Automatic Generation of Finite Element Meshes on Poorly Parameterized Surfaces

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ABSTRACT

The NURBS surfaces are widely employed for exchanging geometric models between different CAD/CAE systems. However if the input surfaces are poorly parameterized, most surface meshing algorithms may fail or the constructed meshes can be ill-conditioned. In this paper presents a new method is presented that can generate well conditioned meshes even on poorly parameterized surfaces by regenerating NURBS surfaces. To begin with, adequate points are sampled on original poorly parameterized surfaces and new surfaces are created by interpolating these points. And then, mesh generation is performed on new surfaces. With this method, models with poorly parameterized surfaces can be meshed successfully.

Keywords: Mesh Generation, Poorly Parametrized Surfaces, Interpolation, Regeneration .

1. INTRODUCTION

The NURBS surfaces are widely employed to represent 3D geometry when geometric models are constructed by using IGES format. However in practice mesh generation on NURBS surfaces sometimes fails or produces ill-conditioned elements. This happens when the parameters that define parametric planes of NURBS surfaces are not uniform and poorly parameterized. This phenomenon occurs when the surfaces are modified several times before completion in CAD system.

In such case new surfaces that have same geometry and well parameterized should be constructed. There are many researches performed so far to produce smooth surfaces and most of the works on surface generation employ the existing point data set to construct surfaces[1-4]. In the meanwhile Razdan[5] proposed surface generation method that construct new surfaces with similar geometry to the given surfaces and the parameters of parametric plane are well conditioned. But in his work four boundary curves are newly generated and too many sampling points are needed to satisfy certain conditions.

Therefore lots of computation time is required in this method, which can be a drawback in practical application.

In this paper in order to solve this problem a mesh generation method based on surface regeneration has been proposed, which employs interpolation to construct geometrically similar surfaces to the original surfaces and meshes are generated on new surfaces.

2. IMPROPER SURFACES FOR MESH GENERATION

Since NURBS surfaces are determined by knot values, control points and weights as well as basis functions, the distances between uniform parameters in parametric planes do not usually correspond to uniform distances in real coordinates. Fig.1 shows an example of a non-uniformly parameterized line. As shown in Fig.1, the real x-coordinates corresponding to uniform parametric u-coordinates are not uniformly distributed.

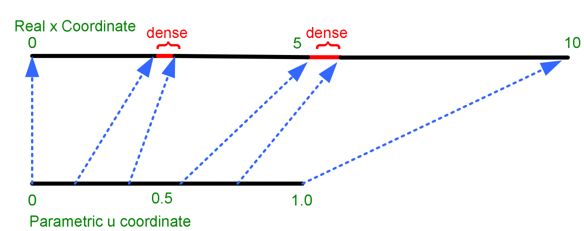
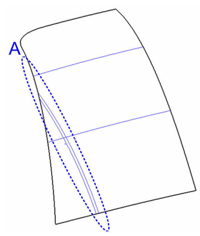
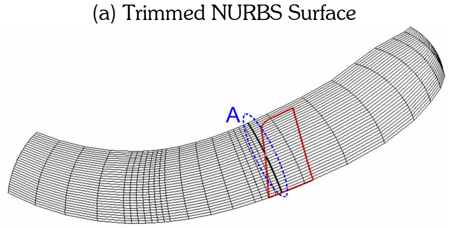


Fig 1. Poorly parameterized line

Even when there are considerable differences in terms of distances between parameters both in parametric planes and corresponding 3D surfaces, the NURBS surfaces still can represent the given geometry without any problem. Razdan[6] and Farin[7] called this type of surface as "poorly parameterized surface". This could be a serious problem when an indirect surface mesh generation is involved. In indirect mesh generation approach, 3D surfaces are transformed into some forms of 2D parametric planes and meshes are constructed in this planes. Finally the constructed 2D meshes are transformed back to original 3D surfaces[8]. Therefore the relationship between parameters in parametric planes and the corresponding real coordinates plays an important role in the quality of meshes to be generated. Fig.2 shows an example of poorly parameterized trimmed NURBS surface and its basis surface.



(a) Trimmed NURBS Surface



(b) Basis NURBS surface

Fig 2. Poorly parameterized surface

The grid lines on basis NURBS surface in Fig 2 correspond to uniformly distributed parameters in parametric planes. As shown in Fig.2 the distances between parameters both in parametric planes and in real 3D surfaces are severely distorted especially in dotted area A. Fig.3 shows meshing result on this surface by using a domain decomposition method with indirect approach. As shown in Fig.3, the constructed meshes are very bad such that severely distorted elements are generated in several areas and many ill-conditioned elements are generated throughout the surface.

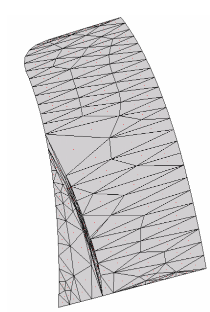


Fig 3. Meshing result on poorly parameterized surface

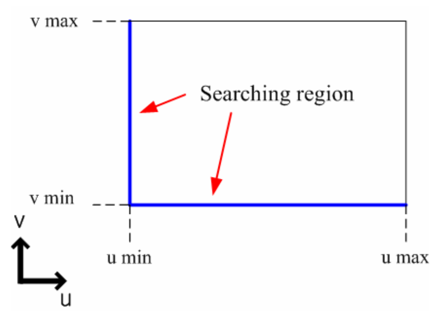
3. REGENERATION OF SURFACE

We propose to regenerate new bicubic B-spline basis surface and trim the newly generated basis surface to overcome the problems described above. Points are sampled from the given NURBS surface and interpolated to regenerate basis surface. The procedure can be described as four steps.

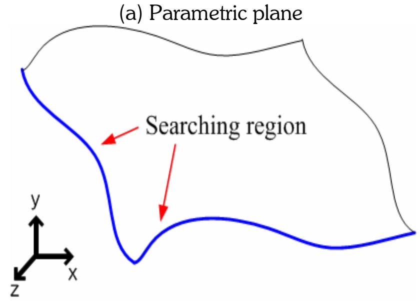
a) Check the given surface for irregular parameterization.
b) Sample points from the given surface.
c) Generate basis surface by interpolating sample points.
d) Generate parameter space curve for trimming boundary.

3.1 Surface Check

It is necessary to check whether the surface given through IGES format is appropriate for mesh generation. The criterion of appropriateness for mesh generation is based on the ratio between distances in parametric space and surface model space. If the ratio goes beyond the preset criterion, the surface is classified as not suitable for meshing. The check procedure is elaborated in detail.



(a) Parametric plane



(b) 3D surface

Fig 4. Searching region on surface

First, (u, v_{min}) and (u_{min}, v) region is selected on the parametric space of the given surface. Selected parametric region is divided into 50 equal intervals. Then corresponding distances in 3D surface domain are computed and saved. In general these 50 distances in surface domain are not uniform. For each divided interval, following test is performed to decide the poorness of the surface for meshing.

In the following test, k is decided through experience. For the test in v direction, only the test region is changed from u direction to v direction.

$$\begin{aligned}&\textit{if } L_i > kC \text{ or } L_i < \frac{C}{k}\\&\quad \textit{then} \text{ surface is poorly parameterized surface}\\&\quad \textit{where } C = (\text{boundary curve length of } v{=}v_{\min})/50,\\&\qquad L_i:\ \text{length on surface between } S(u_i, v_{\min}) \text{ and } S(u_{i-1}, v_{\min})\\&\qquad i = 0,\ldots,50,\ k = 4.5\end{aligned} \quad (1)$$

3.2 Point Sampling for Surface Interpolation

The number of sampling points for interpolation must be decided before points are sampled from the given surface. Points can be sampled on the given surface after the number of points is decided. The interpolated surface will be closer to the original surface if the number of interpolating points is larger. However, there are practical barriers in increasing the number of interpolating points.

First of all, it will take considerable amount of time to interpolate too many points because surface interpolation is accomplished by solving linear system of equations, where the number of equations is proportional to the interpolating points. The second problem is that the number of control points for the new surface is increased in proportion to the number of interpolating points. This implies that the amount of data for each NURBS surface increases. In extreme case, the number of control points for interpolated surface may become thousands while the number of control points for the original surface is less than one hundred. Therefore, smaller number of points must be sampled to approximate the given surface. In this research, the number of sampling points in u and v direction is decided with the parametric continuity information in each direction.

When the basis surface has C^2 or higher than C^2 continuity in u or v direction, equation (2) is used to decide the number of sampling points[9].

$$n = (b-a)\left(\frac{\sup_{a\le t\le b}\left\|C''(t)\right\|}{8\varepsilon}\right)^{1/2} \quad (2)$$

Equation (2) is the maximum error allowed when curve segment, defined in the parameter range $a \le t \le b$, is approximated by straight line segments. The number of points necessary to approximate a curve into straight line segments can be decided with this criterion.

To apply equation (2) for a surface, it is necessary to fix parameter value in the direction that is not being considered for approximation. The equation (3) is a modified one for surface.

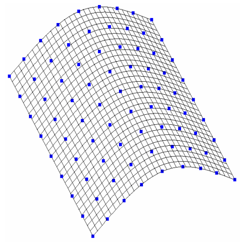
$$\begin{aligned}n &= \left|u_{\max} - u_{\min}\right|\left(\frac{\sup_{u_{\min}\le u\le u_{\max}}\left\|S_{uu}(u, v_{fixed})\right\|}{8\varepsilon_u}\right)^{1/2} \quad \textit{at } u \text{ direction}\\ n &= \left|v_{\max} - v_{\min}\right|\left(\frac{\sup_{v_{\min}\le v\le v_{\max}}\left\|S_{vv}(u_{fixed}, v)\right\|}{8\varepsilon_v}\right)^{1/2} \quad \textit{at } v \text{ direction}\end{aligned} \quad (3)$$

Evaluating the upper bound of second derivatives in Equation (3) is not an easy task. Since using too many sampling points for this evaluation is not appropriate in practice, only limited number of iterations are performed to estimate the maximum of second derivatives. In this paper in order to compute the maximum of second derivatives, the maximum number of bisection iteration at each knot interval was limited to 20. The value of in Equation (3) was set about 1/1000th of the length of an iso-curve. This evaluation method is applied to entire surface region and the results are computed. The final number of sampling points is obtained by averaging these numbers in each direction. Since we are dealing with the shapes of manufacturable physical parts, the maximum number of sampling points obtained was less than 200. The above approach cannot be used when the degree of basis function is 2 or less than 2. Equation (4) is used instead.

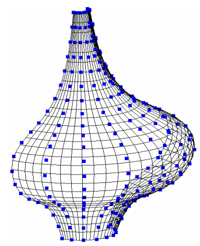
$$\begin{aligned}&(\text{\# of control points at } u \text{ basis function} - p)\times 3 \quad \textit{at } u \text{ direction}\\&(\text{\# of control points at } v \text{ basis function} - q)\times 3 \quad \textit{at } v \text{ direction}\\&\quad p:\ u \text{ direction degree},\ q:\ v \text{ direction degree}\end{aligned} \quad (4)$$

First part of the equation (4), i.e., (# of control points - p), is the number of knot vector intervals that are not repeated at minimum and maximum knot values. At least four points are required for piecewise cubic curve. Considering overlapping points, it was decided to multiply the number of intervals by three. In order to obtain the intervals of sampling points on surfaces as uniform as possible, the locations of sampling points are determined iteratively by using the metric map of iso-curve with one end fixed.

(m+1) and (n+1) points are sampled in u and v direction each after the number of sampling points are determined. These points are selected to have uniform distances in each direction. Fig. 5 shows selected sampling points by this approach.



(a) u direction degree=3, v direction degree=1



(b) u direction degree=4, v direction degree=3

Fig 5. Sampling points on surfaces

3.3 Surface Interpolation

Interpolated surface with the same set of interpolating points may vary depending on the degree of basis functions, knot vectors and end conditions[1-3]. We used bicubic B-spline interpolation that is easier to implement than other methods[7,10]. Surface interpolation is accomplished by determining knot vectors, and by solving simultaneous linear systems of equations to compute control points.

Chord length method was used to determine knot values for control points in our research. This method may yield undesirable result for unevenly spaced points. However, undesirable result is not expected in this case because interpolating points are selected with uniform distance in our approach.

Degree in each direction is set to three for bicubic and all the weights are set to 1.0 for convenience. Control points P are now computed with the equation (5)[7,10]. This concludes the regeneration of bicubic B-spline surface.

$$\begin{aligned} \mathrm{Q}_{k,l} &= S(\bar{u}_k,\bar{v}_l) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(\bar{u}_k)N_{j,q}(\bar{v}_l)\mathbf{P}_{i,j} \\ &= \sum_{i=0}^{n} N_{i,p}(\bar{u}_k)\left(\sum_{j=0}^{m} N_{j,q}(\bar{v}_l)\mathbf{P}_{i,j}\right) = \sum_{j=0}^{m} N_{i,p}\mathbf{R}_{i,l} \end{aligned} \quad (5)$$

where $\mathrm{Q}_{k,l}$: sampling points,

$k = 0,...,n \;\&\; l = 0,...,m$

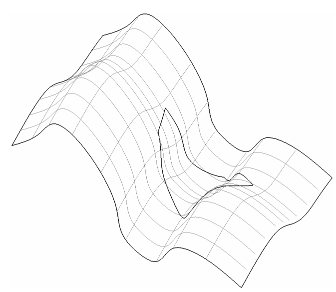
3.4 Regeneration of Parameter Space Curve

Trimmed NURBS surface is composed of NURBS basis surface and parameter space curve (PS curve) for boundary and model space curve (MS curve) for boundary. When NURBS basis surface is regenerated, original boundary PS curve cannot be used anymore. The new boundary PS curve must be generated with new basis surface and original MS curve information.

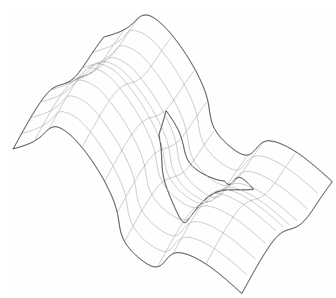
51 points are sampled along each MS curve and closest points on the new basis surface are computed. Corresponding (u, v) coordinates are computed and interpolated to form cubic NURBS curve on parametric space. PS curve obtained with 51 points along MS curve may not be geometrically accurate. However, the meshing algorithm employed in this paper is not sensitive to the accuracy of PS curve. Therefore, it is accurate enough for the use in mesh generation because meshing operation itself is a more serious approximation operation.

3.5 Interpolation Results

Fig.6 and Fig.7 show surface regeneration examples for given surfaces in IGES format. Grids inside the surface are the curves with uniform parametric distance in both directions. When original surface in Fig.6 does not have irregularity in parameter distribution, the interpolated surface is obtained as almost same with the original surface. For original surface with severe irregularity in parameter distribution in Fig.7, regenerated bicubic surface also shows uniformly spaced iso-parametric curves and same geometry with the original surface. From these examples it is seen that interpolated surfaces are well parameterized and almost same with the original surface.

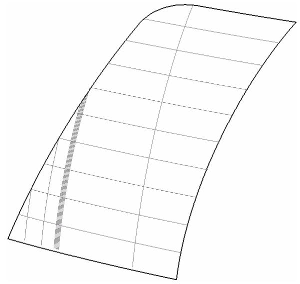


(a) Original surface

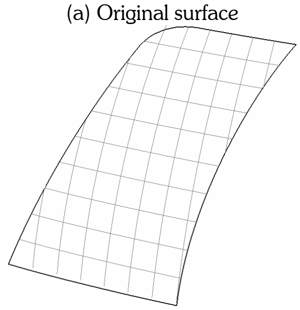


(b) Regenerated surface

Fig 6. Well parameterized surface interpolation result



(a) Original surface



(b) Regenerated surface

Fig 7. Poorly parameterized surface interpolation result

4. MESH GENERATION

Our mesh generation algorithm is based on an indirect meshing approach and a domain decomposition method. The brief procedures are as follows[8].

a) For a given surface, adequate transformation plane is decided. As for transformation planes, quasi-expanded planed planes or projection planes are employed.
b) Nodes are created along boundary curves on the surface.
c) Coordinates of boundary nodes are transformed to transformation plane.
d) Triangular or quadrilateral meshes are generated on transformation plane with a domain decomposition method. For the mesh generation with triangular or quadrilateral elements, a domain decomposition method is employed in which analysis domains are recursively subdivided into subdomains with the use of the best split lines as shown in Fig.8

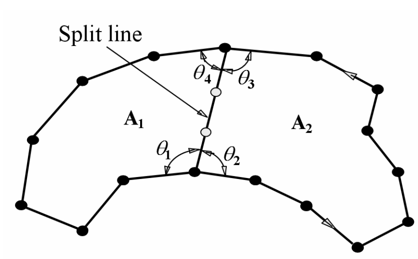


Fig 8. An example of a candidate split line

Among the candidate split lines that connect any two visible nodes in a loop and represent a possible location where the loop could be subdivided, the best split line is determined as in equation (6), which minimize π, a linear combination of four dimensionless parameters involving angles (α), areas (β), lengths (γ), and node placement errors (ε)[11,12].

$$\text{Minimize } \pi = C_1\alpha + C_2\beta + C_3\gamma + C_4\varepsilon \tag{6}$$

where C1, C2, C3, C4 are empirically determined constants. Here, α represents a split angle error, which ideally should be 60o for triangular elements or 90o for quadrilateral elements, and thus be defined as a measure of the deviation of actual split angles (θ_i , i=1, 4) from 60° or 90° as in equation (7).

$$\alpha = \frac{\sum_{i=1}^{4}\left|\theta_i - \frac{\pi}{2}\left(or \ \frac{\pi}{3}\right)\right|}{2\pi} \tag{7}$$

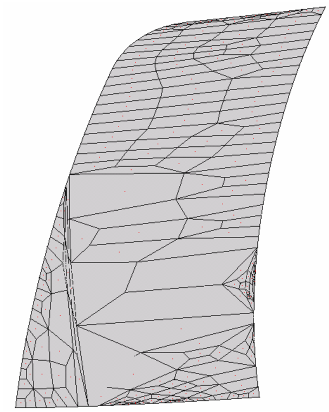
A split area error, β, is defined as a measure of the deviation of the actual areas of subdivided loops from the averaged area. A split line length error, γ, is defined as a nondimensionalized value of the split line length divided by a diagonal line length, a characteristic length of the loop, and thus making the split line length as short as possible. A node placement error, ε, represents a nondimensionalized error value between an ideal split line length and an actual split line length.

e) Meshes are transformed into coordinates in original surface domain.

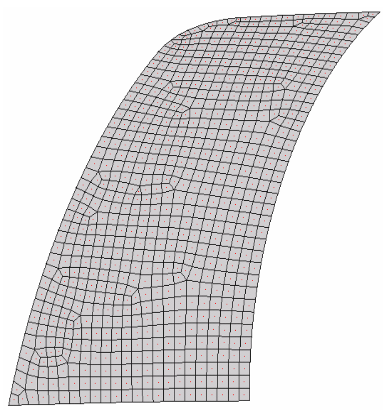
5. RESULTS AND DISCUSSION

The proposed algorithm was implemented with C++ on a PC-based Windows platform. The program reads in IGES format data file and generates two output files. One is IGES file for geometry information and the other is UNV file for meshing result.

Test results of proposed mesh generation approach are shown in Fig. 8 to 10. Left hand side figure of each example is a meshing result on improperly parameterized original surface. Three examples indicate that meshing results are severely distorted, or the algorithm failed. Right hand side figure of each example is a meshing result on regenerated surface that approximates given trimmed NURBS surface.

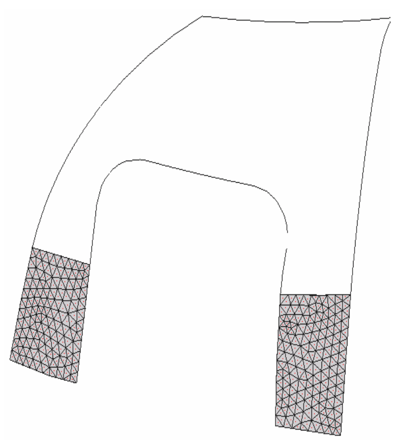


(a) on original surface

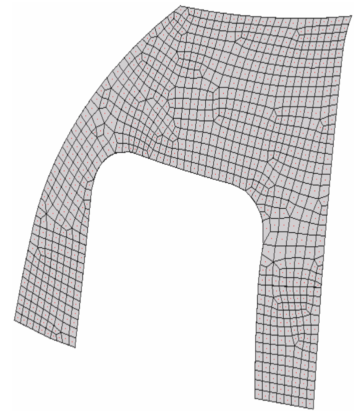


(b) on regenerated surface

Fig 8. Mesh generation example1

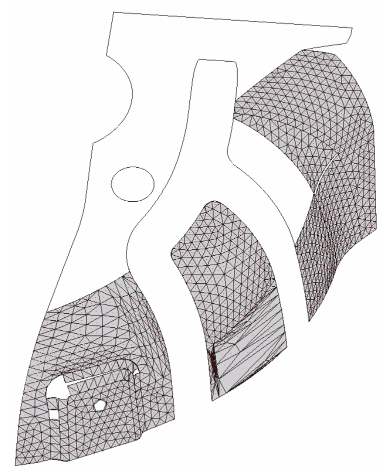


(a) on original surface

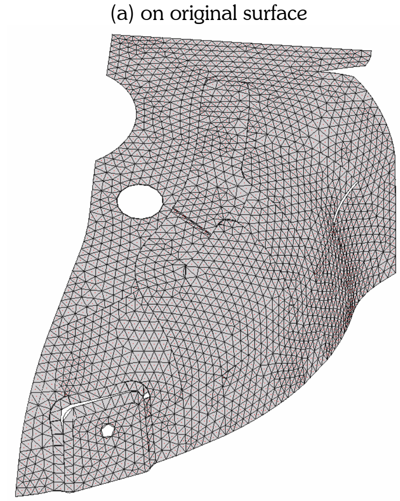


(b) on regenerated surface

Fig 9. Mesh generation example 2



(a) on original surface



(b) on regenerated surface

Fig 10. Mesh generation example 3

6. CONCLUSIONS

A novel approach for generating meshes on trimmed NURBS surface that has desirable geometric shapes, but has improper parameterization on basis surface has been proposed. This is done through generating a bicubic B-spline surface that approximates given basis surface model.

For a given trimmed NURBS surface, it is checked whether there are significant discrepancies between the distance ratios in parametric domain and those in real coordinate domain. If the surface shows large difference between parametric distance ratio and geometric distance ratio, then approximating surface is generated for efficient meshing. Firstly, sampling points are selected after number of sampling points is decided. Then a basis surface is generated with sampled points and parametric space curve is generated with the information from the new basis surface and boundary model space curve.

It is shown that the proposed approach allows us to easily generate meshes for the surfaces that have been difficult to mesh otherwise.

7. REFERENCES

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