Constraint Generation for Alternative Dimensional Specifications

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ABSTRACT

Dimension and tolerance specifications influence greatly the manufacturability of a part. However, initial design specifications usually reflect the functional point of view of dimensional specifications and result in a low manufacturability. To facilitate the required redesign process, a design rewriting system was proposed in an earlier paper to modify the shape of a design dimensional scheme. To complement the modification, to ensure the functionality of the design, and to reallocate tolerance values for the new derived dimensions, a constraint generation methodology is proposed here.

Keywords: manufacturability, tolerance, redesign, and constraint.

1. INTRODUCTION

Design dimensions and tolerances can be satisfied by a process plan in one of two possible ways: directly or indirectly. A dimension and its tolerance are achieved directly if a single machining operation exists that generates the final state of dimension using the design datum(s) of that dimension as manufacturing and location datums. For indirect machining, as opposed to a direct case, there is not a single cut, which can finish a dimension (tolerance) using one of the surfaces constraint by the design dimension as datum to manufacture the other. In the direct machining case, the tolerance stack is minimal as it consists of one operation only and the accuracy and precision of this cut are alone responsible for the compliance with tolerance constraints. Direct machining has the advantage of requiring the least possible process tolerances. Unfortunately, direct machining is often difficult to achieve as designs specify dimensions that cannot be machined, or only under excessive costs. Note that in the following, the term *dimension* encompasses geometric relationship of two entities, as well as the associated tolerance.

However, initial design specifications, including both size and geometric, are often unsuitable for efficient process plans (e.g. direct machining). This is in particular true if the designer has no process planning experiences, insufficient information on the machining environment or a complex design on hands. Process planners are consequently forced to:

- either to introduce tolerance stacks into the plan, which in turn require excessive high machining capabilities,
- to plan with frequent datum and fixture changes, causing high fixture and labour costs.
- or to request a redesign of the part – in particular, to re-dimension it.

The later is - not considering the logistic and work cost - the better option. These disadvantages can be reduced by the presented approach, as it semi-automatically re-dimensions the design. The approach aims at replacing sub-optimal dimensions by alternatives without impact the functionality of the design.

The general idea of alternative design specifications was already presented in [7], though with a limitation to size dimensions. This limitation was addressed in [8] and [12], where the authors outline a systematic procedure to rewrite both geometric and size dimensions.

The design rewriting system proposed in [12] allows to inspect possible alternative design dimensions for any dimension desired to be removed. However, these design rewriting rules handle only geometric dimensions excluding the associated tolerances (see section 2.2). This represents a major difficulty as determining tolerance values of involved GD&Ts is much more involved as compared to basic size tolerances. To address this issue, a mechanism consisting of two relatively independent modules is designed in which one proposes alternative tolerance chain by manipulating types of dimensions and another to determine appropriate tolerance. The later is discussed hereafter.

The whole approach is illustrated in fig. 1. Given the initial design and a set C of constraints on the machining environment (essentially best achievable tolerances for certain processes), the design is step-wise transformed towards a higher machinability. Two procedures take place in parallel: while design dimensions causing a low machinability are replaced one by one, constraints on tolerance values are added to C. These constraints are

such that, if they are fulfilled, the resulting design fulfils the original design intent.
This publication proposes a scheme for the generation of constraints on the tolerances, which is depicted in the right section of fig. 1. In this part of the algorithm, parallel to the use of design rewriting rules, constraints are automatically created to restrict the resulting tolerance values in order to maintain the initial design intent.

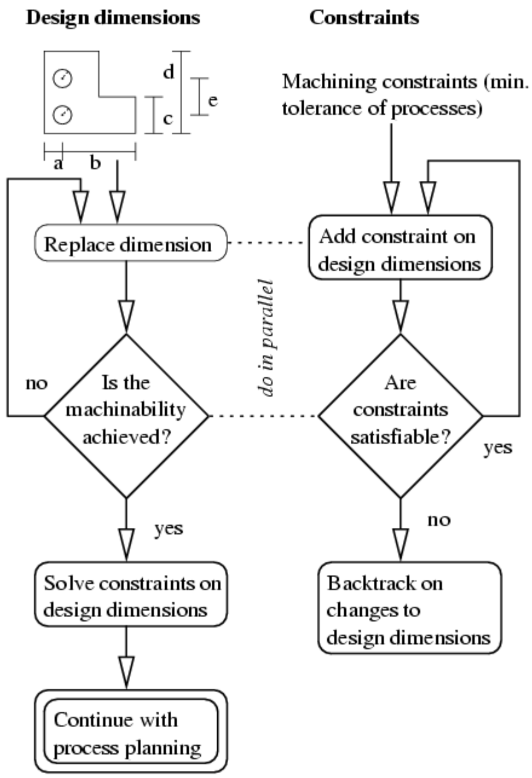

Fig. 1. An illustration of the approach

This publication is less concerned with the choice of alternative design specifications (i.e. which alternative is better), but on providing the tool to transform a design dimension graph into another without compromising the design intent. A possible target of such a transformation is a design permitting *direct machining* [14]. Furthermore, this works does not intend to re-specify tolerance values to new-derived dimensions, but to create constraints which have to be respected during re-specifications. The link between a design and direct machining is discussed in [11]. Note, that perpendicularity, parallelism, position, angularity and size tolerances are the main concern of this project, because other ANSI geometric tolerances are not expected to contribute to the decisions made during machining operation sequencing, set-up and fixture planning [6].

2. BACKGROUND

In this section, the *torsor model* of design tolerances and the *design rewriting system* used to modify the dimensioning scheme of a design are briefly introduced to provide necessary background knowledge.

2.1. Torsor model

Here, tosors are used to mathematically represent geometric tolerance zones and to accommodate the propagation of geometric tolerances. These tolerances are expressed with respect to a reference system (R_i) at the origin of this reference system (point O_i) as defined as [12]:

$$T_{a,b}|_{RiOi} = [w\ v]|_{RiOi} \quad (1)$$

Where $w = [w_x, w_y, w_z]^T$ is a small rotation vector (superscript T means vector transpose), and v the small displacement vector.
Two types of torsors are used [3]:

1. *Deviation torsors* (D-torsor) represent variations between nominal and actual features and
2. *Variation torsors* (V-torsor) denote relative variations between two or more real features of a part.

In order to allow the accumulation of several deviation or variation torsors (using matrix additions), they have to be expressed in a common reference system. To this means, equation (2) expresses the propagation of a torsor from its current reference frame to another reference frame [13]:

$$T_{Rj} = [M_{i,j} w_{Ri}\ \ M_{i,j}(v_{Ri} + w_{Ri} \times O_i O_j)\ ] \quad (2)$$

Where O_iO_j describes the translation of the origin of reference frame R_i to the origin of R_j (relative to the earlier frame's origin), and the matrix $M_{i,j}$ rotates R_i onto R_j.
Note, that this publication adopts the *degree of invariance* concept proposed in [3]. In this concept, elements of the variation and deviation torsors are undetermined as feature-specific rotations of translations leave the feature unchanged. These undetermined elements are denoted as U in the following. If, for example, a sphere is considered with the origin of its local coordinate is its center. As the rotation around the X-, Y- and Z-axis leave the sphere invariant, the D-torsor expressed at its center in its local reference system is

$$T_{sphere} = \begin{bmatrix} U & v_x \\ U & v_y \\ U & v_z \end{bmatrix} \quad (3)$$

2.1.1. Tolerance zones based on torsors

A tolerance zone is a region of the Euclidean space, in which a fabricated surface is positioned. This zone is expressed relative to the ideal design surface [10]. Based on the concepts mentioned above, a tolerance zone can be defined mathematically by a V torsor combined with six interval constraints for components of the torsor. The result of this combination is an *augmented torsor* with 12 elements, in which each of the six components of the original torsor is replaced by the lower and upper value of an interval constraint. In this augmented torsor, the first two columns are the extremes of the rotations of the real feature in either direction from the ideal position as given by the reference system. The third and fourth columns do the same for the translations. For the geometric tolerances considered in this publication, corresponding augmented torsors can be defined to mathematically model tolerance zones.

Suppose $T_{i,j}$ is an augmented torsor, then $-T_{i,j}$ (or $T_{j,i}$) is derived in this manner: multiply -1 to each element of $T_{i,j}$, exchange column 1 with column 2, and at the same time, exchange column 3 with column 4.

In the torsor model, a substituting feature is assumed to reserve the topology of the nominal feature, and used to represent the real feature. In other words, a substituting feature has a perfect form. Therefore, the torsor model alone by far does not support form tolerances [9,15].

2.2. The design rewriting system

The design rewriting system presented in [12] aims at transforming a design in a specific manner, and comprises rewriting rules, which will be explained in greater detail later in this section. The Design rewriting system was developed on the basis of *term rewriting systems* [2].

The purpose of the design rewriting rules is to modify design dimension schemes. These are graphs, also called *overall dimension graph* [7]. Such a graph is modified by a design rewriting system in two steps: first, the left side of a rewriting rule is matched, if possible, with a part of the design. Then, the part of the design is replaced by the right part of the rule. The application of a rule results in a new design dimension scheme. This procedure allows to remove problematic dimensions from the design. This approach is suitable for a computerized tool, as various applications of term rewriting systems show.

Rewriting rules are the core of the system. A *rewriting rule* is an identity of two terms l and r, where l is not a variable and all variables occurring in r must also be in l and vice versa. The rule $l \xrightarrow{cond} r$ means the left-hand side is replaced by right-hand side if the (optional) condition $cond$ is true. In this publication, a condition may make use of the following predicates: ρ(a, b, …) signifies that a, b are planes and L(a, b, …) constrains a, b,… to be lines.

For the sake of the design rewriting system, dimensions are attributed (have the states): Original **O**, Implicit **I**, Removed **R** or Created **C**. A dimension is in the state **O**, if it was specified by the designer, and it may change into state **R**, if it is replaced by some other dimensions. A dimension that is implicit in the original design has the state **I** and may change its state to **C** when it replaces some other dimension(s). By convention, the first parameter of a primitive term is the state of a dimension and variables s and t designate states.

$$//(\mathbf{I}, a, b) \perp (\mathbf{O}, a, c) \perp (\mathbf{I}, b, c) \xrightarrow[L(a,b,c)\vee\rho(a,b,c)]{\rho(a,b)\wedge L(c)\vee} //(\mathbf{C}, a, b) \perp (\mathbf{R}, a, c) \perp (\mathbf{C}, b, c) \quad (4)$$

$$//(\mathbf{O}, a, b)\ //(\mathbf{O}, a, c)\ //(\mathbf{I}, b, c) \xrightarrow{\rho(b,c)\vee L(a,b,c)} //(\mathbf{R}, a, b)\ //(\mathbf{O}, a, c)\ //(\mathbf{C}, b, c) \quad (5)$$

$$//(\mathbf{O}, a, b)\ //(\mathbf{I}, a, c)\ //(s, b, c) \xrightarrow[(\rho(b,c)\vee L(a,b,c))]{s\in\{\mathbf{I},\mathbf{C}\}\wedge} //(\mathbf{R}, a, b)\ //(\mathbf{C}, a, c)\ //(\mathbf{C}, b, c) \quad (6)$$

Three example rules are listed above [12]. Rule (4) is used to replace perpendicularity, while rule (5) and (6) for parallelism. In the following, the meaning of rule (6) is illustrated. It intends to substitutes the initial parallelism between features a and b by two parallelism specifications between features a and c and between features b and c (see [12] for a larger set of rules). This rule is only applicable when the following conditions are satisfied:

1. b and c are planes (and a line or plane), or a, b and c are lines;
2. the parallelism between b and c must be implicit or the result of an earlier application of a rule.

3. TOLERANCE CONSTRAINT

In [12], many more rules suitable to deal with perpendicularity, parallelism, angularity, position and size tolerances were presented. However, the rewriting rules alone are insufficient to maintain the design functionality, as they do not determine the tolerance values of dimensions in state **C**. In the following, the general method to create such constraints is introduced, followed by detailed descriptions on how to define tolerances for parallelism and size dimensions. Other types of dimensions can be handled in a similar manner.

Given a design or a solid model with design specifications conform to ANSI Y14.5M [1], an unequation for a rewriting rule is created as follows:

1. Identify the dimension chain designated by the rewriting rule, and features F_i ($i = 1, 2, \ldots n$) involved in this chain;
2. Suppose features controlled by the original specification are F_1 and F_n. Then, define the augmented V-torsor $T_{1,n}$ representing the tolerance zone of the original specification;

3. Define augmented V-torsor $T_{i,i+1}$, (i =1, 2, … n-1) denoting tolerance zone between features F_i and F_{i+1};
4. Transfer all torsors and their component interval bounds into one common reference system.
5. Generate unequation

$$\sum_{i=1}^{n-1} T_{i,i+1} \le T_{1,n}$$

3.1. Parallelism

The most common geometrical design constraint is parallelism. If such a dimension cannot be machined directly, another plane must be inserted into the tolerance chain to make it possible, which is done by the appropriate rewriting rule.

For example, three planes with local reference frames attached to their nominal position, respectively, are shown in fig. 2. These planes are attached to a common rigid body with reference frame R_0, a convention used throughout in following examples. Plane P1 is of size a1×b1 (length×width), plane P2 of size a2×b2, and plane P3 of size a3 × b3. Suppose that the initial specification is P1//P2, and the specified tolerance value is t.

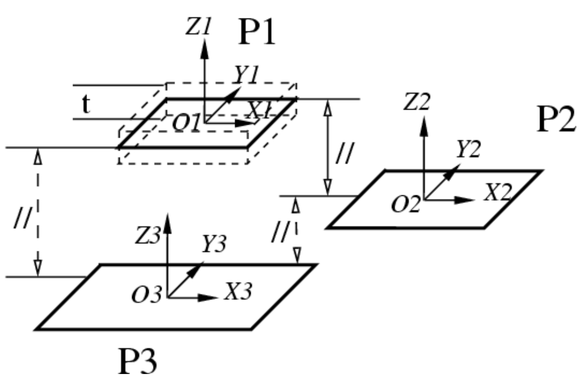

Fig. 2. Planes with parallelism constraints

If rule (6) is selected to rewrite the design, and the variables in the rewriting rule are chosen to be instantiated as: a := P1, b := P2, c := P3, s := **I**. Then the alternative tolerance chain of this modification is //(P2,P3)-//(P3,P1). Obviously, there are three augmented torsors involved in this chain: //(P1,P2), //(P1,P3), and //(P2,P3). The calculation of these three augmented torsors is demonstrated hereafter.

As the parallelism constraint concerns only two rotations of a plane, the D-torsor representing the allowable deviation zone of real plane P1 from its nominal position comprises several undetermined elements. Here, only the rotations around the x- and y-axis are relevant for $T_{1,1^*}$, as shown in equation (7).

$$T_{1,1^*}\Big|_{R_1O_1} = \left\| \begin{bmatrix} r_x & R_x & U & U \\ r_y & R_y & U & U \\ U & U & U & U \end{bmatrix} \right\|_{R_1O_1} \tag{7}$$

Analogously, the augmented D-torsor standing for the allowable deviation zone of P2 is

$$T_{2,2^*}\Big|_{R_2O_2} = \left\| \begin{bmatrix} r_x & R_x & U & U \\ r_y & R_y & U & U \\ U & U & U & U \end{bmatrix} \right\|_{R_2O_2} \tag{8}$$

where 1^* and 2^* are the nominal planes of features 1 and 2. To calculate the stackup of involved torsors, it is necessary to convert them into a common reference system prior to the addition of the matrices. In this case, R_1 is chosen as the common reference system. The reference frame of $T_{2,2^*}|_{R2O2}$ is then changed from R_2O_2 to R_1O_1 by using equation (2), resulting into $T_{2,1^*}|_{R1O1}$. The augmented V-torsor denoting the tolerance zone of //(P1, P2) is:

$$\begin{aligned} & T_{1,2}\Big|_{R_1O_1} \\ & = \left(T_{1,1^*} + T_{1^*,2}\right)\Big|_{R_1O_1} \\ & = \left(T_{1,1^*} - T_{2,1^*}\right)\Big|_{R_1O_1} \\ & = \left\| \begin{bmatrix} r_{x1,1^*} - R_{x2,1^*} & R_{x1,1^*} - r_{x2,1^*} & U & U \\ r_{y1,1^*} - R_{y2,1^*} & R_{y1,1^*} - r_{y2,1^*} & U & U \\ U & U & U & U \end{bmatrix} \right\|_{R_1O_1} \end{aligned} \tag{9}$$

There are eight unknown variables in torsor (9). However, $r_{x1,1^*} - R_{x2,1^*}$, $R_{x1,1^*} - r_{x2,1^*}$, $r_{y1,1^*} - R_{y2,1^*}$ and $R_{y1,1^*} - r_{y2,1^*}$ can be calculated respectively. To do this, several extreme points of plane P1 must be picked out, and create unequations to limit their displacements within the tolerance zone of //(P1, P2). All created unequations are linear, therefore, maximal and minimum bounds of the variables in the generated unequations can be determined by the *simplex*-algorithm, depending on the original tolerance specification as well as the length and width of planar surface P1. The derived bounds for the rotation around x- and y- axis are worst cases. The substituting plane of the actual planar surfaces may deviate up and down from the nominal position of the plane.

Similarly, the augmented V-torsors for the tolerance zones of //(P1, P3) and //(P2, P3) are:

$$T_{1',3'}\Big|_{R_1O_1} = \left(T_{1',1^*} + T_{1^*,3'}\right)\Big|_{R_1O_1} \tag{10}$$

$$T_{2',3'}\Big|_{R_1O_1} = \left(T_{2',1^*} + T_{1^*,3'}\right)\Big|_{R_1O_1} \tag{11}$$

where 1', 2', and 3' represent real surfaces after re-design. They use different notations instead of 1, 2 and

3, because real surfaces before and after redesign may have different D-torsors, and have to be distinguished.

$$\left(T_{1',3'} + T_{3',2'}\right)\Big|_{R_1O_1} \leq T_{1,2}\Big|_{R_1O_1} \qquad (12)$$

Equation (12) constrains the tolerances zones: the sum of the two involved torsors in the alternative chain has to be inferior to the torsor representing the initial tolerance zone. Provided this constraint is satisfied, the allowable deviation region can be allocated to the two V-torsors according to optimization criteria, as, for example, those proposed in [4] and [5].
Similar procedures can be applied to other dimensions like perpendicularity, angularity, and position tolerances.

3.2. Size dimensions

A size dimension can be specified between any combination of planes, lines, or points.
Fig. 3 shows the most common case, two planar surfaces constrained by a size dimension. Suppose that rule (13) is used to rewrite the design in fig. 3, where a := P1, b := P2, c := P3. Then the combined tolerance between surfaces P1, P2 and P3 has to be inferior or equal to the original design tolerance.

$$\mathscr{S}(\mathbf{O}, a, b)\ \mathscr{S}(s, b, c)\ \mathscr{S}(\mathbf{I}, c, a) \xrightarrow{s\in\{\mathbf{O},\mathbf{C}\}\wedge//(a,b,c)} \mathscr{S}(\mathbf{R}, a, b)\ \mathscr{S}(s, b, c)\ \mathscr{S}(\mathbf{C}, c, a) \qquad (13)$$

This constraint for size dimensions can be expressed by the means of the augmented torsors. The D-torsor standing for the deviation zone of plane P1 is

$$T_{1,1^*}\Big|_{R_1O_1} = \left\|\begin{bmatrix} r_x & R_x & U & U \\ r_y & R_y & U & U \\ U & U & t_z & T_z \end{bmatrix}\right\|_{R_1O_1}$$

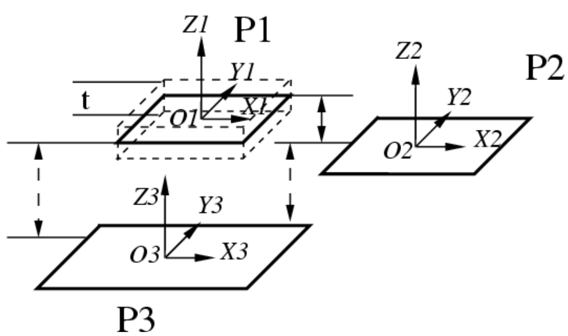

Fig. 3. Size dimension and tolerance

Analogously, the one for P2 is

$$T_{2,2^*}\Big|_{R_2O_2} = \left\|\begin{bmatrix} r_x & R_x & U & U \\ r_y & R_y & U & U \\ U & U & t_z & T_z \end{bmatrix}\right\|_{R_2O_2}$$

Note that, although the situation is similar to replacing parallelism constraint, the D-torsors for P1 and P2 are different. For size tolerances, an additional constraint is added (the translation along z-axis). The augmented V-torsor represents the tolerance zone of dimension $\mathscr{S}$(P1, P2) is

$$T_{1,2}\Big|_{R_1O_1} = \left(T_{1,1^*} + T_{1^*,2}\right)\Big|_{R_1O_1}$$

The alternative chain implied by rule (13) is $\mathscr{S}$(P2, P3)—$\mathscr{S}$(P3, P1), for which the augmented torsors for $\mathscr{S}$(P2, P3) and $\mathscr{S}$(P3, P1) are:

$$T_{2',3'}\Big|_{R_2O_2} = \left(T_{2',2^*} + T_{2^*,3'}\right)\Big|_{R_2O_2}$$

$$T_{1',3'}\Big|_{R_1O_1} = \left(T_{1',1^*} + T_{1^*,3'}\right)\Big|_{R_1O_1}$$

resulting in the constraint:

$$\left(T_{1',3'} + T_{3',2'}\right)\Big|_{R_1O_1} \leq T_{1,2}\Big|_{R_1O_1}$$

4. EXAMPLE

The generation of constraints is demonstrated by the means of the part in fig. 4. Assume that the part is cut to shape except for the slot, the top surface, the step and the hole. Then, in practice, the machine tool is usually set such that the step surfaces 12Y and 4Z, as well as top surface 1Z are milled parallel to the control surface 5Z defined by a 3-point locator. Surface 10X is adopted as the 2-point location surface to meet dimension 10X-9X. Surface 13Y is used as the 1-point location surface in order to achieve size dimension 13Y-12Y.
This set-up results into a tolerance stack for the perpendicularity tolerance between surfaces 8X-4Z, and the position tolerance for the hole 14XY, due to both machining and setting variations. The design is such that any single set-up causes excessive tolerance stacks.
Direct machinability of these three features in a single setup can be achieved by applying two rewriting rules [12], however, only one is examined here. After instantiation, rule (4) reads:

$$//(\mathbf{I}, 4Z, 5Z) \perp (\mathbf{O}, 8X, 4Z) \perp (\mathbf{I}, 8X, 5Z) \longrightarrow //(\mathbf{C}, 4Z, 5Z) \perp (\mathbf{R}, 8X, 4Z) \perp (\mathbf{C}, 8X, 5Z)$$

This rule replaces the original dimension 8X-4Z and adopts the alternative chain 8X-5Z-4Z.
As argued in the previous sections, this redesign must be complemented by unequations limiting the tolerances of the dimensions in states **O** and **C** on the right side of he rules.

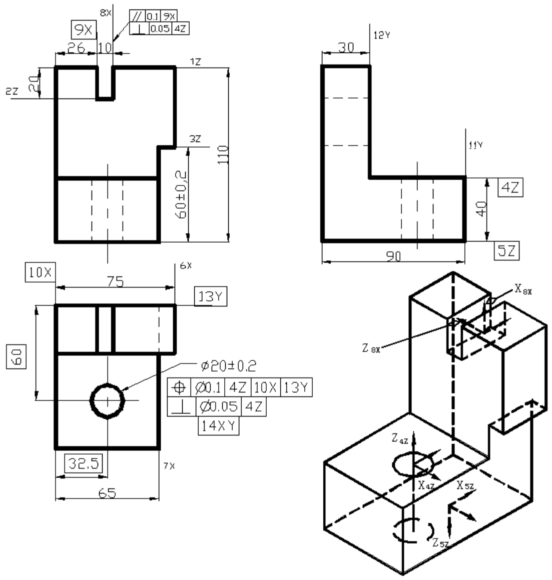

Fig. 4. Drawing and 3-D model of the example part

The local reference systems of features 4Z, 5Z and 8X (R_{4Z}, R_{5Z} and R_{8X}, respectively) are as indicated in the 3-D model of fig. 4. It can be inferred from this figure that the rotations and translations in equations (14) to (17) permit the adoption of a common reference system.

$$M_{4Z,8X} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \quad (14)$$

$$O_{4Z}O_{8X} = \begin{bmatrix} 3.5 & 45 & 60 \end{bmatrix}^T \quad (15)$$

$$M_{5Z,8X} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \quad (16)$$

$$O_{5Z}O_{8X} = \begin{bmatrix} 30 & 3.5 & -100 \end{bmatrix}^T \quad (17)$$

Based on the information given and the procedure to construct augmented D-torsor presented above, the augmented V-torsor for ⊥ (8X, 4Z) derived is

$$T_{8X,4Z}\Big|_{R_{8X}O_{8X}} = \left(T_{8X,8X*} - T_{4Z,8X*}\right)\Big|_{R_{8X}O_{8X}} = \begin{bmatrix} U & U & U & U \\ m & n & U & U \\ U & U & U & U \end{bmatrix}\Bigg|_{R_{8X}O_{8X}}$$

where $m = r_{y8X,8X*} - R_{y4Z,8X*}$, and $n = R_{y8X,8X*} - r_{y4Z,8X*}$. According to the simplex algorithm, $m=-0.025$, and $n=-0.025$. Therefore,

$$T_{8X,4Z}\Big|_{R_{8X}O_{8X}} = \begin{bmatrix} U & U & U & U \\ -0.025 & 0.025 & U & U \\ U & U & U & U \end{bmatrix}\Bigg|_{R_{8X}O_{8X}} \quad (18)$$

And the resulting constraint is

$$\left(T_{8X',5Z'} + T_{4Z',5Z'}\right)\Big|_{R_{8X}O_{8X}} \le T_{8X,4Z}\Big|_{R_{8X}O_{8X}} \quad (19)$$

Substituting equation (18) into unequation (19), it is clear that for rotations around the y-axis of the two augmented torsors on the left side of unequation (19), the sum of their ranges cannot exceed [-0.025, 0.025]. How to allocate this range to two augmented torsors corresponding to tolerance zones in the alternative tolerance chain depends on the optimization criteria selected. One possible way is to distribute it equally, i.e. each gets [-0.0125, 0.0125].

5. CONCLUSION

A methodology to create constraints for new derived geometric dimensions & tolerances is presented. It allows for a semiautomatic redesign process, while maintaining the functionality of the changed design. The approach is exemplified for parallelism and dimensional constraints and uses the torsor model to mathematically represent geometric tolerance zones. Tolerance chains are manipulated and unequations are created to maintain the design intent.

This constraint generation approach is a part of a design rewriting system, targeting a better manufacturability of a design through redesign. Notably, the method is independent from a particular machinability measure, as well as of the tolerance accumulation model, and can therefore be considered generic.

6. REFERENCES

[1] ANSI Y14.5M. *Dimensioning and tolerancing*. The American Society of Mechanical Engineers, NY, 1994. American National Standard.

[2] Franz Baader and Tobias Nipkow. *Term Rewriting and All That*. Cambridge University Press, 1998.

[3] P. Bourdet and E. Ballot. *Computer aided tolerancing*, chapter 8 Geometrical behavior laws for computer-aided tolerancing, pages 119–131. Chapma & Hall, 1996.

[4] Z. Dong, W. Hu, and D. Xue. New production cost-tolerance models for tolerance synthesis. *Journal of Engineering for Industry*, 116:199–206, May 1994.

[5] J.-R. He and P. R. Gibson. Computer aided geometric dimensioning and tolerancing for process-operation planning and quality control. *International Journal of Advanced Manufacturing Technology*, (7):11–20, 1992.

[6] Y. S. Hong and T.-C. Chang. Tolerancing algebra: a building block for handling tolerance interactions in design and manufacturing part 2: tolerance interaction. *International Journal of Production Research*, 41(1):47–63, 2003.
[7] Lin Jietong, G. Thimm, and G. A. Britton. Alternative design specifications of prismatic parts for manufacturability. *Proc Instn Mech Engrs Part B, J. of Engineering Manufacture*, 217(4):569–572, 2003.
[8] Lin Jietong, G. Thimm, and G. A. Britton. Alternative geometric design specifications for manufacturability. In Y Qin and N Juster, editors, *Advances in Manufacturing Technology - XVII: Proceedings of the First International Conference on Manufacturing Research (ICMR2003)*, pages 265–269. University of Strathclyde, Professional Engineering Publishing Limited, Bury St Edmunds and London, UK, September 2003.
[9] T. M. Kethara Pasupathy, Edward P. Morse, and Robert G. Wilhelm. A survey of mathematical methods for the construction of geometric tolerance zones. *Journal of Computing and Information Science in Engineering*, 3(1):64–75, March 2003.
[10] D. Teissandier, Y. Couetard, and A. Gerard. A computer aided tolerancing model: proportioned assembly clearance volume. *Computer-Aided Design*, 31:805–817, 1999.
[11] G. Thimm, G. A. Britton, and S. C. Fok. A graph theoretic approach linking design and process planning, part 1: design to process planning. *International Journal of Advanced Manufacturing Technology*, 2003. to appear.
[12] Georg Thimm and Lin Jietong. Redimensioning parts for manufacturability: A design rewriting system. *International Journal of Advanced Manufacturing Technology*, 2003. accepted.
[13] F. Villeneuve, O. Legoff, and Y. Landon. Tolerancing for manufacturing: a three-dimensional model. *International Journal of Production Research*, 39(8):1625–1648, 2001.
[14] Hsu-Pin Wang and Jian-Kang Li. *Computer-aided process planning*. Elsevier, Amsterdam-Oxford-New York-Tokyo, 1991.
[15] Daniel E. Whitney, Olivier L. Gilbert, and Marek Jastrzebski. Representation of geometric variations using matrix transforms for statistical tolerance analysis in assemblies. *Research in Engineering Design*, (6):191–210, 1994.