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The Polar-aesthetic Curve and Its Applications to Scissors Design

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#### Abstract

The origin of scissors is very old and a pair of scissors which was probably made in ancient Greece around B.C. 1000 is discovered. In Japan a new type of scissors was produced with newly designed blades and has become one of hit products in 2012. The shape of their blades is a Bernoulli curve. The Bernoulli curve is the logarithmic spiral and can be classified as a type of the log-aesthetic curves. One of its properties is that the angle between its tangent vector and the radial axis from the origin is kept constant. This is the reason why the logarithmic spiral is also called the equiangular spiral. Consequently if the blades of a pair of scissors are given by a logarithmic spiral, the cutting angle is always constant.

In this paper, we extend the logarithmic spiral to make the cutting angle of a pair of scissors a linear function of the rotation angle of the cutting edges and newly define the polar-aesthetic curve and discuss about conditions for the monotonicity of its curvature. Furthermore we analyze cutting torques of the scissors and show that the torque of the scissors whose blade is a polar-aesthetic curve can be controllable to make the users feel easier to cut a sheet of paper.


Keywords: polar-aesthetic curve, logarithmic spiral, log-aesthetic curve, scissors design.

## 1. INTRODUCTION

The origin of scissors is very old and a pair of scissors which was probably made in ancient Greece around B.C. 1000 is discovered. In Japan a new type of scissors was produced with newly designed blades and has become one of hit products in 2012[8]. The scissors use a logarithmic spiral for their blade. The features of the scissors are as follows:

1) From the start and end cutting positions, the cutting angle of the blades can be kept constant at 30 degrees since they use the newly developed blade of a Bernoulli curve shape.
2) Thanks to the constant cutting angle, the user's force necessary to cut is reduced to $1 / 3$ of that for conventional products.

The Bernoulli curve is the logarithmic spiral and can be classified as a type of the log-aesthetic curves [2,4,5,6,7]. One of its properties is that the angle between its tangent vector and the radial axis from the origin is kept constant. This is the reason why the logarithmic spiral is also called the equiangular spiral. Consequently if the blades of a pair of scissors
are given by a logarithmic spiral, the cutting angle is always constant. Figure 1. shows the cutting and opening angles of the scissors.

In this paper, we extend the logarithmic spiral to make the cutting angle of a pair of scissors a linear function of the rotation angle of the cutting edges and newly define the polar-aesthetic curve and discuss about conditions for the monotonicity of its curvature. Furthermore we analyze cutting torques of the scissors and show that the torque of the scissors whose blade is a polar-aesthetic curve can be controllable to make the users feel easier to cut a sheet of paper.

## 2. CONSTANT CUTTING ANGLE

In this section, we derive conditions for the blades of a pair of scissors to have a constant cutting angle. We define a curve by using the polar coordinate system and use $s$ for its arc length. A point on the curve $P(S)$ is given by $(r(s) \cos \theta(S), r(s) \sin \theta(s))$ where $\theta(s)$ is an azimuth angle and the direction angle there is denoted by $\phi$. The tangent vector of the curve $t(s)$ is (c) 2015 CAD Solutions, LLC, http://www.cadanda.com


Fig. 1: Blades of scissors.
given by

$$
\begin{align*}
\boldsymbol{t}(s) & =\frac{d}{d s}(r \cos \theta, r \sin \theta) \\
& =\left(\frac{d r}{d s} \cos \theta-r \sin \theta \frac{d \theta}{d s}, \frac{d r}{d s} \sin \theta+r \cos \theta \frac{d \theta}{d s}\right) \tag{2.1}
\end{align*}
$$

Hence

$$
\begin{equation*}
\tan \phi=\frac{\frac{d r}{d s} \sin \theta+r \cos \theta \frac{d \theta}{d s}}{\frac{d r}{d s} \cos \theta-r \sin \theta \frac{d \theta}{d s}}=\frac{\frac{d r}{d s} \tan \theta+r \frac{d \theta}{d s}}{\frac{d r}{d s}-r \tan \theta \frac{d \theta}{d s}} \tag{2.2}
\end{equation*}
$$

We define $c=\phi-\theta$, then $\phi=\theta-\mathcal{c}$ and

$$
\begin{equation*}
\tan \phi=\frac{\tan \theta+\tan c}{1-\tan \theta \tan c} \tag{2.3}
\end{equation*}
$$

By rewriting Eqn. (2), we obtain

$$
\begin{equation*}
\tan \phi=\frac{\tan \theta+\frac{r \frac{d \theta}{d s}}{\frac{d r}{d s}}}{1-\tan \theta \frac{r \frac{d \theta}{d s}}{\frac{d r}{d s}}} \tag{2.4}
\end{equation*}
$$

Therefore, $\tan c=r \frac{d \theta}{d s} / \frac{d r}{d s}$. Hence if the ratio of $r d \theta / d s$ to $d r / d s$ is constant, $\tan c$ is constant, i.e. $c$ is constant. Then the angle formed by the straight line from the origin to the point $\mathbf{P}(s)$ and the tangent vector will be constant.

The factor of proportionality between $r d \theta / d s$ and $d r / d s$ is assumed to be $k$, so $d r / d s=k r d \theta / d s$. We solve this equation by separation of variables and obtain $r=C \exp (k \theta)$ where $C=\exp c_{0}$.

A logarithmic (equiangular) spiral can be expressed using the imaginary unit $i$ as follows:

$$
\begin{equation*}
C(t)=r(t) \exp (i \theta(t))=C \exp (a t) \exp (i b t) \tag{2.5}
\end{equation*}
$$

Hence $r(t)=C \exp (a t)$ and $\theta(t)=b t$. So $r=C \exp \left(\frac{a}{b} \theta\right)$ and $c$ becomes constant

## 3. POLAR-AESTHETIC CURVE

Based on the discussions above, we define the polaraesthetic curve in this section. A curve is assumed to be given by $\mathbf{C}(t)=r(t) \exp (i \theta(t))$. The angle difference between the direction angle $\phi$ and the azimuth angle $\theta$ is defined as $\theta_{0}=\phi-\theta$. Then $\tan \theta_{0}=r \frac{d \theta}{d s} / \frac{d r}{d s}$. We regard $\theta_{0}$ to be a function of $\theta$. The differential equation which the curve should satisfies is given by

$$
\begin{equation*}
\frac{d r}{d \theta}=\frac{r}{\tan \theta_{0}} \tag{3.1}
\end{equation*}
$$

We call a curve which satisfies the above equation the polar-aesthetic curve. The reasons why we call it an aesthetic curve are 1) its curvature varies monotonically with simple conditions as explained in Section 4, and 2 ) it is derived from the logarithmic spiral, which is one of the typical aesthetic curves. Similar to the log-aesthetic curve [4,5], we would like to propose a new type of aesthetic curves and we hope we can use it for aesthetic design. But in this paper we concentrate our discussions on scissors design because its primary usage will be for blade design.

### 3.1. In Case where $\boldsymbol{\theta}_{0}$ is a Linear Function of $\boldsymbol{\theta}$

Here we assume that $\theta_{0}$ is a linear function and $\theta_{0}=$ $a \theta+b$ for some constants $a(\neq 0)$ and $b$. The constant $a$ is assumed not to be equal to 0 because if $a=0$, the curve will be a logarithmic spiral. In this case Eqn. (3.1) is rewritten as follows:

$$
\begin{equation*}
\frac{d r}{r}=\frac{d \theta}{\tan (a \theta+b)} \tag{3.2}
\end{equation*}
$$

Hence

$$
\begin{align*}
\log r & =\frac{\log |\sin (a \theta+b)|+c_{2}}{a} \\
r^{a} & =C^{\prime}|\sin (a \theta+b)| \tag{3.3}
\end{align*}
$$

where $C^{\prime}=\exp c_{2}$. Therefore

$$
\begin{equation*}
r=C|\sin (a \theta+b)|^{\frac{1}{a}} \tag{3.4}
\end{equation*}
$$

where $C=C^{\prime \frac{1}{a}}$.

### 3.2. In Case where $\boldsymbol{\theta}_{\boldsymbol{0}}$ is an Arbitrary Function of $\boldsymbol{\theta}$

 We assume that $\theta_{0}$ is a linear function and $\theta_{0}=f(\theta)$. Then Eqn. (3.1) is given by$$
\begin{equation*}
\frac{d r}{d \theta}=\frac{r}{\tan f(\theta)} \tag{3.5}
\end{equation*}
$$

Hence

$$
\begin{equation*}
\log r=\int \frac{d \theta}{\tan f(\theta)} \tag{3.6}
\end{equation*}
$$



Fig. 2: top: logarithmic spiral $\left(a_{0}=6, b_{0}=-10\right)$, bottom: polar-aesthetic curve $\left(a_{1}=0.2, b_{1}=\pi / 24\right)$.

Here we put $u=\sin f(\theta)$, then $\sin ^{-1} u=f(\theta)$, i.e. $\theta=$ $f^{-1}\left(\sin ^{-1} u\right)$ and $d u=\cos (f(\theta)) \frac{d f(\theta)}{d \theta} d \theta$. Then

$$
\begin{equation*}
\cos (f(\theta)) d \theta=\frac{d u}{\frac{d f(\theta)}{d \theta}} \tag{3.7}
\end{equation*}
$$

Therefore Eqn. (3.6) is rewritten as follows:

$$
\begin{equation*}
\log r=\int \frac{\cos f(\theta)}{\sin f(\theta)} d \theta=\int \frac{d u}{u \frac{d f(\theta)}{d \theta}} \tag{3.8}
\end{equation*}
$$

Hence

$$
\begin{equation*}
r=C \exp \left(\int \frac{d u}{u \frac{d f(\theta)}{d \theta}}\right)=C \exp \left(\int \frac{d u}{u^{\frac{d f\left(f^{-1}\left(\sin ^{-1} u\right)\right)}{d \theta}}}\right) \tag{3.9}
\end{equation*}
$$

where $u$ is determined by the constant of integration.

## 4. MONONICITY OF CURVATURE

In this section we discuss monotonicity of curvature of the polar-aesthetic curve whose $\theta_{0}$ is given by a linear function of $\theta$. The tangent vector of the curve $C(t)=r(t) \exp (i \theta(t))$ is given by

$$
\begin{align*}
\boldsymbol{t}(t) & =\frac{d r}{d t} \exp (i \theta)+i r \frac{d \theta}{d t} \exp (i \theta) \\
& =\frac{d r}{d t} \cos \theta-r \frac{d \theta}{d t} \sin \theta+i\left(\frac{d r}{d t} \sin \theta+r \frac{d \theta}{d t} \cos \theta\right) \tag{4.1}
\end{align*}
$$

The curvature $\kappa(t)$ is given by

$$
\begin{equation*}
\kappa(t)=\frac{2 \dot{r}^{2} \dot{\theta}+r \dot{r} \ddot{\theta}-r \ddot{r} \dot{\theta}+r^{2} \dot{\theta}^{3}}{\left(\dot{r}^{2}+r^{2}\right)^{\frac{3}{2}}} \tag{4.2}
\end{equation*}
$$

where for example, $\dot{r}$ denotes the derivative of $r$ with respect to $t$. When $\theta=t$, we obtain

$$
\begin{equation*}
\kappa(t)=\frac{2 \dot{r}^{2}-r \ddot{r}+r^{2}}{\left(\dot{r}^{2}+r^{2}\right)^{\frac{3}{2}}} \tag{4.3}
\end{equation*}
$$

Hence if $2(1-a) / a>0$, i.e. $0<a<1$, then when $\sin (a t+b)$ monotonically increases, the curvature decreases monotonically. If $a<0$ or $a>1$, when
$\sin (a t+b)$ monotonically increases, the curvature also increases monotonically. When $a=1$,

$$
\begin{align*}
C(t) & =\sin (\theta+b)(\cos \theta, \sin \theta) \\
& =\frac{1}{2}(\sin (2 \theta+b)+\sin b,-\cos (2 \theta+b)+\cos b) \tag{4.4}
\end{align*}
$$

then the curve is a circular arc whose center is at $((\sin b) / 2,(\cos b) / 2)$ and radius is equal to $1 / 2$.

The left of Figure 2 shows a logarithmic spiral $C_{0}(t)=\exp \left(a_{0} t+b_{0}\right) \exp (i t)$ in green color and its mirror image about the $x$-axis in blue. Its right shows a polar-aesthetic curve $C_{1}(t)=\left|\sin \left(a_{1} t+b_{1}\right)\right|^{\frac{1}{a_{1}}} \exp ($ it $)$ in green and its mirror image about the $x$-axis in blue. The tangent vectors at their intersection points are depicted in these figures. We notice that for the logarithmic spiral its cutting angle is constant, but the cutting angle of the polar-aesthetic curve changes gradually.

## 5. FORCE AND TORQUE APPLIED TO A PAIR OF SCISSORS

In this section, we discuss force and torque applied to a pair of scissors based on Mahvash et al [3].

### 5.1. Contact Force

Figure 3 shows a pair of scissors, its coordinate systems with a thin plate. The origin of the coordinate system is located at the pivot and the symmetrical axis of the scissors is on the $x$-axis. We assume that both the scissors and the plate do not move during the cutting process. The opening angle of the pair of scissors is denoted by $\theta$ and the position of a crack generated by it is denoted by $x_{c}$.

The blades locally deform the plate around a crack. This deformation is composed of bending, stretching, compression, or their combination. During deformation, the upper end of the crack is displaced from $\left(x_{\mathcal{C}}, h / 2\right)$ to ( $x_{\mathcal{C}}, h / 2-\delta$ ), where $\delta$ is a displacement length (see Fig. 1.). In response to deformation of the plate, the force is $f_{n}$ applied to the upper blade along the normal to the blade's edge at point $\left(x_{C}, h / 2-\delta\right) . f_{n}$
(a)

(b)
$t+d t$


Fig. 3: Coordinates of a pair of scissors [3].
is given by

$$
\begin{equation*}
f_{n}=g(\delta) \tag{5.1}
\end{equation*}
$$

where $g(\delta)$ is a linear function of the tip displacement, obtained by measurement or material properties.

When the blades of the scissors can be assumed to be a straight line and $\alpha$ is the angle between the blade's edge and the centerline of a blade as shown in Fig. 3., the torque caused by $f_{n}$ at the pivot is given by

$$
\begin{equation*}
\tau=x_{c} f_{n} \cos (\alpha) \tag{5.2}
\end{equation*}
$$

where $\alpha$ is the angle between the blade's edge and the centerline of the blade. Since scissors' blades are slightly tapered as shown in Fig. 3., $\alpha$ is usually not zero.

The force which the user feels at the handle is calculated by

$$
\begin{equation*}
f_{u}=\frac{\tau}{R}=\frac{x_{C}}{R} f_{n} \cos (\alpha) \tag{5.3}
\end{equation*}
$$

where $R$ is the distance between the pivot and the handle.

### 5.2. Blade Edge Curve

We define the curve of the edge of the upper blade in the coordinate system of the scissors as

$$
\begin{equation*}
y=\phi(x, \theta) \tag{5.4}
\end{equation*}
$$

Where $(x, y)$ is a point on the edge of the blade and $\phi(x, \theta)$ is a nonlinear function of $x$ and $\theta$. From the above equation, the displacement $\delta$ caused by a blade with curve is obtained by

$$
\begin{align*}
\frac{h}{2}-\delta & =\phi\left(\chi_{C}, \theta\right) \\
\delta & =\frac{h}{2}-\phi\left(\chi_{\mathcal{C}}, \theta\right) \tag{5.5}
\end{align*}
$$

### 5.3. Cutting Force by Sharp Blade

Figure 3 shows two sequential time steps at $t$ and $t+$ $d t$ of a scissor cutting process. During the time interval $d t$, the opening angle of the scissors is changed
from $\theta$ to $\theta+d \theta$, and the upper end position of the crack is moved from $x_{c}$ to $x_{c}+d x_{c}$. The area of crack extension is given by $d A=h d x_{c}$.

An energy-based approach for fracture mechanics is adopted to estimate the torque and the upper end position of the crack during cutting. If we consider the principle of conservation of energy, we obtain

$$
\begin{equation*}
d W_{e}=d W_{A}+d U \tag{5.6}
\end{equation*}
$$

where $d W_{e}$ is the external work applied by the scissors, $d W_{A}$ is the irreversible fracture work, and $d U$ is the change in elastic potential energy stored in the plate.

If quasi-static operation is assumed and we ignore inertia terms, the external work $d W_{e}$ can be calculated by

$$
\begin{equation*}
d W_{e}=-\tau d \theta \tag{5.7}
\end{equation*}
$$

If the blade is very sharp, it does not cause cutting burrs and the work of fracture for separating the area is given by [1]

$$
\begin{equation*}
d W_{A}=J_{C} d A=J_{C} h d x_{C} \tag{5.8}
\end{equation*}
$$

where $J_{c}$ is the fracture toughness. Substituting Eqns. (5.7) and (5.8) into Eqn. (5.6) yields

$$
\begin{equation*}
-\tau d \theta=J_{c} h d x_{C}+d U \tag{5.9}
\end{equation*}
$$

Then

$$
\begin{equation*}
\tau=-J_{c} h \frac{d x_{c}}{d \theta}-\frac{d U}{d \theta} \tag{5.10}
\end{equation*}
$$

If the deformation pattern around the upper end of the crack does not significantly change when its upper end is displaced, the change of the potential elastic energy stored in a plate during sharp cutting can be ignored, so we can assume that $d U / d \theta=0$. This and

Eqn. (5.10) yield

$$
\begin{equation*}
\tau_{\mathcal{C}}=-J_{c} h \frac{d x_{c}}{d \theta} \tag{5.11}
\end{equation*}
$$

where $\tau_{c}$ is the torque applied to the scissors during sharp cutting. Differentiating Eqn. (5.5)

$$
\begin{equation*}
\frac{d \delta}{d \theta}=\frac{1}{2} \frac{d h}{d \theta}-\frac{d \phi\left(x_{c}, \theta\right)}{d \theta}=0-\frac{\partial \phi}{\partial \theta}-\frac{\partial \phi}{\partial x_{c}} \frac{d x_{c}}{d \theta} \tag{5.12}
\end{equation*}
$$

If we ignore the change of displacement $\delta$ during cutting and assume $d \delta / d \theta=0$, then

$$
\begin{equation*}
\frac{d x_{c}}{d \theta}=-\frac{\partial \phi / \partial \theta}{\partial \phi / \partial x_{c}} \tag{5.13}
\end{equation*}
$$

By substituting the above equation into Eqn. (5.10) we obtain

$$
\begin{equation*}
\tau_{C}=J_{c} h \frac{\partial \phi / \partial \theta}{\partial \phi / \partial x_{c}} \tag{5.14}
\end{equation*}
$$

In case where the edge curve of the blade is given by a parametric curve, i.e. there is a parameter $u$ such that

$$
\begin{equation*}
(x, y)=(x(u, \theta), y(u, \theta)) \tag{5.15}
\end{equation*}
$$

at the upper end of the crack $\left(x_{c}, h / 2\right), \mathrm{x}\left(u_{c}, \theta\right)=x_{c}$ and $\mathrm{y}\left(u_{c}, \theta\right)=h / 2$ are satisfied. By solving $\mathrm{y}\left(u_{c}, \theta\right)=$ $h / 2, u_{c}$ is determined and we can obtain $x_{c}=x\left(u_{c}, \theta\right)$. If the plate is very thin like paper, we can assume that $h=0$. By solving $y\left(u_{c}, \theta\right)=0, u_{c}$ is determined and we can obtain $x_{C}=x\left(u_{C}, \theta\right)$.

### 5.4. Logarithmic Spiral

Here we use a logarithmic spiral to represent the upper blade. For comparison to the polar-aesthetic curve, we define the curve using parameter $u$ as follows:

$$
\begin{equation*}
\boldsymbol{C}(t)=\exp (a u+b) \exp (i u) \tag{5.16}
\end{equation*}
$$

where $a$ and $b$ are constants. Note that $\exp (a u+b)=$ $\exp (b) \exp (a u)$. We can regard $\exp (b)$ as a scaling factor for the whole curve.

In order to take the rotation about the scissors' pivot into consideration, we assume that the curve is at the initial position when $t=0$ and the direction angle of the blade at the initial position is $\theta_{s}$. If the clock-wise rotation angle according to the elapsed time is $\theta_{r}$, then the relationship of the direction angle $\theta$ to $\theta_{r}$ is given by

$$
\begin{equation*}
\theta=\theta_{s}-\theta_{r} \tag{5.17}
\end{equation*}
$$

The direction angle $\theta$ is a half of the opening angle in Fig. 1. The upper blade after rotation is given by

$$
\begin{equation*}
C^{\prime}(u)=C(u) \exp \left(-i \theta_{r}\right)=\exp (a u+b) \exp \left(i\left(u-\theta_{r}\right)\right. \tag{5.18}
\end{equation*}
$$

We assume that the plate is very thin like paper and ignore thickness $h$. Then $y_{c}=0$ and from $\sin \left(u-\theta_{r}\right)=$

0 , we obtain $u=\theta_{r}$. Hence

$$
\begin{equation*}
x_{c}=\exp (a u+b)=\exp \left(a \theta_{r}+b\right)=\exp \left(a\left(\theta_{s}-\theta\right)+b\right) \tag{5.19}
\end{equation*}
$$

From Eqn. (5.10),

$$
\begin{equation*}
\tau_{c}=-2 J_{c} h \frac{d x_{c}}{d \theta}=2 J_{c} h a \exp \left(a\left(\theta_{s}-\theta\right)+b\right) \tag{5.20}
\end{equation*}
$$

A factor 2 is multiplied to Eqn. (5.10) because the direction angle $\theta$ here is a half of the opening angle $\theta$ in Eqn. (5.10).

### 5.5. Polar-aesthetic Curve

Here we use a polar-aesthetic curve to represent the upper blade. We represent a polar-aesthetic curve using parameter $u$ as follows:

$$
\begin{equation*}
C(t)=C \sin ^{\frac{1}{a}}(a u+b) \exp (i u) \tag{5.21}
\end{equation*}
$$

where $a, b$ and $C$ are constants.
Similar to the logarithmic spiral case, the upper blade after rotation is given by

$$
\begin{equation*}
C^{\prime}(u)=C(u) \exp \left(-i \theta_{r}\right)=C \sin ^{\frac{1}{a}}(a u+b) \exp \left(i\left(u-\theta_{r}\right)\right. \tag{5.22}
\end{equation*}
$$

The plate is assumed to be very thin like paper and we ignore thickness $h$. Then $y_{c}=0$ and from $\sin \left(u-\theta_{r}\right)=$ 0 , we obtain $u=\theta_{r}$. Hence

$$
\begin{equation*}
x_{c}=C \sin ^{\frac{1}{a}}(a u+b)=C \sin ^{\frac{1}{a}}\left(a\left(\theta_{s}-\theta\right)+b\right) \tag{5.23}
\end{equation*}
$$

From Eqn. (5.10),

$$
\begin{align*}
\tau_{C}= & -2 J_{c} h \frac{d x_{c}}{d \theta}=2 J_{c} h C \sin ^{\frac{1}{a}-1}\left(a\left(\theta_{s}-\theta\right)+b\right) \\
& \times \cos \left(a\left(\theta_{s}-\theta\right)+b\right) \tag{5.24}
\end{align*}
$$

### 5.6. Torque Calculation Examples

In this subsection we show several examples of torque calculation.

### 5.6.1. Logarithmic spiral

We show several torque calculation examples for logarithmic spirals in Fig. 3. Since the purpose of this subsection is to illustrate how the torque changes according to rotation angle $\theta$, the parameters of the plate, material toughness $J_{c}$ and thickness $h$ are assumed to be simple values, i.e. $J_{c}=h=1$.

Figure 4(a) shows a change of torque with respect to $\theta$ from 0 to $\pi / 2$ for the logarithmic spiral whose $(a, b)=\left(1 / \tan \left(\frac{\pi}{6}\right), 1\right) \approx(1.732,1) . \theta$ is the angle between the centerline of the upper blade and the $x$-axis and the angle interval used for cutting is from $\pi / 12$ to 0 . Since the position of the handle is fixed, the force felt by the user is proportional to the torque. Figures 4(b) and


Fig. 4: Torque of various logarithmic spiral scissors.


Fig. 5: Torque of various polar-aesthetic curve scissors.

4(c) show the torques for the logarithmic spirals whose $(a, b)=\left(1 / \tan \left(\frac{\pi}{12}\right), 1\right) \approx(3.732,1)$ and $(a, b)=$ $\left(1 / \tan \left(\frac{\pi}{24}\right), 1\right) \approx(7.593,1)$, respectively. For the scissors with logarithmic spiral blades, the torque is increasing rapidly as $\theta$ gets closer to 0 .

### 5.6.2. Polar-aesthetic curve

We show several torque calculation examples for polar-aesthetic curves in Fig. 4. The parameters of the plate, material toughness $J_{C}$ and thickness $h$ are also assumed that $J_{C}=h=1$.

Figure 5(a) shows a change of torque with respect to $\theta$ from 0 to $\pi / 2$ for the logarithmic spiral whose $(a, b, C)=\left(0.2, \frac{\pi}{24}, 1\right) \approx(0.2,0.1309,1)$. Figures 5(b) and 5(c) show the torques for the logarithmic spirals whose $(a, b, C)=\left(0.3, \frac{\pi}{24}, 1\right) \approx(0.3,0.1309,1)$ and $(a, b, C)=\left(0.45, \frac{\pi}{24}, 1\right) \approx(0.2,0.1309,1)$ respectively. For the scissors with polar-aesthetic curve blades, the torque can be controlled by selecting appropriate parameters to avoid rapid increases as $\theta$ approaches to 0 and we can obtain flatter change of torque than that of logarithmic spiral. That means the torque will be independent of the opening angle of the scissors and we will be able to cut a sheet of paper with almost a constant force.

## 6. DESIGN OF SCISSORS

In this section at first we analyze the shape of actual product scissors using a logarithmic spiral for their blade shape. Then we show a design example with a polar-aesthetic curve as well as that with a straight line.

### 6.1. Fitcut Curve Scissors: SC-175S

The scissors SC-175S produced by Plus Corp. in Japan is sold under the name of Fitcut Curve Scissors. The scissors use a logarithmic spiral for their blade. The specifications of the scissors are as follows:

- Length of the centerline of the blade $=74 \mathrm{~mm}$
- Opening angle of the configuration when the cutting starts $=40$ degrees
- Position of the configuration when the cutting starts $=10 \mathrm{~mm}$
- Cutting angle (angle between the two blades) = 19 degrees

In our measurement the cutting angle is 19 degrees instead of 30 degrees [8] and the shape of the blade becomes almost identical if we adopt 19 degrees to determine its shape. The shape and torque of the scissors with the above specifications are shown in Fig. 6.

### 6.2. Scissors with a Polar-Aesthetic Curve

We design a pair of scissors using a PA curve for their blade shape. Its specifications are as follows:

- length of the centerline of the blade $=74 \mathrm{~mm}$
- Opening angle of the configuration when the cutting starts $=40$ degrees
- Position of the configuration when the cutting starts $=10 \mathrm{~mm}$
- Cutting angle $=$ linearly changes from 12 degrees (opening angle $=40$ degrees) to 30 degrees (opening angle $=0$ degrees)


Fig. 6: Commercial Scissors, SC-175S: (a) its shape and (b) torque.


Fig. 7: Designed Scissors with PA curve: (a) its shape and (b) torque.


Fig. 8: Designed Scissors with straight curve: (a) its shape and (b) torque.

The shape and torque of the scissors with the above specifications are shown in Fig. 7.

### 6.3. Scissors with a Straight Line

To evaluate effects of the blade shape, we design a pair of scissors using a straight line instead of a smoothly bending curve. Its specifications are as follows:

- length of the centerline of the blade $=74 \mathrm{~mm}$
- Opening angle of the configuration when the cutting starts $=40$ degrees
- Position of the configuration when the cutting starts $=10 \mathrm{~mm}$
- The centerline intersects the blade at its end.

The shape and torque of the scissors with the above specifications are shown in Fig. 8.

Although the torque of SC-175 rapidly increases as the opening angle approaches to 0 , that of the scissors with a PA curve is almost linearly decreases. That of the scissors with a straight line seems to increase exponentially as the opening angle approaches to 0 . We can expect that we will be able to cut a sheet of paper and other slabs of material easily with scissors with a PA curve.

## 7. CONCLUSIONS

In this paper, we have proposed a new aesthetic curve named the polar-aesthetic curve as another type of aesthetic curves in addition to the log-aesthetic curve.

We use "aesthetic" for its name because 1) it is derived from the logarithmic spiral, which is a typical aesthetic curve and 2) we can guarantee the monotonicity of curvature under simple conditions. The curve is basically fair and it can be used to be a shape of the blade of scissors to control the torque to cut a slab of material. For future work, we will try to extend it into 3 dimensional space.

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