimensional facial surfaces represented by sets of radial curves. The average accuracy, evaluated on BU4DFE, was 93.83%. The lower RR was obtained for the disgust expression (91.54%) which was confused with angry and fear ones [15]. Riemann geometry theory was also the basis of the work of Zeng et al., who developed a 3D FER framework to detect the six expressions within the BU-3DFE database. Average accuracy was 68.15% [16].

This work proposes an automatic 3D facial AUs and emotion recognition algorithm for identifying Ekman's action units and the six main expressions. The proposed method is based on descriptors from Differential Geometry background which are mapped point-by-point on facial surfaces, angles, Euclidean, and geodesic distances that are evaluated between automatically localized landmarks. The features support the definition of inter- and intra-expression variations quantities, thus fostering the identification of the proper AUs and emotion on the probe/query face(s).

The system is designed for safety applications. The application scenario is Intelligent Drive for different purposes involving safe and user-friendly driving: facilitating a more natural human-computer interaction between driver and car dashboard; detecting surprise/fear behaviours of the driver, i.e. "study of reactions", when a unexpected event happens on the road; detecting micro sleeps, stress, fatigue, workload, and general attention of the driver. Besides general safety and enhancement of interaction between vehicles and environment, these applications are also aimed at the implementation of procedures for special driving such as assisted or autonomous driving systems.

2 METHOD

Action units are given by facial muscle movements and subsequent facial morphological changes at skin level. To conceptualize every AU on soft-tissues, compact features are to be extracted from the face to allow analysis and comparisons. The features used in this work are Euclidean and geodesic distances, and angles between 17 automatically extracted landmarks relying on previously developed techniques [17] [18] [19] [20] shown in Figure 1, and geometrical descriptors [21].

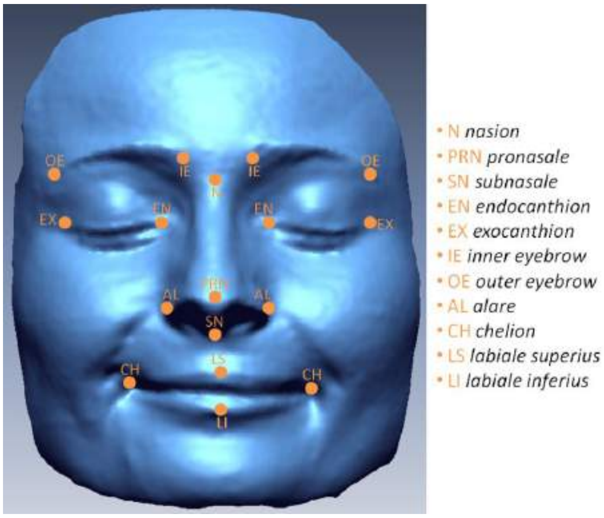


Figure 1: The 17 landmarks adopted in this study to evaluate Euclidean, geodesic distances, and angles. OE outer – eyebrow, IE inner – eyebrows, EX exocanthions, EN endocanthions, N nasion, AL alae, PRN pronasal, SN subnasal, LS labrum superius, CH chelions, and LI labrum inferius.

The method accepts in input two facial 3D models of the same person: the serious pose and an expressive face, which is the query/probe face whose AUs are to be recognized by the algorithm. Features are evaluated both on the serious and the emotioned face. They will be respectively called *basic* and *emotion* features. Basic features involve distances and angles, while emotion ones also involve geometrical descriptors. Comparisons between basic and emotion features are made, and involved geometrical descriptors are evaluated to identify the AUs of the probe face, which are the first output of the method. Relying on the identified AUs, the global emotion acted by the query face is identified. Figure 2 shows the method scheme.

2.1 Action Unit identification

The geometrical descriptors of this study are chosen among a set of twelve descriptors previously investigated [21]: the six coefficients of the first and second fundamental forms; the mean and Gaussian curvatures; the principal curvatures; the shape index and curvedness introduced by Koenderink and van Doorn [22]. In particular, this work only adopts the third coefficient of the second fundamental form, called g, and curvedness, identified by C. Both g and C rely on the derivatives and focus on the description of the surface curvature. These are formulas adopted in the algorithm:

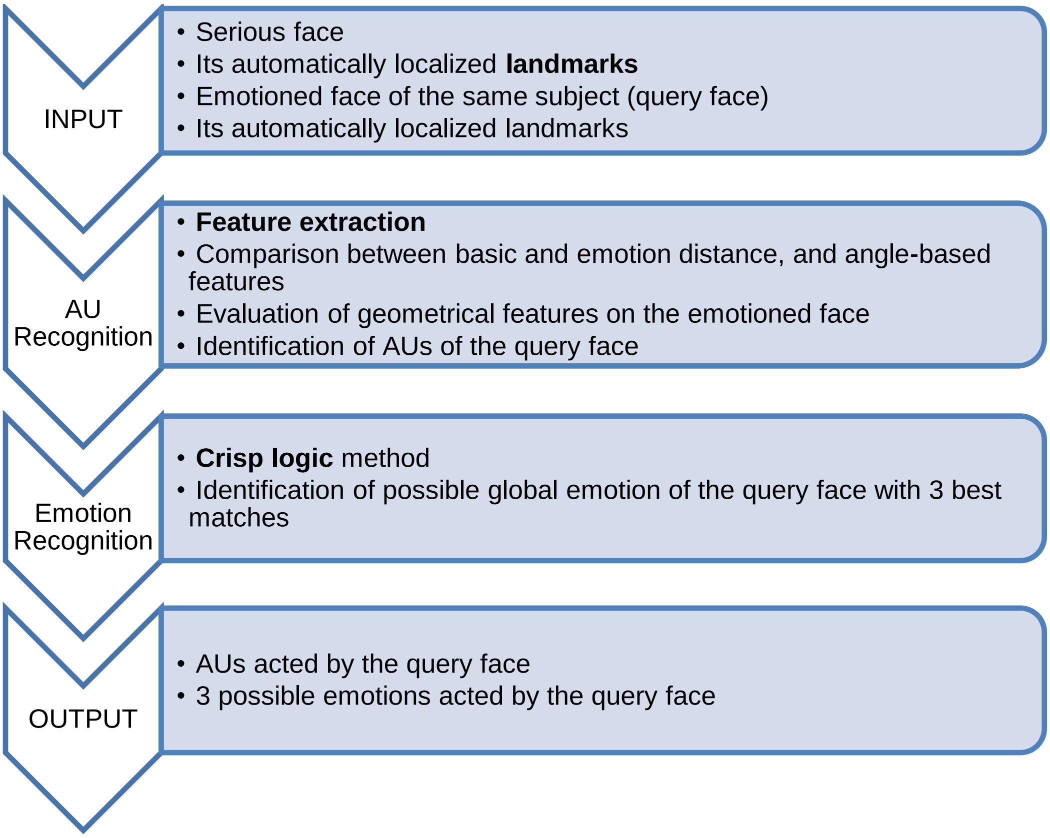


Figure 2: Scheme of the proposed method.

$$g = \frac{h_{yy}}{\sqrt{1 + h_x^2 + h_y^2}}$$

$$C = \sqrt{\frac{k_1^2 + k_2^2}{2}}$$

where hx, hy,... are the derivatives with respect to x and y, k1 and k2 are given by

$$k_1 = H + \sqrt{H^2 - K}$$

$$k_2 = H - \sqrt{H^2 - K}$$

where K is the Gaussian curvature and H is the mean curvature, given by

$$K = \frac{h_{xx} h_{yy} - h_{xy}^2}{\left(1 + h_x^2 + h_y^2\right)^2}$$

$$H = \frac{(1+h_x^2)h_{yy} - 2h_x h_y h_{xy} + \left(1+h_y^2\right)h_{xx}}{\left(1+h_x^2+h_y^2\right)^{3/2}}$$

These two descriptors mapped point-by-point on a facial 3D model acquired via laser scanner are reported in Figure 3.

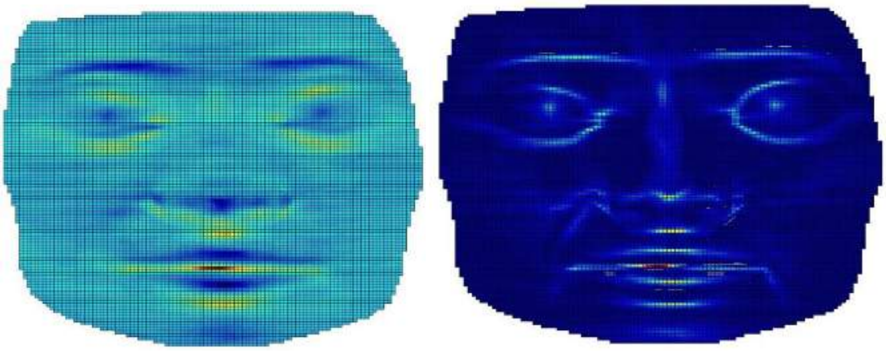


Figure 3: From left to right: descriptor g mapped point-by-point on a 3D face model; descriptor C mapped on the same face.

The chosen AUs to be analysed and considered in this study are those strictly connected on the six basic emotions theorised by Paul Ekman [1] [2] [3] [4] (anger, disgust, fear, joy, sadness, surprise) and presented in Table 1 in conjunction with the related facial expressions.

emotion	Action Units
anger	AU4 AU23 AU24
disgust	AU9 AU10 AU16
fear	AU1 AU2 AU27
joy	AU6 AU12
sadness	AU1 AU15
surprise	AU1 AU2 AU26

Table 1: AUs which compose every expression.

The method follows a similar structure for all AUs, but is shaped differently according to each AU specificity. A training set of 140 faces of the public Bosphorus database including 10 males and 10 females, each with 6 expressions plus the serious one, is used to experimentally design the methodology in terms of threshold and weight settings. The algorithm has been fully developed in Matlab®.

AU1 is "inner brow raiser". Thinking about the movement of the eyebrow representing the AU as a vector from the initial and final locus of inner eyebrow (IE) landmarks, the movement connected to this AU can be tracked on the skin/soft-tissue. Three features are used to map this movement:

- Euclidean distances between IE – EN, both on left and right side of the face;
- geodesic distances between IE – EN, on both sides;
- angles described by landmarks IE – N – EN, on both sides.

These features are shown in Table 2, together with the features used to identify all other AUs. All these measures are calculated using the coordinates of landmarks. Geodesic distances are computed using a Dijkstra-based algorithm. For this AU, no geometrical descriptors are adopted.

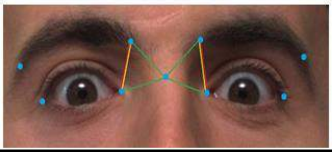
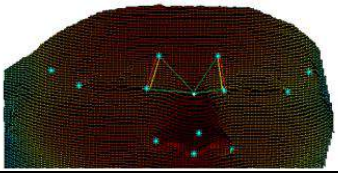
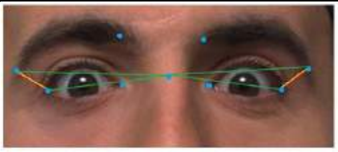
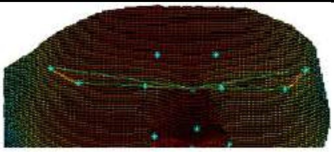
The process of identification of this AU is based on the final numerical value AU1 variable has. For each distance (two Euclidean and two geodesic in this case), if the distance evaluated on the emotioned face is greater than that on serious one, value 0.25 is added to AU1 variable. When all distances are evaluated, if AU1 variable is lower than or equal to 0.5, angles are evaluated; otherwise AU1 is set to 1, meaning the AU1 has been identified on the probe face. Angle evaluation is made by comparing the angles of the emotioned and serious face. For each angle (two in this case), if the angle of the emotioned face is greater than that on the serious one, value 0.25 is added to AU1 variable. If the final numerical value of the AU1 variable is greater than or equal to 0.75, AU1 variable is set to 1, meaning that this AU has been identified on the query face; otherwise AU1 variable is set to 0 and this AU has not been identified. The pseudo code of this identification is reported in Figure 4.

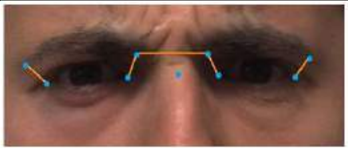
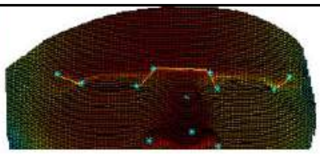
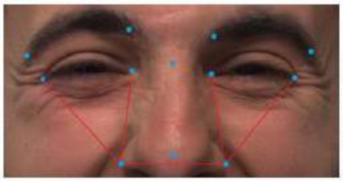
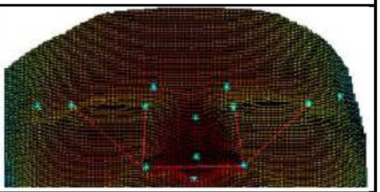
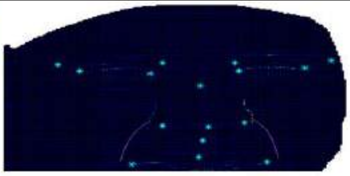
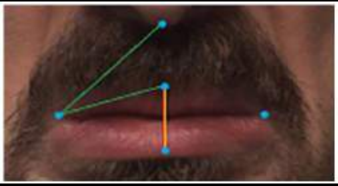
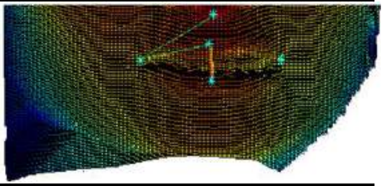
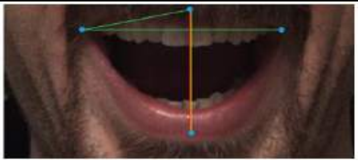
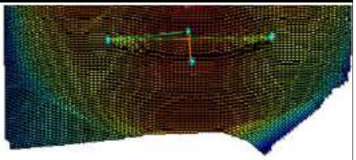
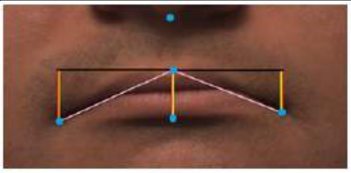
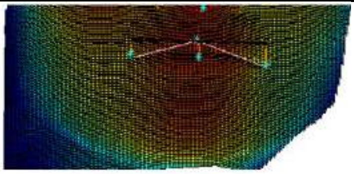
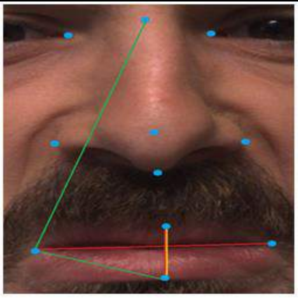
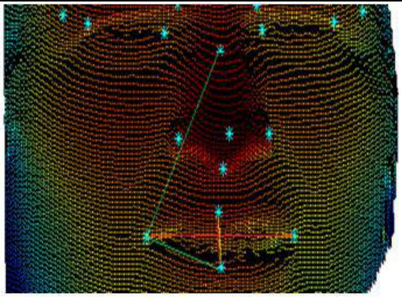
```
AU1 = 0
for all distances   % both Euclidean and geodesic
    if distance_emotion > distance_serious
    AU1 = AU1 + 0.25
if AU1 <= 0.5
then
    for all angles
        if angle_emotion > angle_serious
        AU1 = AU1 + 0.25
else AU1 = 1

if AU1 >= 0.75
    then AU1 = 1  % AU1 is recognized
    else AU1 = 0  % AU1 is not recognized
```

Figure 4: Pseudo code of the process of identification of AU1.

A similar feature evaluation based on distances and angles is made for the other AUs. The choice of features for each AU identification process is reported in Table 2. Similar assumptions, not elaborated here, are made for the other AUs.

AU	features	features on a face	
AU1 *inner brow raiser*	Euclidean distances IE – EN geodesic distances IE – EN angles IE – N – EN		
AU2 *outer brow raiser*	Euclidean distances OE – EX geodesic distances OE – EX angles OE – N – EX		

AU4 *brow lowerer*	Euclidean distances OE – EX IE – EN intraIE geodesic distances OE – EX IE – EN intraIE angles OE – N – EX IE – N – EN geometrical descriptor *g*		
AU6 *cheek raiser*	Euclidean distances OE – AL intraAL geodesic distances OE – AL		
AU9 *nose wrinkler*	geometrical descriptor *C*		
AU10 *upper lip raiser*	Euclidean distances LS – SN geodesic distances LS – SN angles SN – CH – LS		
AU12 *lip corner puller*	Euclidean distances LS – LI geodesic distances LS – LI angles CH – CH – LS geometrical descriptor *g*		
AU15 *lip corner depressor*	Euclidean distances LS – LI CH – LS oblique CH – LS geodesic distances LS – LI CH – LS geometrical descriptor *g*		
AU16 *lower lip depressor*	Euclidean distances LS – LI intraCH geodesic distances LS – LI angles N – CH – LI		

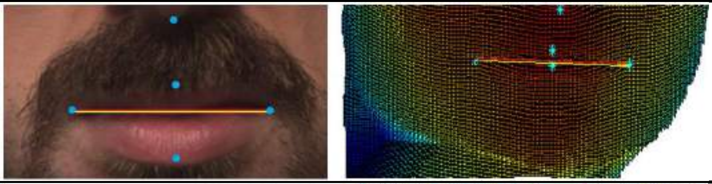
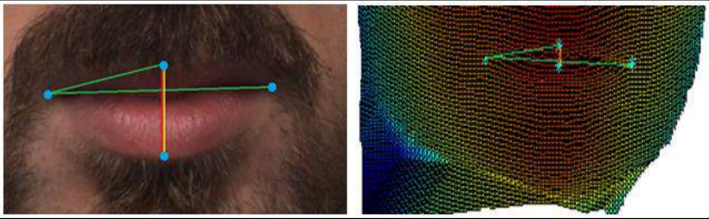
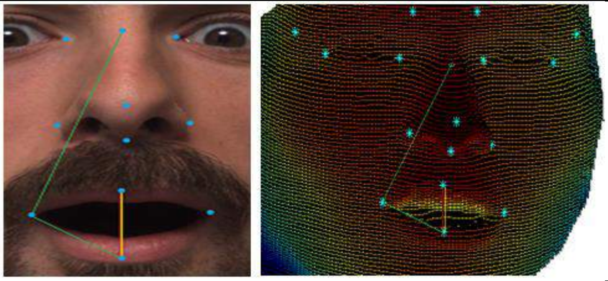
AU23 *lip tightener*	Euclidean distances intraCH geodesic distances intra CH	
AU24 *lip pressor*	Euclidean distances LS – LI geodesic distances LS – LI angles CH – CH – LS	
AU26 *jaw drop* & AU27 *mouth stretch*	Euclidean distances LS – LI geodesic distances LS – LI angles N – CH – LI	

Table 2: Features adopted to identify each AU. In the images shown on the third column, a face of the Bosphorus database, displayed both in 2D and 3D, is used to show distances and angle on the face. Euclidean distances are represented red-coloured; geodesic distances are yellow-coloured; angles are green.

The identification process of AU9 is elaborated here to understand the adoption of geometrical descriptors.

AU9 is "nose wrinkler". The significant skin-level aspect of this AU are the wrinkles laying on both sides of the nose branching off till mouth sides, shown in Figure 5. It is known for being typical of the disgusted expression.

The identification of this AU starts by mapping descriptor C, representing curvedness, on the facial map. Then, a region of interest is selected which could focus on the wrinkles area. A binary mask is applied to the selected region of interest; points with $C \geq 0.4$ are put equal to 1, while others are null. Matlab function bwconncomp is used to separate different connected components. Finally, area and orientation properties of regionprops function are used to select the connected components with area > 90, meaning number of points > 90; orientation is evaluated on the components reaching this threshold. Orientation property computes the angle ∊ [-90°; 90°] between the *x*-axis and the major axis of the ellipse having the same moment of planar inertia of the region. If this angle is major than 30°, AU9 is identified and its variable is set to 1.

2.2 Emotion recognition

When all AUs are tested and related variables set to 0 or 1, the algorithm evaluates the possible emotions of the query face. This evaluation, which gives only one best match among the six basic emotions (anger, disgust, fear, joy, sadness, surprise), relies on *crisp logic* techniques. Similarly to the AU identification process, emotions are analysed one by one. For each emotion, the final decision about whether the emotion is likely or not is made by assigning importance weights to each AU composing the emotion (Table 1). These weights have been set by examining the expressive faces composing the training dataset. Depending on which and how many AUs are identified, a likelihood is assigned to the emotion: 0.99 if the emotion is extremely likely to be the emotion of the person of the query face; 0.66 if there is a medium possibility; 0.33 if the likelihood is low; 0 if there is not any probability. Figure 6 shows weights of the AUs and related emotion probabilities for each expression evaluation.

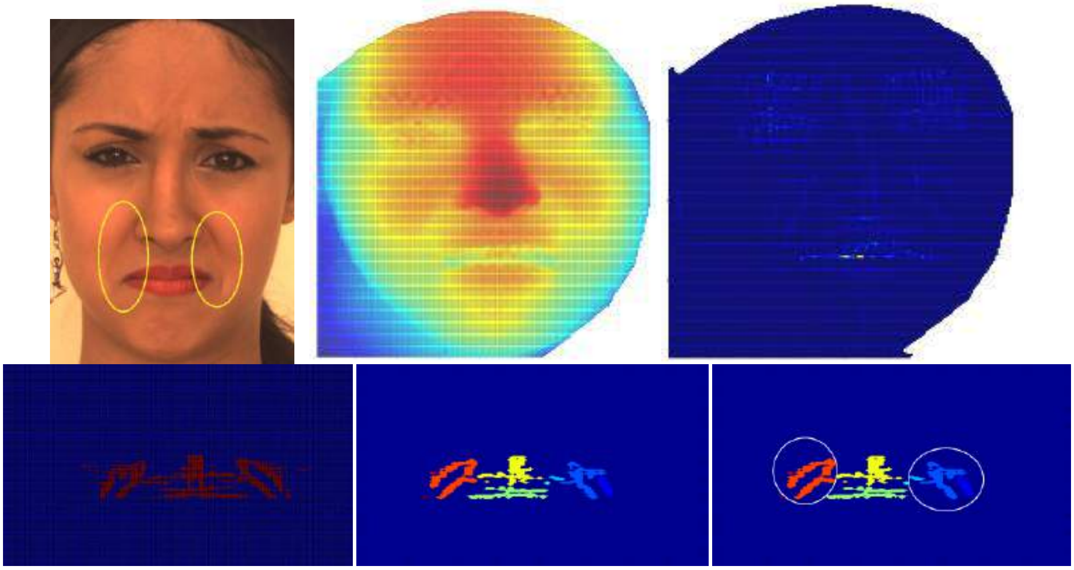


Figure 5: From left to right. Above: disgusted expression of a face of the Bosphorus database with highlighted nose wrinkles; the same face in the form of 3D model; descriptor curvedness *C* mapped on the face (wrinkles area has higher values of descriptor C). Below: binary mask of the region of interest created by selecting points with $C \geq 0.4$; identification of the connected components with function bwconncomp, each coloured differently; identification of the components for which the area was major than 90 thanks to function regionprops.

Relying on these weight and probabilities, the system defines one to three possible emotions performed by the subject of the query face.

3 RESULTS

Experimentations in the testing phase have been carried out on 1539 complete and non-occluded faces of the Bosphorus database [23]. Among these faces, 618 faces have serious and expressive states, while 921 are faces representing specific AUs, in particular those addressed in this study, with the exception of AU6 which is not present in the Bosphorus database. The algorithm works with two faces per time: a serious face and an expressive/AU-based face of the same subject with an unknown emotion/AU to be identified by the system.

Global AU recognition rate (RR) among AU-based faces is 82.53%. AU15 "lip corner depressor" reached the highest RRs (100%), followed by AU1 “inner brow raiser” (97.73%), AU27 "mouth stretch" (95%), and AU26 "jaw drop" (91.18%). The lowest RR (52.17%) was gained by AU9 "nose wrinkler".

Global AU RR among expression-based faces is 75%. AU1 “inner brow raiser” was the action unit which was identified the most (92.30% RR) and AU16 “lower lip depressor” was the least (56.25%

RR). In this set, a trend has been observed in these RRs regarding the involvement of the mouth. Action units concerning mouth muscular movements gain lower RRs when other facial movements (global facial expressions) are present. The mouth is the most moving parts of the face; thus, the behaviour of geometrical descriptors changes more in the mouth area than in any other facial zone. This is the reason why the lower RRs concern the mouth in expression-based faces.

AU4	AU23	AU24	OUTPUT
1	1	1	HIGH
1	1	0	MEDIUM
1	0	1	MEDIUM
0	1	1	MEDIUM
1	0	0	LOW
0	1	0	LOW
0	0	1	LOW
0	0	0	NULL

AU9	AU10	AU16	OUTPUT
1	1	1	HIGH
1	1	0	MEDIUM
1	0	1	MEDIUM
0	1	1	MEDIUM
1	0	0	LOW
0	1	0	LOW
0	0	1	LOW
0	0	0	NULL

AU1	AU2	AU27	OUTPUT
1	1	1	HIGH
1	1	0	MEDIUM
1	0	1	MEDIUM
0	1	1	MEDIUM
1	0	0	LOW
0	1	0	LOW
0	0	1	LOW
0	0	0	NULL

ANGER DISGUST FEAR

AU1	AU2	AU26	OUTPUT
1	1	1	HIGH
1	1	0	MEDIUM
1	0	1	MEDIUM
0	1	1	MEDIUM
1	0	0	LOW
0	1	0	LOW
0	0	1	LOW
0	0	0	NULL

AU6	AU12	OUTPUT
1	1	HIGH
0	1	MEDIUM
1	0	LOW
0	0	NULL

AU1	AU15	OUTPUT
1	1	HIGH
0	1	MEDIUM
1	0	LOW
0	0	NULL

JOY SADNESS SURPRISE

Figure 6: Selection rules for each expression. For each emotion, specific weights assigned to each AU support the definition of the presence probability of that emotion. HIGH is 0.99; MEDIUM is 0.66; LOW is 0.33; NULL is zero.

Overall, 79.18% is the global RR for the whole testing database including 1539 faces. Table 3 sums up the RRs for each AU.

AU	RRs		
	AU-based	expression-based	tot
# faces	921	618	1539
AU1	97.73%	92.30%	95.01%

AU2	85.86%	76.92%	81.39%
AU4	82.83%	62.58%	72.70%
AU6	na	87.56%	87.56%
AU9	52.17%	62.52%	57.35%
AU10	65.67%	60.00%	62.84%
AU12	93.00%	87.51%	90.26%
AU15	100.00%	87.54%	93.77%
AU16	83.08%	56.25%	69.66%
AU23	73.13%	56.27%	64.70%
AU24	70.77%	81.25%	76.01%
AU26	91.18%	87.51%	89.34%
AU27	95.00%	87.52%	91.26%
TOT	82.53%	75.83%	79.18%

Table 3: Recognition rates (RRs) of each Action Unit (AU).

Regarding expression recognition, the testing has been carried out on the 618 expression-based faces of the Bosphorus dataset. An overall 73.62% RR is obtained.

In terms of computational time, each image is elaborated in approximately 40 seconds, in which a 15% is dedicated to expression recognition; 35s are required only for AU detection, as geodesic distance evaluation is responsible for most of this time.

A direct comparison between these results and those obtained in current literature is not possible, due to the different conditions of experimentations in terms of database, adopted expressions, and results form. Globally, the proposed methodology gave about 79% and 73% for action unit recognition and expression recognition, respectively. Taking into consideration the contributions relying on geometry, these results match the state of the art accuracy, which ranges between 70% RR [11] and 96.3% [12][13]. The novelty of this work relies on the adoption of geometrical descriptor *g,* distances and angles as key features and on the development of a deterministic methodology based on connected facial points to identify AUs and emotions. This is something new in the branch of 3D FER.

4 CONCLUSION

This work introduces a semi-automatic algorithm for detecting 13 Action Units and recognizing the six basic emotions. The proposed method is based on descriptors coming from Differential Geometry, which are mapped point-by-point on facial surfaces, angles, Euclidean and geodesic distances between 17 automatically localized landmarks. For each query expressive face, the method compares its features to the respective features of the serious face of the same subject; specific geometrical evaluations are made to detect relevant soft-tissue surface behaviours which define the AUs. Then, a crisp logic technique is adopted to recognize the emotion. Experimentations carried out on the 3D Bosphorus facial database brought to a 79% RR for AU recognition and a 73% RR for the expression recognition.

Although the method is still preliminary, it discloses a vein to automatic FER techniques. Improvements of the presented methodology would involve: the integration of other techniques such as neural networks and statistic techniques; the adoption of newly designed (geometrical) features; the enlargement of the experimental facial dataset, including AU specific faces of the Bosphorus database and other 3D databases like FRGC and BU-3DFE; the management of camouflages and holes (occlusions); the analysis of other AUs.

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