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An extended B-Rep solid modeling kernel integrating mesh and NURBS faces

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ABSTRACT
In several application contexts, virtual solid models require to integrate portions of polygonal meshes with synthetic models, designed by traditional parametric/analytical multipatches systems. The paper reports the research aiming at covering the theoretical and numerical aspects connected with an extended geometric solid modeling system, focusing on the B-Rep models and introducing the new paradigm of Extended B-Rep (EB-Rep), which is able to integrate mesh-faces as part of a B-rep model. This paradigm introduces a notion of continuity between parametric and discrete representations, regularized Boolean Operations, a join operator and an approach to represent a valence semi-regular mesh as an EB-Rep structure. A prototype of the geometric solid modeling kernel has been realized and tested in the OpenCascade library environment.

KEYWORDS
Solid modeling kernel; polygonal meshes; B-Rep; regularized boolean operations

1. Introduction

In several application fields, ranging from computer graphics to industrial design, 3D modeling and production systems, users face the design and editing of 3D virtual geometric models. 3D models can be classified in two major families: polygonal models, that represent real shapes at different levels of approximation, and analytical models, which are representations of nominal mathematical shapes, usually formulated as geometrical primitives and Non Uniform Rational B-Splines (NURBS) [6, 16].

Each family has its own modeling pipeline. Polygonal models could be the result of a conversion of an analytical model, an acquisition process through 3D scanners, the extraction of the external boundary of voxel-based representations, such as tomography applications, virtual sculpturing systems and topological optimization software. Oppositely, analytical models are typically created by conventional computer aided modeling systems, which are nowadays disseminated and consolidated on the market to transpose shape concepts into 3D models.

Up to now, only limited research efforts have been devoted to put together these two families of models [2], while the common practice is the conversion of one type into the other. However, the conversion of geometric primitives often implies expensive computations and possible loss of information. Given the usual complexity of the shapes represented by polygons, their conversion to analytical models is onerous and often unfeasible [4, 11]. On the other hand, converting continuous surfaces into polygonal meshes through tessellation prevents the representation from easy editing process in the design stages and loss of explicit geometrical information.

The paper reports a research aiming at integrating designed models, represented by continuous surfaces, and digital models, represented by polygonal meshes, in a unique 3D model in which entities such as NURBS analytical surfaces and meshes coexist. This would close the gap existing between designed and digital models and it would simplify many processes that nowadays require the conversion of one representation into the other.

To support this new modeling paradigm an Extended Solid Modeling System (ESM System) is proposed. It relies on Boundary Representation (B-Rep) of solid models where faces are described by different kinds of representations, both continuous and discrete, i.e. NURBS surfaces and meshes. The representations coexist, interact and, since they do not have to be converted into a common form, they always keep their shape features and properties. The regions of the model represented by meshes maintain their faithful compliance to the real data, while those represented by continuous models are easily editable.

The possible applications of such ESM System spread through cultural heritage, medical science, passing through industrial design and engineering applications. In the cultural heritage applications, analytical models representing regular architectonic elements are often combined with decorative patterns and artistic shapes,

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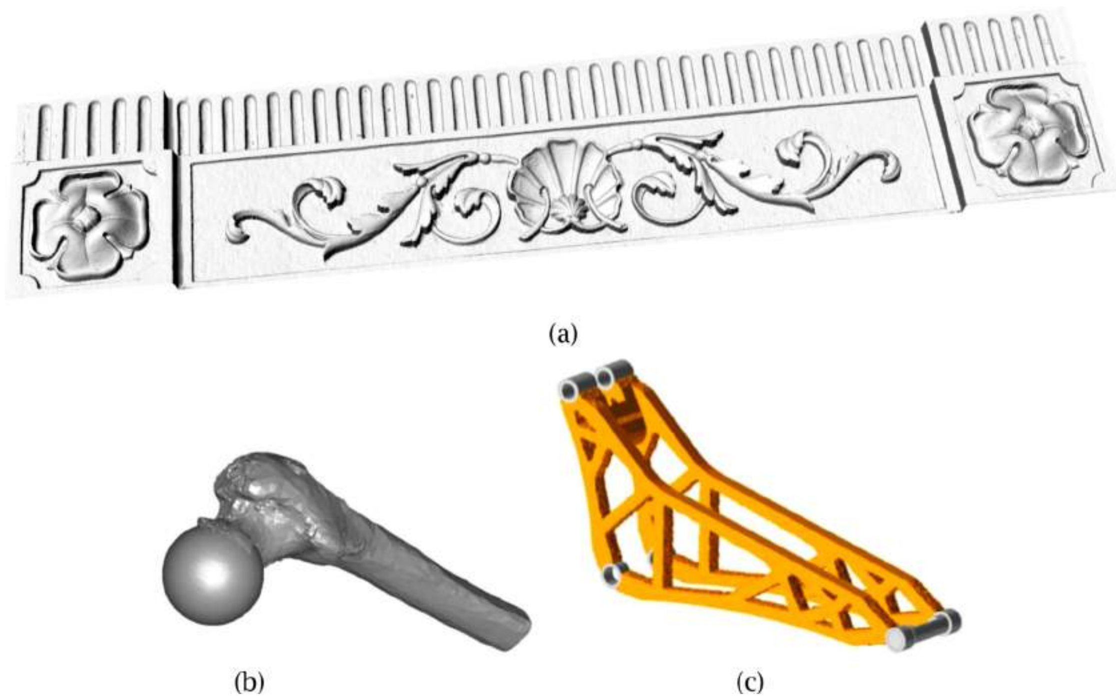

Figure 1. Examples of possible uses of extended solid models: (a) combination of regular shapes with scanned ornaments in the cultural heritage field, (b) integration of the shape of a femur bone model obtain from TC (c) model of a stirrup obtained by topological optimization combined with hinge connections modeled as analytic solids.

acquired by 3D scanning systems (Fig. 1(a)). In the biomechanical field, for instance in the implant design, plastic surgery or maxillofacial surgery, the ESM System could be used to integrate the digitalized parts of the human patient with parts modeled by a biomedical designer, such as in a prosthesis or dental implant (Fig. 1(b)). This would makes the design and prototyping processes quicker, cheaper and more efficient. Finally, in the emerging field of the Additive Manufacturing (AM), topological optimization is an effective way to design parts minimizing material consumption and then part weight. However, such algorithms produce freeform polygonal shapes from the intrinsic voxel-based representation, which must be combined with analytic shapes at the interfaces with other mechanical components (Fig. 1(c)).

The research has been focused on the following goals:

- The study and design of a theory for the new paradigm of the ESM System;
- The development of tools to extend a traditional solid modeling kernel towards an ESM System, aimed at integrating the new primitives and the new paradigm.

Since digital models are represented by meshes, the first goal aims at extending the representation scheme to include a new entity, i.e. the Mesh-Face and a new B-Rep description of a mesh. Then, an extended tool to perform Boolean Operation is introduced which involves the mesh/NURBS intersection problem, in order to realize new mixed models in which NURBS and meshes coexist.

To control the quality of the model, especially in joining operations between faces of different type, it has been necessary to include a notion of continuity for Extended models, called Approximate Geometric continuity (AG), and a set of conditions to guarantee that extended models be manipulated keeping prescribed continuity constraints between their constituting patches.

2. The extended solid modeling system

The structure of a solid modeling system can be subdivided in three parts: the representation data structure, the mathematical foundations and the algorithms necessary for the applications. The representation data structure is the scheme used to represent a virtual 3D model. The mathematical foundations are all those abstract concepts that allow a physical object to be idealized and approximated. Such concepts consider the shape as a perfect and homogeneous 3D point set, ignoring internal structures and boundary imperfections. Furthermore, continuity between geometric representations is considered in order to join solid objects. These are the basis for the algorithms necessary to model the object, i.e. tools to represent, modify and interrogate solid objects.

2.1. Mesh-Face in an extended B-Rep

In the B-Rep model, a face is the topological element connected to a geometric entity in the form of a plane, an analytic surface or a NURBS surface [12, 14]. The proposed Extended B-Rep (EB-Rep) exploits all these kinds of primitives, adding a new type, i.e. the Mesh-Face. A Mesh-Face consists in an arbitrary topology mesh of polygonal facets (quads and/or triangles) with boundaries associated to other faces of the EB-Rep representing a solid object.

The proposed scheme includes both the trivial cases, where the solid is described by only one Mesh-Face, and the more general cases, where the Mesh-Face, delimited by a closed polyline, represents one face of the EB-Rep. The Mesh-Face is handled exactly as a standard face in a B-Rep data structure. The loop of the face is defined by the polygonal boundary of the Mesh-Face.

Therefore, the following definition can be given:

Definition 1. *(Extended B-Rep). An Extended B-Rep is a representation scheme* $B_e = (G_e, T_e)$ *where* T_e *is the set of topological entities and* $G_e = (V_e, E_e, F_e)$ *is the set of geometric entities, in which the set of surfaces* F_e *admits also Mesh-Faces.*

The topological structure $T_e = (V_t, E_t, F_t)$ consists of: V_t a set of pointers to the corresponding vertices in V_e, E_t a set of pointers to V_t to define the topological edge extrema, and pointers to the associated geometrical edges in E_e, and analogously, F_t is a set of pointers to V_t to define the topological face corners, and pointers to the associated geometrical faces in F_e.

Examples of EB-Rep models are shown in Fig. 1.

This new data structure has to maintain the same properties of the standard B-Rep, in particular the same topology T_e, and to provide the same tools while holding the new potential for Mesh-Face primitives. The geometric entities in G_e, are points, curves and surfaces. In particular, the set of surfaces admits analytical surfaces, NURBS surfaces and Mesh-Faces.

2.2. EB-Rep form of a valence semi-regular mesh

The most trivial way to represent a mesh surface as an EB-Rep is to associate a plane to every face of the mesh. We propose an alternative approach, suitable for valence semi-regular quad meshes.

A quad mesh is said to be valence semi-regular if the number of extraordinary vertices (EV), i.e. interior vertices with valence different from 4 and boundary vertices with valence different from 3, is limited [3]. The EVs allow for defining the boundary curves of a coarse partitioning of the model. Each of the coarse regions is described by a regular submesh. The proposed algorithms rely on this idea. The original mesh is subdivided into submeshes with rectangular topology. Then the E-BRep structure is created associating a Mesh-Face to each submesh.

In case of medium resolution triangular meshes, a preliminary conversion into a quadrilateral valence semi-regular mesh can be applied, see [15], [10] and the references therein. For this conversion, valence of vertices is considered. Triangles that share a 4-valence vertex are unified in a quadrilater and then subdivided in 4 quadrilaters, then other triangles are coupled in order to minimize the angular variance form a rectangle and then subdivided in 4 quadrilaters. Finally remaining triangles are splitted in 3 quadrilaters. For high resolution triangular meshes, as those acquired by 3D scanners, a preliminarily simplification step is required before conversion to quad-meshes.

We propose two algorithmic variants, called Quad Mesh Patching (QMP) and Quad Mesh T-Patching (QMTP). The first method creates rectangular patches without T-junctions, while the second one allows T-junctions. Given a quadrilateral mesh $M = (V,E,F)$ with or without boundary, the methods create an EB-Rep $B_e = (G_e, T_e)$, $G_e = (V_e, E_e, F_e)$ where:

- $V_e \subset V$
- $E_e = \{\, \tilde{e} : \tilde{e} = \bigcup e_j,\ e_j \in E \,\}$
- $F_e = \{\, \tilde{f} : \tilde{f} = \bigcup f_j,\ f_j \in F \}$

The EB-Rep geometrical structure G_e has vertices that are vertices of the mesh M, edges that are obtained gluing adjacent edges in E of M, and each Mesh-face in F_e is associated to a submesh with rectangular topology formed by adjacent faces in F. B_e is the EB-Rep description of the original mesh M. In the following description of the methods we will denote by $EV \subset V$ the set of extraordinary vertices of V, by $BV \subset V$ the set of boundary vertices of V that are not in EV.

Both the QMP and QMTP methods are realized by the following steps:

- Step 1: Select the set of extraordinary vertices EV in M.
- Step 2: Create the set of edges E_e and vertices V_e.
- Step 3: Create the set of the faces F_e of the EB-Rep with vertices V_e and edges E_e, and generate the topological structure T_e.

Step 1

Collect all the extraordinary vertices EV of the original mesh M. If M is defined by all extraordinary vertices,

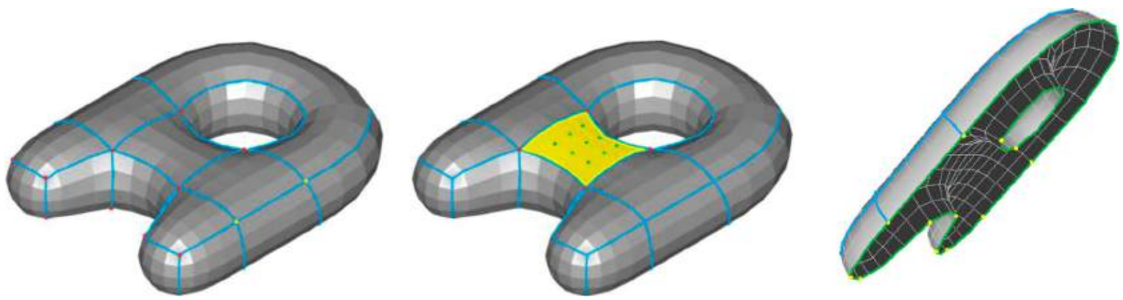

Figure 2. QMP results obtained on closed and open meshes "A."

the method is stopped because there is no possibility to decrease the number of faces, that is $|F_e| = |F|$. If no extraordinary vertex is found, the mesh is represented with a unique Mesh-Face.

Step 2

Starting from each vertex $v_i \in EV$, we add it to V_e, we generate a poly edges $\tilde{p}$ for each incident edge at v_i, and we trace every $\tilde{p}$ by following adjacent edges in E. In order to follow each edge once their end vertices are marked as visited. Each poly edge $\tilde{p}$ is built as follows:

- $\tilde{p}$ starts from $v_i \in EV$, follows a *straight direction,* and ends when it meets:
  - another vertex $v_j \in EV$;
  - a boundary vertex $bv \in BV$ and v_i is an inner vertex;
  - an already visited vertex $v_k \in V$.

  In the last two cases, bv and $v_k \in V$ are added to V_e. In case 2 bv is added to EV in order to be sure that also boundary edges are created.
- If $\tilde{p}$ arrives at a previously visited inner vertex $v_k \in V$, that is $\exists \tilde{e}_i$, delimited by vertices $\tilde{v}_i$ and $\tilde{v}_f$ such that $\tilde{p} \cap \tilde{e}_i \neq \emptyset$, we proceed as follows:
  - the vertex v_k is added to V_e
  - $\tilde{e}_i$ is split into two edges: $\tilde{e}_i$ is only updated since now is delimited by $\tilde{v}_i$ and v_k, while a new edge $\tilde{e}_j$ delimited by v_k and $\tilde{v}_f$ is generated
  - $\tilde{p}$ and $\tilde{e}_j$ are added to E_e

Moreover

- for QMTP: $\tilde{p}$ ends at v_k,
- for QMP: a new poly edge $\tilde{p}$, which has v_k as starting vertex, is generated.

Step 3

The faces $\tilde{f}$ in F_e are created in the last step of both the methods. Starting from V_e and E_e this step first determines the boundary edges for each face in F_e, then adjacent faces f in F insides the edge boundaries are merged to define the rectangular faces $\tilde{f}$ of the EB-rep.

In Fig. 2 we illustrate an example of QMP algorithm applied to the closed (left, center) and open (right) meshes "A". In Fig. 2(left) the extraordinary vertices of the mesh "A" are red-colored, the ordinary vertices in V_e are yellow-colored, and the edges in E_e are marked in blue. Fig 2(center) shows one of the mesh faces in F_e. Fig. 2 (right) shows the result on an open mesh with the boundary vertices in V_e colored in yellow.

Experimental results of the QMP and QMTP algorithms will be provided in section 3.

2.2.1. Remarks

Beyond the design of a new Extended Solid Modeling System, that allows the Extended B-Rep paradigm to be realized as a new data structure, the instruments to integrate the innovative proposal in actual systems based on a classical B-Rep paradigm has been investigated, giving the possibility to enlarge their potentialities without strongly modifying their geometric kernel. In this case, the data structure cannot be modified and thus finding an alternative way to represent a mesh in a standard B-Rep data structure becomes necessary. At this aim, we first apply the described algorithms QMP and QMTP to the original mesh in order to subdivide it into submeshes with rectangular topology. Then we create an E-BRep structure with mesh faces obtained by associating to each submesh a bilinear NURBS surface. The NURBS control points are trivially the vertices of the original submesh. This allows the exact representation of each Mesh-Face as a NURBS surface.

2.3. Continuity for an EB-Rep model

To control the quality of the EB-rep models in the proposed ESM System it is necessary to set conditions to guarantee that the extended models be manipulated while maintaining continuity constraints between their constituting patches. To this aim, how to handle the join between Mesh-Face and NURBS entities is investigated. In this section, a new concept of continuity between smooth and discrete entities will be proposed. Meshes are piecewise linear approximations, under a given tolerance, of analytic surfaces, thus it is only possible to impose certain less restrictive continuity conditions in order to obtain a join that is smooth under a given tolerance.

The preliminary concept introduced in [2] of continuity between discrete and continuous entities, is here formalized and defined as Approximated Geometric (AG^0 and AG^1) continuity.

Definition 2. *(AG^0 continuity). Given a surface s(u,v) with a boundary curve c and a mesh M with a boundary polyline p, we say that s and M join with* ***AG^0 continuity*** *along the boundaries c and p, according to a given tolerance tol, if and only if the bivariate Hausdorff distance between p and c satisfies:*

$$\delta_H(p,c) := \max(\overline{\delta}_H(p,c), \overline{\delta}_H(c,p)) < \mathrm{tol}$$

where

$$\overline{\delta}_H(A,B) = \max_{a \in A} \min_{b \in B} d(a,b)$$

is the univariate Hausdorff distance between curves and d(a,b) is the Euclidean point distance.

This definition implies that the distance between all points of p from c and the distance of all points of c from p has to be under a given tolerance tol.

Fig. 3 shows on the left an example where p is shorter than c, so $\overline{\delta}_H(p,c) < \mathrm{tol}$ but $\overline{\delta}_H(c,p) > \mathrm{tol}$, since $d(c_0,p) > \mathrm{tol}$. Moreover, for every point $p_i \in p$ there is a point $c^* \in c$ such that $d(p_i c*) = \overline{\delta}_H(p,c)$ and for every point $c_i \in c$ there is a point $p^* \in p$ such that $d(c,p*) = \overline{\delta}_H(c,p)$. We observe that if c^* is the nearest point to p_i on c, this does not imply that p_i is the nearest point to c^* on p. Fig. 3 shows on the right an example where c^* is the nearest point to p_2, but p_2 is not the nearest point to c^*.

Definition 3. *(AG^1 continuity). Given a surface s(u,v) with a boundary curve c and a mesh M bounded by a polyline p; let M and s be joined with AG^0 continuity along c and p respectively. We say that s and M join with* ***AG^1 continuity*** *along c and p respectively, according to a given tolerance tol_1, if and only if the bivariate angular Hausdorff distance satisfies:*

$$\delta_{H\alpha}(p,c) := \max(\overline{\delta}_{H\alpha}(p,c), \overline{\delta}_{H\alpha}(c,p)) < tol1$$

where

$$\overline{\delta}_{H\alpha}(A,B) = \max_{a \in A} \vec{n}_A|_a \cdot \vec{n}_B|_b^*$$

with $b^* \in B$ such that $b^* = \arg\min_{b \in B} d(a,b)$ *and* $\vec{n}_A|_a$ *represents the normal vector at a w.r.t. the entity whose boundary is A, and $\overline{\delta}_{H\alpha}(A,B)$ is the univariate angular Hausdorff distance.*

This definition implies that the angle between the normal vectors at two loosest points respectively on c and on p is smaller than a given tolerance.

The univariate angular Hausdorff distance is computed considering for every point $a \in A$, the nearest point $b \in B$.

Finally, another condition of almost continuity between Mesh-Face and NURBS is introduced.

Definition 4. *(G^1-AE continuity). Given a surface s(u,v) with a boundary curve c and a mesh M bounded by a polyline p with vertices $p_1, \ldots, p_n$; let M and s be joined C^0 along p and c. M and s join with G^1-Almost Everywhere Continuity along c and p if and only if they are G^1 connected along all points of c except at $p_1, \ldots, p_n$, where c cannot be differentiable.*

An example of G^1-AE Continuity is illustrated in Fig. 4 where a NURBS surface, on the left, and a Mesh-Face on the right, join with G^1-AE continuity. The surfaces have C^0 continuity along the boundary curve and G^1 continuity everywhere except at the vertices of the mesh on the boundary polyline.

The definitions of AG^0 and AG^1 coincide with the main idea of numerical approximation. Instead, the

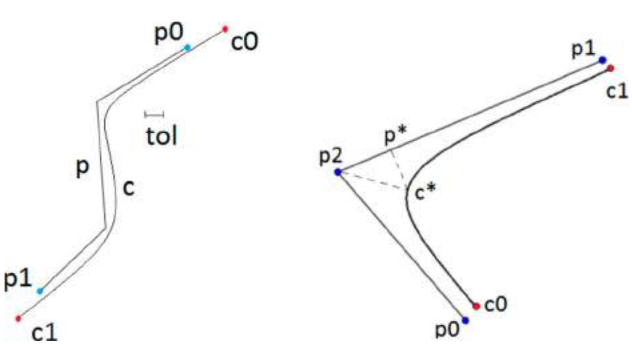

Figure 3. Curve c and polyline p for the definition of AG^0 Continuity.

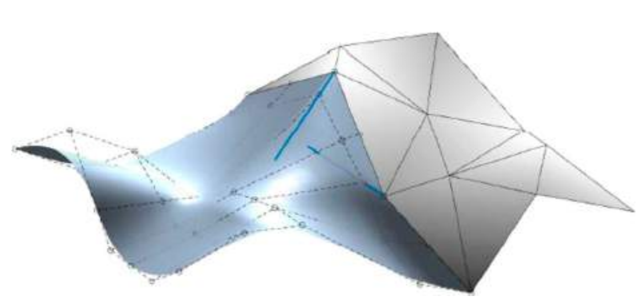

Figure 4. Result of a join between a NURBS and a Mesh-Face with G1-AE Continuity.

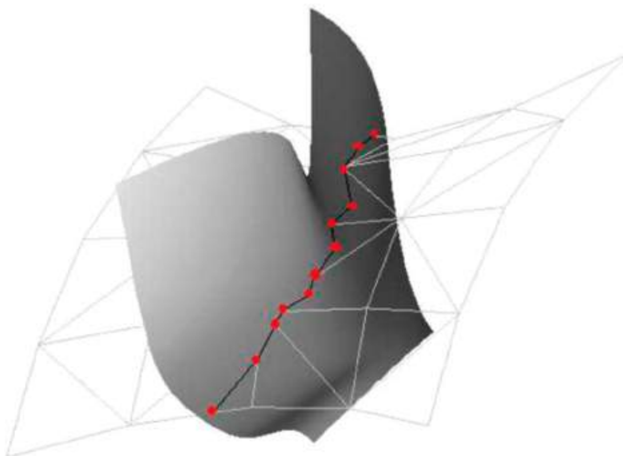

Figure 5. Intersection between a NURBS and a Mesh-Face.

introduced G^1-AE Continuity definition allows joining operators to be smooth except for a finite number of points and in these points the two tangent vectors have a distance angle that is related with the dihedral angle between the adjacent polygons.

2.4. Operators for the extended B-Rep

In this section, Boolean Operations, Cutting and Join Operators are introduced for the EB-Rep, allowing for basic solid modeling functionalities.

The concept of Regularized Boolean Operation is maintained from the traditional approach [7], in which the result of the operations are the closure of the operation between the interior of the two processed solids in order to eliminate the remaining lower-dimensional structures. Similarly, also the cutting operation between a solid and a surface, which results in two distinct solids, is maintained.

The main issue targets the need to handle more general intersection cases, i.e. NURBS-NURBS, mesh-mesh and NURBS-Mesh. While traditional algorithms can be employed for handling the first two cases [1, 5, 9], the NURBS-Mesh intersection introduces a new problem to be considered. In particular, the resulting surfaces are trimmed NURBS and trimmed Mesh-Faces (Fig. 5). The intersection curves between a Mesh-Face M and a NURBS surface s are respectively a polyline p on the Mesh-Face and a NURBS curve c on the NURBS surface. The polyline is C^0 with the Mesh-Face, while is AG0 with the NURBS surface. The precision of the intersection polyline respects the tolerance used in the surface-surface intersection algorithm and depends on the tolerance associated to the AG^0 continuity.

2.5. Join operator

The Join operator attaches two face entities of an EB-Rep changing one or both entities according to AG^0/AG^1 or G^1-AE continuity. The possibility of modifying the entities makes this tool differ from union Boolean Operation, in which both entities are fixed. We define a *1-1 Face-Join* operation that matches two surfaces along an edge, creating a connected surface, a *1-n Face-Join* which closes the hole of an open solid with a surface, and *n-m Face-Join* that matches two open solids closing a hole on both entities. In the following, we will analyze in details the *1-1 Face-Join* and the *1-n Face-Join, considering* the C^0 and the AG^0 cases. The reader can refer to [8] for further details on the other cases.

2.5.1. 1-1Face-Join

Given a Mesh-face M and a NURBS surface s, in order to realize the *1-1 Face-Join* we distinguish two cases:

- Case I: M is fixed, s is modified,
- Case II: s is fixed, M is modified,

In Case I, the boundary curve c of s(u,v) is adapted to join exactly M along a boundary polyline p.

In particular, the *Adapt NURBS to Mesh* (ANTM) algorithm is applied following these steps:

- Multiple knot insertions are applied to subdivide c in a piecewise curve with m pieces, where m is the number of edges in p.
- The control points of c are moved to match points on the polyline p. In particular, control points that are exactly on c are matched with the correspondent vertices of p, while the control points that are not on c are moved onto the correspondent segment delimited by two consecutive vertices of p. Thus, c is matched with p.

The steps of the ANTM algorithm are shown in Fig. 6, while an example of 1-1 Face-Join is shown in Fig. 7. In particular, Fig. 7(b) shows the NURBS surface and the Mesh-Face, which are joined C0.

Case II relies on the preliminary construction of a blending surface $s_M(u,v)$. In particular, the method creates a blending NURBS surface such that:

- one boundary coincides with the boundary curve c of s,
- the opposite boundary matches the polyline p of M,
- the degree of $s_M(u,v)$ is the degree of c in one direction and 1 in the other direction.

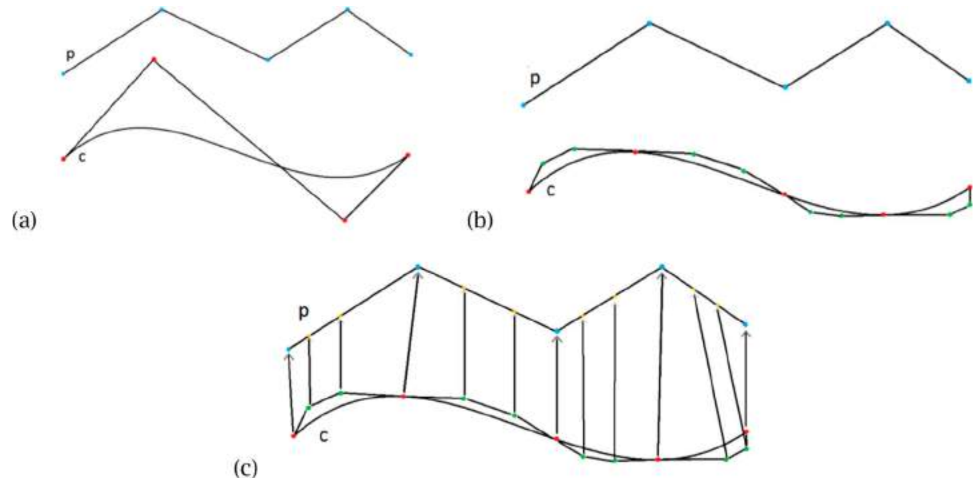

Figure 6. ANTM algorithm: a) curve c and polyline p, b) multiple knot insertions applied to c, c) control points of c are attached to points on p.

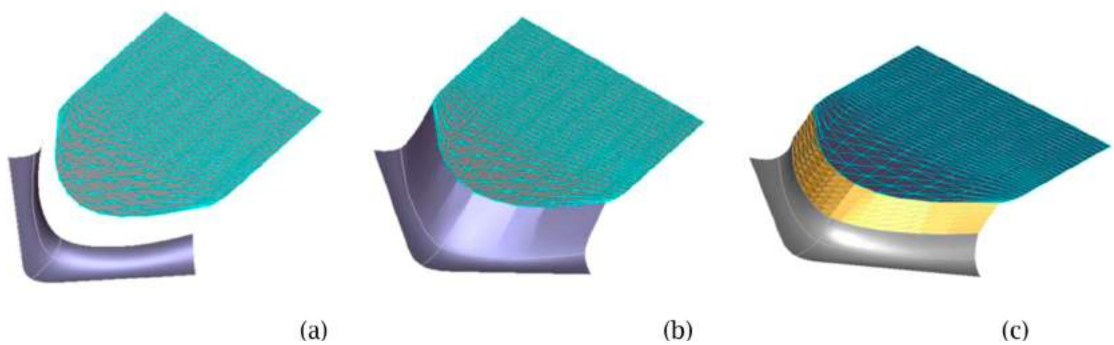

Figure 7. Example of 1-1 Face-Join: (a) Mesh-Face (top) and NURBS surface (bottom); (b) Case I, result of C^0 join; (c) Case II, result of AG^0 join.

Finally, a tessellation procedure is applied to the blending surface in order to create a mesh that extends M and joins AG^0 with the NURBS surface. An example is shown in Fig. 7(c). The Mesh-Face in yellow represents the tessellated surface s_M created under a tolerance tol $= 0.005$. Accuracy AG^0 will be illustrated in section 3.

2.5.2. 1-n Face-Join

The *1-n Face-Join* operation closes a hole of an open EB-Rep solid with a face. F denotes the face which is supposed to have a closed outer boundary W. Instead, the open solid C is supposed to have a bounded hole H which is shared by n faces of C. Case I and Case II are considered as before.

In Case I we extend of the *1-1 Face-Join* C^0 method. In particular, the ANTM algorithm is applied to every couple of correspondent shared edges of W and H. An example is shown in Fig. 8.

In Case II a blending surface $s_M(u,v)$ is created. In this case, the blending NURBS surface is created such that it coincides on one side with the boundary of the NURBS entity and it matches the boundary polyline of M on the other side. Then a tessellation algorithm is applied to $s_M(u,v)$ in order to extend M and join the NURBS face with AG^0 continuity.

The described methods can be applied both in case C is described by NURBS surfaces and F by a Mesh-Face, and in the opposite case, where C is composed by Mesh-Faces and F is a NURBS surface.

3. Implementation and results

A preliminary prototype of the geometric solid modeling kernel has been implemented in the OpenCascade [13] environment. The proposed algorithms have been tested on some models demonstrating the feasibility of the approach.

3.1. Results of EB-Rep form of a valence semi-regular mesh

We evaluated the algorithms QMP and QMTP on a large set of meshes with and without boundaries. By the way of illustration, we report some EB-Reps results obtained by

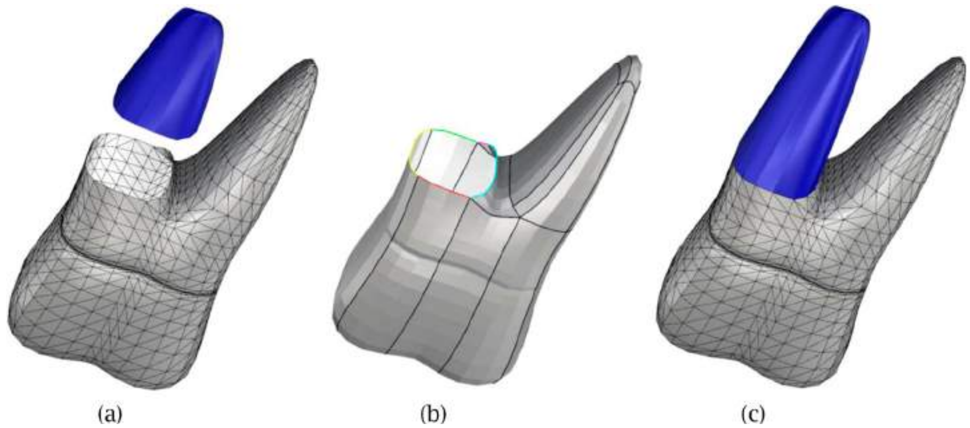

Figure 8. Example of *1-n Face-Join* with n = 5, Case I : a) NURBS surface above the open EB-Rep, b) Topology of the open EB-Rep c) Result of *1-n Face-Join*.

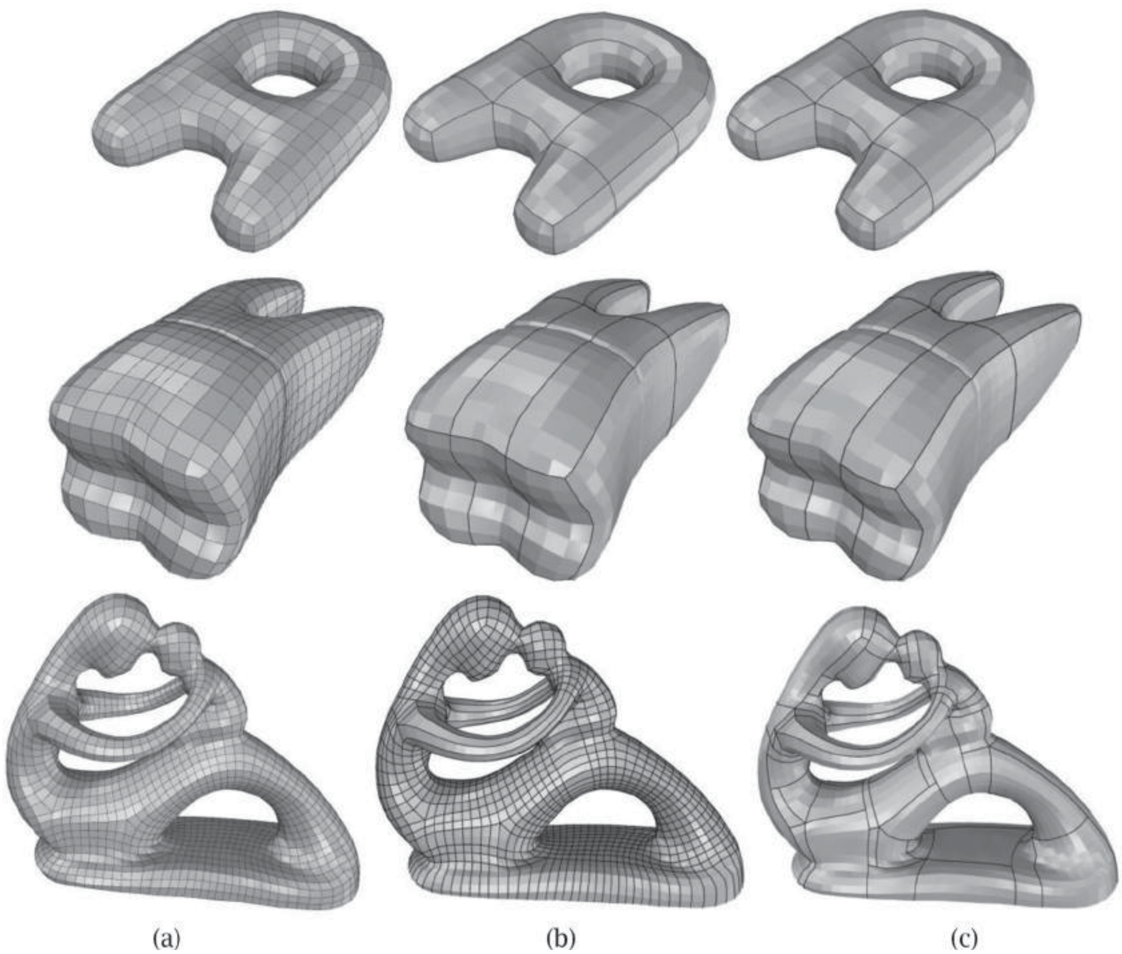

Figure 9. (a): Initial meshes M, (b) EB-Reps obtained by QMP, (c) EB-Reps obtained by QMTP.

applying QMP and QMTP methods to the meshes shown in Fig. 9 that are valence semi-regular quad meshes without boundaries. The obtained results are reported in Tab. 1, where from left to right it can be found: mesh name, number of faces, vertices, extraordinary vertices of M, number of faces and vertices in T obtained by QMP, number of faces and number of vertices in T obtained by QMTP. As expected, the final number of faces and vertices necessary for efficiently representing a valence semi-regular mesh depends on the number of extraordinary vertices of the mesh. The less is the number of EV in the original M, the strongest is the reduction in terms

Table 1. EB-Rep form of Quadrilateral Meshes for meshes illustrated in Fig. 9.

Mesh	|F|	|V|	|EV|	$\|F_e\|$ QMP	$\|V_e\|$ QMP	$\|F_e\|$ QMPT	$\|V_e\|$ QMPT
A	512	512	16	24	24	18	20
Tooth	1132	1134	16	30	32	28	32
Fertility	3357	3351	48	2271	2265	132	191

of number of final faces in the EB-Rep. Moreover, when the EB-Rep structure allows for T-Junctions as in case of QMTP, the reduction achieved is improved.

3.2. Results of 1-1 Face-Join

Let us consider the Case II of 1-1 Face Join where s is fixed and M is modified described in section 3.5.1.

In the example illustrated in Fig. 6(c) the mesh is a 9×5 grid. The AG^0 join between the blending surface and the NURBS for a given tolerance tol $= 0.0015$ is obtained by tessellating the blending surface to a 21×11 grid. The tolerance is chosen so that it is 0.1% wrt the length of the boundary curve c of s(u,v).

The intermediate steps of the process before reaching the fixed tolerance produced the distances $\delta_H(p,c)$ reported in Tab. 2, according to *D*ef. 2.

Table 2. Accuracy according AG^0 join

Mesh grid	$\delta_H(p,c)$
9×5	0.0047
13×7	0.0023
17×9	0.0017
21×11	0.0014

3.3. Results of Boolean Operations with EB-Reps

The Boolean Operations between EB-Rep solids have been implemented in the Extended Solid Modeling System. Few examples are reported of Boolean Operations between a solid represented by NURBS surfaces and a solid described by Mesh-Faces.

A first example is shown in Fig. 10, where a NURBS cylinder, a Mesh-Face representing a tooth and a NURBS cube are used to create a prosthesis for a dental

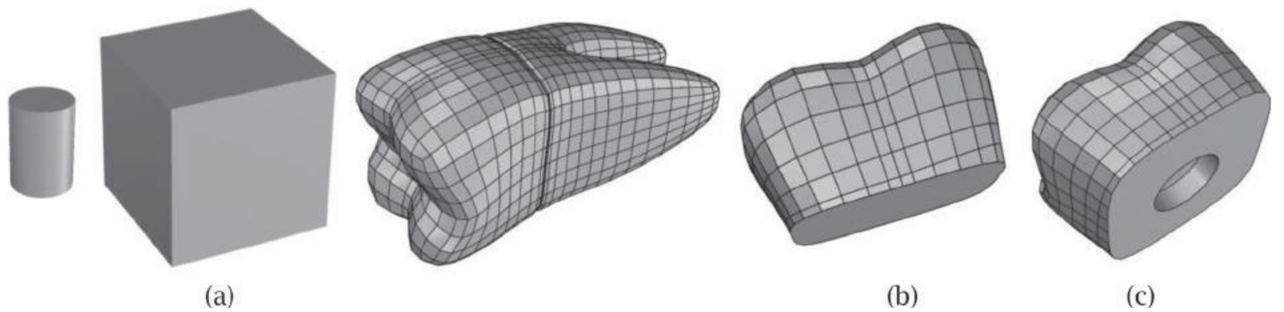

Figure 10. Example of Boolean operation: a) a B-Rep (NURBS) cylinder, an EB-Rep (Mesh-Face) tooth and a NURBS cube, b) difference between tooth and cube. c) difference between result of operation in b) and cylinder.

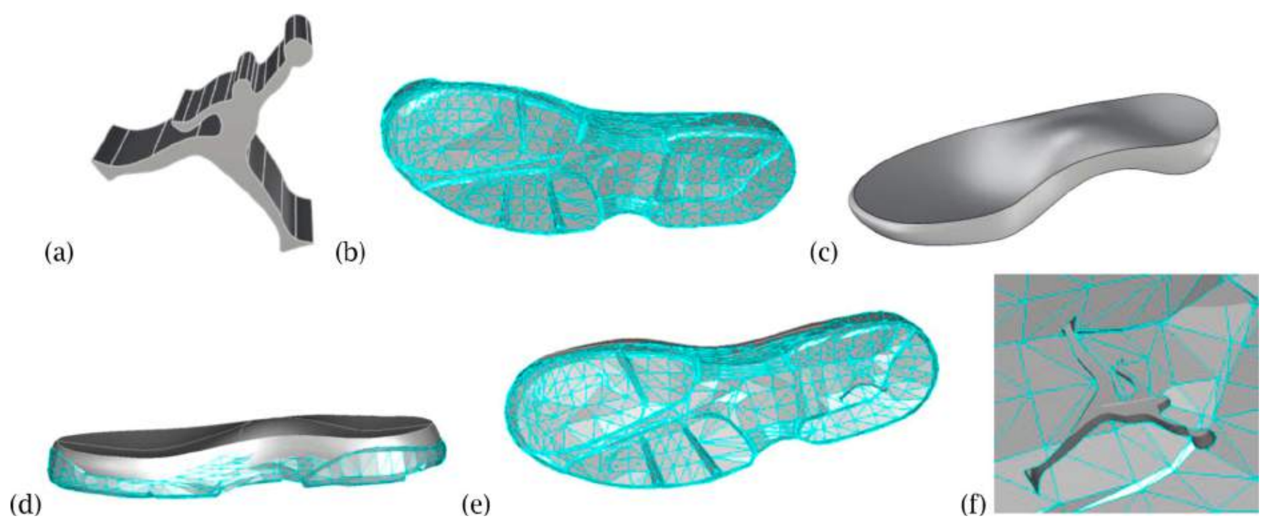

Figure 11. Example of Boolean operation: (a) a B-Rep (NURBS) solid representing the “Air Jordan logo”, (b) an EB-Rep (Mesh-Face) representing a sole, (c) NURBS insole, (d) union between insole and sole, (e) difference between the previous solid and the logo, (f) Detail of the Boolean operation

implant. Fig. 10(a) shows the three objects. Fig. 10(b) shows the result of the Boolean Difference between the tooth and the cube. Fig. 10(c) shows the result of the difference between EB-Rep obtained in Fig. 10(b) and a NURBS cylinder. In this case, the new EB-Rep obtained is bounded by a trimmed Mesh-Face, a trimmed NURBS plane, and a trimmed NURBS cylinder. The intersection curves are closed polylines that join AG^0 with both the NURBS and the Mesh-Face surfaces.

The second example is illustrated in Fig. 11, where the lower part of a shoe is created using Boolean operations between EB-Reps with both Mesh-Faces and NURBS surfaces. Fig. 11(a-b-c) show respectively the B-Rep NURBS solid representing the "Air Jordan logo", a Mesh-Face representing a sole and a NURBS solid representing an insole. In particular, a preliminary representation of the sole triangular mesh as an EB-rep structure is required in order to be handled in our prototype extended solid modeling system.

In Fig. 11(d) the result of Boolean Union between the sole and the insole is shown. Fig. 11(e) show the difference between EB-Rep in Fig. 11(d) and the "Air Jordan logo". A detail is shown in Fig. 11(f). In addition, in this case the new EB-Rep obtained is bounded by trimmed Mesh-Faces and trimmed NURBS. Intersection curves join with AG^0 continuity with both the NURBS and the Mesh-Face surfaces.

4. Conclusions and further work

The paper presents the Extended Solid Modeling System to manage both Mesh and NURBS entities and represents solids obtained by modeling these entities using an Extended B-Rep scheme. In order to realize this new system it was necessary to introduce some definitions to handle the notion of continuity between Mesh-Face and NURBS entities and operators to perform join and Boolean operations.

The proposed approach has been experimented in an existing solid modeling kernel, enhancing the concept of valence semi-regular mesh to formulate mesh faces as linear NURBS. Future work is focusing on the implementation of a kernel to handle natively mesh faces in order to operate on mesh-analytic hybrid models and to be experimented on application test cases.

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