# CAD based simulation of ball end mill manufacturing 

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#### Abstract

Geometry of micro ball end mill is quite complex, as helical cutting edges are formed on a hemispherical surface. The edge must have proper rake and clearance faces for effective cutting and appropriate radius to strengthen it. Manufacturers of micro tools do not reveal the details, as they are proprietary in nature. In the present work, a CAD approach is proposed to simulate the generation machining of micro ball end mill using appropriate kinematic relations. Different geometries of micro ball end mill can be generated by the approach proposed in the present work. A critical study of virtual micro ball end mills will be useful to understand their performance. The study allows visualization of the cutting angles and edge radius along different transverse sections, as these geometrical parameters influence mechanics of micro cutting.


## KEYWORDS

Ball end mill; generation machining; CAD approach; edge radius

## 1. Introduction

Performance of a micro ball end mill is influenced by its geometry. Besides rake and clearance angles along cutting edge, cutting edge radius is imparted on the cutting edge so that it gets strengthened. In a micro ball end mill, the study of cutting edge is quite complex. The helical cutting edge lies on hemispherical surface and hence position of normal plane changes from the ball tip to outer surface. The edge radius has to be maintained at a specified value in normal plane along the helical edge. It is essential to understand the geometry of ball end mill and assess edge radius so that cutting performance of micro ball end mill can be modeled.

The geometry of micro ball end mill has been analyzed using orthographic views or solid models. However, these geometrical approaches do not consider the manufacturing aspects. In practice, different features provided on ball end mill are generation machined by imparting suitable kinematic motions. Traditionally, analytical or graphicanalytical methods are used to arrive at generated profile from a given cutter profile [12]. With the advent of computer and enhancement of its graphics capability, these 2-D profiling approaches have been extended to 3-D surface generation. In general, method of co-ordinate transformation [2-5] and enveloping theory based methods $[10,11]$ are used to generate the desired tool geometry. Majority of ball end mill geometries were developed analytically based on coordinate transformation concept, for example, grinding of ball geometry with equal rake
and clearance angle [2], ball with tooth offset center [3], tapered ball with torus shape cutter [4] and ball end mill with chamfered edge [5]. Few approaches followed combined envelop and differential geometry methods to develop ball end mill geometry; examples being grinding of ball end mill with new engagement conditions [10] and grinding of a 5 -axis ball end mill [11]. Tandon et al. [13] discussed a method of creating different end mill geometry by sweeping sectional profile along the axis. In some attempts, the 3-D model is verified and necessary CNC programs are written to manufacture the required geometry. Attempts are made to validate CNC programs through simulation, but these are not documented here as they are outside the scope of the present work.

With solid modeling approach, it has been shown that generated surface can be obtained as a result of Boolean subtraction. Mohan and Shunmugam [8] demonstrated the use of Boolean operation in helical grooving simulation in CAD environment. They also verified the results by comparing with that obtained by the analytical approach using differential geometry and envelope theory [8]. Following such CAD based simulation approach, other researchers have simulated generation machining of rake face of ball end mill [1], flutes on flat end mill [6], helical groove on flat end mill [7] and spherical gear surface [9]. In the present work, CAD based simulation is followed to generate rake and relief surfaces of the micro ball end mill along with radiused edge. This virtual model of ball end mill can be used to view the transverse
section at any distance from center of hemisphere and edge radius at the corresponding section. In other words, it is possible to virtually section the developed ball end mill by a plane perpendicular to the axis of ball end mill. In the present work, Boolean subtraction in CAD package is used to section the geometry. The sectioned tool can be used to visualize basic features of the ball end mill.

The current work presents a simple CAD based ball end mill generation simulation model based on a single reference parameter. This model is generic, but the results are presented here for a specific ball end mill of 0.4 mm radius, $30^{\circ}$ ( 0.5236 radian) helix angle and zerorake angle. This CAD model can be used for visualizing the complex geometry of ball end mill. Authors have used the developed model to observe included angle between rake and clearance face and edge radius in a transverse plane by virtual sectioning.

## 2. CAD based ball end mill manufacturing simulation

Generation machining of ball end mill is different from that of flat end mill. The basic difference between flat and ball end mill geometry generation is the number of axes involved in the generation process. In ball end mill, geometry of helical flute on the hemisphere is generated by the simultaneous motions of five different axes. The relative motions between cutter and workblank are decided based on the geometrical parameters of both ball end mill and cutter wheel. The cutting edge curve on the ball end is similar to a helix lying on a hemispherical surface. The schematic diagram shown in Fig. 1 gives better idea about the geometrical parameters of ball end mill cutter.


Figure 1. Geometrical details of a ball end mill.

Helical edge can be defined by the Eqn. (2.1).

$$
\begin{align*}
\vec{r} & =\left\{\begin{array}{l}
R \phi \cot \alpha \\
R \sqrt{1-\left(\phi^{2} \cot ^{2} \alpha\right)} \cos \phi \\
R \sqrt{1-\left(\phi^{2} \cot ^{2} \alpha\right)} \sin \phi
\end{array}\right.  \tag{2.1}\\
\phi & =\tan \alpha \sin \beta \tag{2.2}
\end{align*}
$$

where, $R$ is the radius of the cutter, $\phi$ (Eqn. (2.2)) and $\beta$ are angular parameters used to define the ball geometry in spherical coordinate system and $\alpha$ is the helix angle at the joining point of hemisphere and cylinder. The local helix angle ( $\alpha_{x}$ ) on the hemispherical portion is defined as the angle between a vector tangent to a point on helical cutting edge and the vector tangent to the generator curve at the same point. The local helix angle at any point on the edge is given by Eqn. (2.3).

$$
\begin{equation*}
\alpha_{x}=\cos ^{-1} \frac{\cot \alpha}{\sqrt{\cot ^{2} \alpha+\left(1-\phi^{2} \cot ^{2} \alpha\right)^{2}}} \tag{2.3}
\end{equation*}
$$

### 2.1. Rake face generation

In the present CAD simulation, rake face on the hemispherical part is created as successive surfaces generated by cutter solid at fine intervals. These surfaces are obtained by the relative motion between solids and Boolean subtraction operation. The relative movements between workblank and cutter are decided based on the kinematics motions. The relative motions are expressed as a function of single reference parameter, $m$. Implicit representation of the family of surfaces is given in Eqn. (2.4) [8].

$$
\begin{equation*}
q(x, y, z, m)=0 \quad m_{\min } \leq m \leq m_{\max } \tag{2.4}
\end{equation*}
$$

In this work, the simulation process is presented for generation of a ball end mill with 0.4 mm radius, $30^{\circ}$ ( 0.5236 radian) helix, zero-normal rake angle and $12^{\circ}$ ( 0.2094 radian) normal clearance angle. Initial workblank is defined as a cylinder with a hemispherical end and the cutter wheel as a tapered solid disc. When using actual dimensions, it is seen that the resolution causes certain limitation in simulation. Hence, the geometrical parameters of both cutter and work blank are scaled up by 10X for convenience. Scaled up dimensions of both work blank and cutter wheel are given in Fig. 2. Schematic diagram of virtual simulation process is shown in Fig. 3.

Ball end generation involves machining of rake and clearance face. Material removal in the generation process is realized by the movement of workblank and cutter wheel and subtraction of the interface volume from the work blank. The desired shape is obtained on the work blank by a series of Boolean subtraction operation. Generation machining of ball end mill cutter is simulated in AUTOCAD 2016 software by executing Autolisp program. Different views of virtually generated micro ball end mill are shown in Fig. 4. Rake and clearance faces are clearly marked on the virtually generate tool. Procedures followed to generate both the faces on the workblank are discussed below.

(a) Work blank

(b) cutter wheel [10]

Figure 2. Geometrical details of solids considered in the present work.


Figure 3. Schematic diagram of ball end mill generation process.

The desired rake face shape is created by the rotation of workblank about two different axes and linear movements of cutter wheel. Kinematic motions and
orientation of solids for this generation machining are determined based on the reference parameter, $m$. Both the rotational angles involved in this simulation are interdependent. Therefore, in the current work, increment for one of the rotation parameter is set to a constant value and other is expressed as a function of the chosen parameter using Eqn. (2.2) and (2.3). Linear movements of the cutter solid from its home position to the required positions have to be computed for the given geometrical parameters of solids by considering the above mentioned rotational parameters.

Fig. 5. shows the coordinate frames used for the generation simulation. $\mathrm{S}_{\mathrm{o}}:\left(\mathrm{X}_{\mathrm{o}}, \mathrm{Y}_{\mathrm{o}}, \mathrm{Z}_{\mathrm{o}}\right)$ is fixed coordinate system considered for the entire simulation process. It is also taken as home position for the cutter wheel. $\mathrm{S}_{\mathrm{w}}:\left(a_{w}, b_{w}, c_{w}\right)$ is a fixed frame for the workblank. $\mathrm{S}_{\mathrm{c}}:\left(a_{c}\right.$, $\left.b_{c}, c_{c}\right)$ and $\mathrm{S}_{\mathrm{a}}:\left(a_{a}, b_{a}, c_{a}\right)$ are the intermediate coordinate frames for cutter and workblank respectively. $\varsigma_{c, a, w}$, $\rho_{c, a, w}$ and, $\delta_{c, a, w}$ are rotational parameters in coordinate systems $S_{\mathrm{c}}, S_{\mathrm{a}}$ and $\mathrm{S}_{\mathrm{w}}$ respectively. The point ( $a_{w}=-$ $100, b_{w}=-200, c_{w}=50$ ) is set as the home position of the work blank in the fixed coordinate frame. An axis $\left(\mathrm{Y}_{\mathrm{r}}\right)$ parallel to $\mathrm{Y}_{\mathrm{o}}$ and passing through point $\mathrm{O}_{\mathrm{r}}(-70,-$ 200,50 ) on the fixed coordinate frame is used to give a rotational degree of freedom $\left(\rho_{r}\right)$ to the workblank. In this rake face generation process the reference parameter $m$ is varied from 0 to $n=40$ by an increment of 1 . At each step, cutter wheel is copied from its initial position to the desired position $\left(a_{c}, b_{c}, c_{c}\right)$ on the fixed coordinate frame. At the same time, workblank is rotated about $Y_{r}$ and $X_{a}$ axes to trace the required helix. The workblank is first rotated by an angle of $\rho_{r}=\alpha_{x}$ about $Y_{r}$ axis in clockwise direction. Then work blank rotates anti-clockwise direction by a constant angular increment of $\mathrm{d} \varsigma_{a}=0.825^{\circ}$ ( 0.0144 radian) about $\mathrm{X}_{\mathrm{a}}$ axis (axis of workblank). In each step, workblank rotation


Figure 4. Different views of virtual model of ball end mill.


Figure 5. Coordinate systems used for generation simulation.
about $\mathrm{Y}_{\mathrm{r}}$ axis starts from 0 to $\rho_{r}$ radian and at the end of the step it returns back to zero. The value of $\rho_{r}$ is calculated by substituting $\phi=m\left(\mathrm{~d}_{\varsigma_{a}}\right)$ in Eqn. (2.3). Details of linear and rotational motions used for the simulation is given in Tab. 1. In each step, the copied geometry of cutter wheel is subtracted from the work blank to create the desired shape. Detailed procedure of rake face generation is given in Fig. 6. Initial positions
of cutter and workblank for the current simulation are shown in Fig. 7 (a). Images of generation process are captured from different viewing angle during step number 20 (Fig. 7 (b)) for getting better idea about the process. Fig. 7 (c) is the zoomed view of portion near to the rake face and this helps to see the overlapping of cutter and workblank geometries during generation of rake face.

Table 1. Parameters used for tool rake face generation.

| Solids | Motions |  | Distance, mm or angle, rad |
| :---: | :---: | :---: | :---: |
| Work blank | Rotation about $\mathrm{X}_{\mathrm{a}}$ axis | $S_{\text {a }}, \mathrm{rad}$ | 0 to 0.5774 |
|  | Rotation about $\mathrm{Y}_{\mathrm{r}}$ axis | $\rho_{\mathrm{r}}, \mathrm{rad}$ | 0 to 0.5236 |
| Cutter wheel | Translation along $\mathrm{X}_{0}$ axis | $\mathrm{a}_{\mathrm{c}}, \mathrm{mm}$ | $\begin{aligned} & 59.2 \sin (0.1745+0.01745 m)-70- \\ & \quad\left(10-1.732 R m S_{a}\right) \cos \rho_{r} \end{aligned}$ |
|  | Translation along $\mathrm{Y}_{0}$ axis | $\mathrm{b}_{\mathrm{c}}, \mathrm{mm}$ | $\begin{gathered} 59.2 \cos (0.1745+0.01745 m)-200+R(1-m / n) \\ -0.5 \sqrt{1-1.732 m d} \varsigma_{a} \end{gathered}$ |
|  | Translation along $\mathrm{Z}_{0}$ axis | $c_{c}, \mathrm{~mm}$ | $50-\left(10-1.732 R m \mathrm{~d} \varsigma_{a}\right) \sin \rho_{r}$ |



Figure 6. Algorithm for generation of rake face.

### 2.2. Clearance surface generation

Clearance surface is generated with a normal clearance angle of $12^{\circ}$ ( 0.2094 radian). The same coordinate frames discussed in section 2.1 are used for generating clearance face. In clearance surface generation, one rotational freedom is given to workblank and other one is given to the cutter wheel. At the same time cutter wheel also has translational degrees of freedom. In this generation process, rotation about $\mathrm{Y}_{r}$ axis is suppressed ( $\rho_{r}=0$ ). That means the coordinate frame $S_{a}$ kept same as $S_{w}$. Total 90 number of steps are present in this clearance face generation
process and $m$ is varied from 0 to 90 by an increment of 1 . In each step, workblank is rotated anti-clockwise from 0 to an angle of $\zeta_{w}=0.5774\left(\sin \delta_{c}\right)$ about $\mathrm{X}_{\mathrm{W}}$ axis and cutter rotates $\delta_{c}$ about $\mathrm{Z}_{\mathrm{c}}$ axis to trace the cutting edge curve created in rake face generation process. At the end of each step workblank rotates back to initial position. The cutter wheel is copied from its home position to desired position ( $\mathrm{a}_{\mathrm{c}}, \mathrm{b}_{\mathrm{c}}, \mathrm{c}_{\mathrm{c}}$ ) first. Then rotated the cutter anti-clockwise by an angle of $\varsigma_{c}=112^{\circ}$ (1.955 radian) about $\mathrm{X}_{\mathrm{c}}$ axis to set the clearance angle. Cutter wheel is again rotated clockwise by an angle of $\delta_{c}=m\left(\mathrm{~d} \delta_{c}\right)$ about $\mathrm{Z}_{\mathrm{c}}$ axis. The value


Figure 7. Positions of workblank and cutter wheel during rake face generation at step no. 20.
Table 2. Parameters used for tool clearance surface generation.

| Solids | Motions |  | Distance, $m m$ or angle, rad |
| :--- | :---: | :---: | :---: |
| Work blank | Rotation about $X_{w}$ axis | $\varsigma_{w}$, rad | 0 to 0.5774 |
| Cutter wheel | Rotation about $Z_{c}$ axis | $\delta_{c}$, rad | 0 to 1.5708 |
|  | Translation along $X_{o}$ axis | $\mathrm{a}_{c}, \mathrm{~mm}$ | $R \sin \left(m \delta_{c}\right)-80$ |
|  | Translation along $\mathrm{Y}_{0}$ axis | $\mathrm{b}_{c}, \mathrm{~mm}$ | $R \cos \left(m \delta_{c}\right) \cos \left(\zeta_{w}\right)-200$ |
|  | Translation along $\mathrm{Z}_{0}$ axis | $\mathrm{c}_{\mathrm{c}}, \mathrm{mm}$ | 50 |

of increment angle $\mathrm{d} \delta_{c}$ is taken as $1^{0}$ ( 0.01745 radian). Then the copied geometry of the cutter wheel is subtracted from the work blank to create the desired shape on the clearance side of the work blank. Details of translational and rotational degrees of freedom used to generate clearance surface is given in Tab. 2. Algorithm used for
this simulation is shown in Fig. 8. Images of intermediate positions of work blank and cutter wheel and the closer view of cutter and workblank during clearance surface generation simulation are shown in Fig. 9. For a twofluted ball end mill, the steps explained in sections 2.1 and 2.2 are repeated for the second cutting edge.


Figure 8. Algorithm for generation of clearance face.

(a) Intermediate position of cutter and workblank
(b) Closer view

Figure 9. Position of work blank and cutter wheel during clearance face generation at step no. 20.

## 3. Cutting edge radius on transverse and normal planes

Selection of the assessment plane is an important aspect to obtain cutting edge radius. This geometric feature is to be assessed in a plane normal to the cutting edge. Sectioning or scanning along a plane normal to the cutting edge involves extreme positioning difficulty. Practically, it is convenient to produce a section plane (transverse) perpendicular to the axis of the tool. Therefore, it is proposed
to assess the radius in the transverse plane and suitably derive the actual value with the help of their geometric relationship. A rectangular block is created at the selected transverse plane such that it encloses the portion of ball end mill to be removed. Sectioning is done by performing Boolean subtraction operation between the ball end mill and the rectangular block.

Schematic diagram of the planes considered on the ball end mill is shown in Fig. 10 (a). Plane considered


Figure 10. Planes considered on virtual ball end mill at 1.0 mm below center of hemisphere.
to assess the cutting edge radius (perpendicular to the axis of tool, $\mathrm{P}_{\mathrm{t}}$ ) and the plane which contains the specified edge radius of a ball end mill cutter $\left(\mathrm{P}_{\mathrm{n}}\right)$ are shown in this figure. Applying helix angle correction for the edge radius in normal plane can be expressed as,

$$
\begin{equation*}
r_{n^{\prime}}=r_{t} \cos \alpha_{x} \tag{3.1}
\end{equation*}
$$

In Eqn. (3.1), $r_{t}$ is the edge radius obtained from transverse plane and $r_{n}$, is the calculated edge radius in normal plane. The generated tool is used to study the relationship between edge radiuses at transverse and normal sections. Analysis on the virtual tool is started after checking the correctness of the geometry. The tool geometry is verified by assessing the included angle between the rake and clearance face. The generated CAD tool model is transversely sectioned at an axial position of 1 mm from the center of hemisphere as shown in Fig. 10 (b). Positions of transverse and corresponding normal planes passing through the cutting edge point are shown clearly on the CAD geometry. The included angle assessed from a transverse section $\left(\varepsilon_{t}=79.5^{\circ}\right)$ at a height of 1 mm below the center of the hemisphere is shown in Fig. 10 (c). In the present work, the following equation is used to find included angle in normal plane $\left(\varepsilon_{n^{\prime}}\right)$. This relation is valid only in case of zero-rake angle tool geometry.

$$
\begin{equation*}
\tan \varepsilon_{n^{\prime}}=\tan \varepsilon_{t} \cos \alpha_{x} \tag{3.2}
\end{equation*}
$$

The included angle in the normal plane is calculated on the basis of Eqn. (3.2) as $78.1^{\circ}$ (1.363 radian) which is closer to the specified included angle for zero-rake angle ( $\varepsilon_{n}=90-12=78^{\circ}$ or 1.361 radian) given on the virtual tool. The included angles for different heights below the center are given in Tab. 3.

Generation simulation produces a virtual ball end milling cutter with sharp cutting. This ball end mill model is then exported to another CAD software (Cimatron E-10.0) for radiusing. The cutting edge rounding was performed using "round" command with a specific radius value of $r_{n}=4 \mu \mathrm{~m}$ (equivalent of $0.4 \mu \mathrm{~m}$ ) in rolling-ball mode. For assessment of edge radius, the virtual ball end mill is transversely sectioned at different x -distances $(x=0,1,2$ and 3 mm$)$, from the center of ball towards the ball tip. Edge radius obtained on a virtual tool at a transverse plane 1.0 mm below the center of hemisphere is shown in Fig. 11.


Figure 11. Edge radius obtained on a virtual tool at a transverse plane 1.0 mm below the center of hemisphere.

Cutting edge radius, obtained at different transverse sections, are scaled down by 10 X and presented in Tab. 3. Parameter $R_{x}$ (in Tab. 3.) is the radius of hemispherical end portion along the axis at a distance $x$ from the center of the hemisphere. The calculated values of edge radius and included angle are closer to the nominal values. The consolidated result in Tab. 3 gives a better idea about variations in results and it is observed a maximum variation of $0.021 \mu \mathrm{~m}$ in edge radius and $0.01^{\circ}$ ( 0.002 radian) in included angle from the nominal values. Further, the analysis carried out on CAD model strongly suggests the requirement of helix angle corrections to obtain both edge radius and included angle in normal plane.

The major advantages of presented generation machining are discussed below. The relative motions of cutter and workblank in the current generation process is controlled using a single reference parameter. This eliminates the use of complex equations in geometrical modeling of ball end mill. Absence of complex equations helps to reduce total computational time. The algorithm presented in this paper can be easily converted in to CNC codes for actual machining. The developed virtual ball end mill model and CAD based analysis can be considered as a next step for solving the issues related to the measurement of cutting edge radius of micro cutting tools.

Table 3. Edge radius and included angles at different sections.

| $X \mathrm{~mm}$ | $R_{x} \mathrm{~mm}$ | $\phi \mathrm{rad}$ | $\alpha_{x} \mathrm{rad}$ | Edge radius |  |  | Included angle |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $r_{n} \mu \mathrm{~m}$ | $r_{t} \mu \mathrm{~m}$ | $r_{n^{\prime}} \mu \mathrm{m}$ | $\varepsilon_{n} \mathrm{rad}$ | $\varepsilon_{t} \mathrm{rad}$ | $\varepsilon_{n^{\prime}} \mathrm{rad}$ |
| 0 | 0.4 | 0 | 0.52 | 0.4 | 0.462 | 0.4 | 1.361 | 1.388 | 1.361 |
| 0.1 | 0.387 | 0.14 | 0.50 | 0.4 | 0.457 | 0.402 | 1.361 | 1.388 | 1.363 |
| 0.2 | 0.346 | 0.29 | 0.41 | 0.4 | 0.446 | 0.409 | 1.361 | 1.379 | 1.362 |
| 0.3 | 0.265 | 0.43 | 0.25 | 0.4 | 0.391 | 0.379 | 1.361 | 1.369 | 1.363 |

## 4. Conclusions

The major conclusions from present work are

- A simulation of generation machining process for the development of a micro ball end mill has been attempted in this work. Using repeated Boolean subtraction operation on CAD based software platform, virtual model of ball end mill could be successfully developed.
- The proposed approach opens up several possibilities of generating different ball end mill geometries using different cutter geometry and kinematic relations.
- The virtual model of micro ball end mill can be used to study the cutting angles and edge radius along transverse and normal planes.
- The development of virtual ball end mill model and subsequent analysis forms a basis for addressing the issues related to the cutting edge radius measurement.


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