Generating the Mid-Surface of a Solid using 2D MAT of its Faces

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ABSTRACT

Mid-surface of a part is currently used as an idealization of 3D shape for purposes of analysis/simulation of injection molding and other near net shape processes. The mid-surface has also been proposed as an intermediate representation for feature extraction, and feature suppression for analysis.

This paper describes a completely automatic procedure to determine the mid-surface of an object using 2-D Medial Axis Transform (MAT) of each of its faces. Earlier efforts to generate the mid-surface either rely on user intervention and expensive ray tracing procedure to identify pairs of faces that define mid-surface patches or work on a very restricted domain. In the proposed approach, the 2-D MAT of each face is used to define *mid-curves* for each face. These mid-curves are used to obtain the topology of the mid-surface. Pairs of faces corresponding to each mid-surface patch are identified using the mid-curves. The mid-curves also form the boundary of the mid-surface patch in some special cases. Results of implementation on typical objects are presented and scope for further work has been identified.

Keywords: Mid-surface, Skeleton, Medial axis, Abstraction

1. INTRODUCTION : SKELETONS

A type of solid representation called "skeleton/skeletal representation" is receiving much attention of late as an abstraction of 3D shape that can be used for design and engineering tasks. Any skeleton (or an abstracted model) should have the following properties [11].

- The skeleton should have no interior (in the sense of dimensional reduction [11]) in the dimension of the object of space.
- The skeleton should have homotopic equivalence to the object - that is, number of holes, enclosed voids should remain the same.
- The shape of the skeleton should abstract the shape of the object – that is, locations and relative dimensions of features should be the same as in the object.

The dimensional reduction inherent in the skeleton enables abstraction of the 3D model of the part to obtain a model that can be used in the simulation/analysis programs typically used in injection molding and die casting. In these programs only shell or beam elements are used so that the third dimension needs to be suppressed. For the analysis to be correct it is important that the abstraction follow the local topology of the part as closely as possible [4]. Use of skeleton representations are also being explored in applications such as process planning, robot path planning, shape description and finite element modeling [3].

Various skeletal representations [1] are possible depending on the norm used to derive the skeleton. Among these, Medial Axis Transform (MAT) [2] is the most widely studied and perhaps used as well. Other skeleton representations such as the box skeleton [11], have also been proposed for applications like mesh generation and feature recognition.

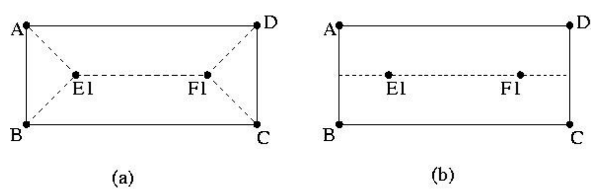


Fig. 1. Medial Axis and Mid-curve.

Mid-surface [4], a variant of MAT, reflects the topology of an object to a greater extent than MAT and has been shown to be useful in various applications. The 2D counterpart of the mid-surface is referred to as the mid-curve [7]. While the MAT is formally defined [2,10], no formal definition exists for either mid-curve or mid-surface. Fig. 1(a). shows referred to as the mid-curve [7]. While the MAT is formally defined [2,10], no formal definition exists for either mid-curve or mid-surface. Fig. 1(a). shows the MAT and Fig. 1(b). shows the mid-curve of a 2D domain. As can be seen in the figure, the MAT falls short in its ability to reflect the local topology of the part exactly. This is because of the extraneous portions and non-linear entities that occur due to convex and concave corners in the domain. On the other hand, the skeleton (mid-curve) in Fig. 1(b). resembles the original geometry to a greater extent when compared to MAT.

Based on the properties of a skeleton listed above, the mid-surface can perhaps be defined as follows.

Definition 1 Mid-surface is an aggregation of surface patches (where each patch corresponds to a pair of non-adjacent surface patches (faces) in the object that are closest to each other) that form a closed and connected set and that satisfy homotopy.

Though the mid-surface has desirable properties as an abstraction of 3D shape, a formal definition of the mid-surface is not available. This makes it difficult to develop algorithms for generating the mid-surface for an object. Also, unlike MAT, the mid-surface is not unique for a given object and this adds to the difficulties in realizing the mid-surface.

There have been very few efforts reported for the construction of mid-surface [4,6,7]. These approaches can be classified into direct and indirect approaches.

The direct approach [4,6] involves constructing the 3D mid-surface for a part model by connecting/sewing the mid-surface patches obtained for 'pairs of surfaces'. This requires a 'pairing strategy' that has thus far required human intervention. Connecting various mid-surface patches require 3D Boolean operations. Rezayat's [4] algorithm appears to work only for objects that are a combination of a few geometric configurations such as L, RIB, LRIB, and RAMP.

The indirect approach involves generating the mid-curves first and then obtaining the mid-surface using mid-curves. However generation of mid-curve has been addressed only for the restricted domain whose boundary consists of two boundary curves. In this case, the mid-curve can be constructed in a straight forward manner based on a predefined norm. Some post-processing may be required due to the formations of undesirable 'loops' [7] which should be eliminated to get a valid mid-curve between two boundary curves. When the domain consists of more than two boundary curves, correct pairing of the edge pairs is required to generate the mid-curve. This then becomes a direct approach albeit in 2D.

The generation of mid-surfaces from mid-curves has been addressed only for 2.5D objects generated through extrusion of the 2D profile [7]. However even for this class of solids, the result is correct only if the span of extrusion is above a threshold.

The mid-surface construction technique described in this paper is also an indirect approach in that the mid-surface is constructed from the mid-curves of the faces in the object. In contrast to the earlier indirect approaches, the mid-surface for a general object has been addressed and not restricted to extruded/revolved objects only. The 2D MAT of the faces in the object are used to define the mid-curve for each face in the object. The problem of identifying the correct pair of edges in a face to define the mid-curve is solved by the use of the MAT. The mid-curves in turn are used to identify the pairs of faces that define the mid-surface patches. As the MAT is formally defined and there are formal algorithms to generate the MAT (atleast in 2D [10]) the process of identifying correct face pairs is completely automatic. It must be mentioned that the MAT not only facilitates automatic pairing but the mid-curve consists largely of MAT segments.

The rest of this article is structured as follows: section 2 presents some preliminaries followed by an overview of the algorithm. Details of the construction are provided next. Results of implementation are presented followed by discussion. The paper concludes with a discussion on directions for future work.

2. MAT

The Medial Axis (MA) of the set D, denoted M(D), is defined as the locus of points inside D which lie at the centers of all closed discs (or balls in 3-D) which are maximal in D, together with the limit points of this locus. A closed disc (or ball) is said to be *maximal* in a subset D of the 2D (or 3D) space if it is contained in D but is not a proper subset of any other disc (or ball) contained in D. The radius function of the MA of D is a continuous, real-valued function defined on M(D) whose value at each point on the MA is equal to the radius of the associated maximal disc or ball. The Medial Axis Transform (MAT) of D is the MA together with its associated radius function. The boundary and
the corresponding MA of an object is shown in Fig. 1(a).

2.1 Points on MAT

Points on the MAT can be classified based on the properties of their maximal disks [5]. A point whose maximal disc touches exactly two separate boundary segments is called *normal point*. Point N (or any point on the line segment (A,E1) excluding the end points A and E1) in Fig. 2(a). is a normal point. Its underlying maximal disk is shown in Fig. 2(b).

A point whose maximal disc touches the domain boundary in three or more separate segments is called *branch point*. Points E1 and F1 in Fig. 2(a). are branch points. Fig. 2(c). shows the maximal disk corresponding to the branch point E1.

A point whose maximal disc touches the boundary in exactly one contiguous set is called an *end point*. Fig. 2(a)., shows the end points A,B,C and D. These points touch the boundary at a point and the corresponding maximal disc is of radius zero.

A point of contact with the domain boundary, of the underlying disk of a point on the MAT is called the *footpoint* of the point on the MAT. From the definition of the point types in a MAT, a normal point will have two footpoints (Fp1 and Fp2 in Fig. 2(b).), a branch point will have three or more (Fp1, Fp2, and Fp3 in Fig. 2(c).) and an end point will have one or more footpoints.

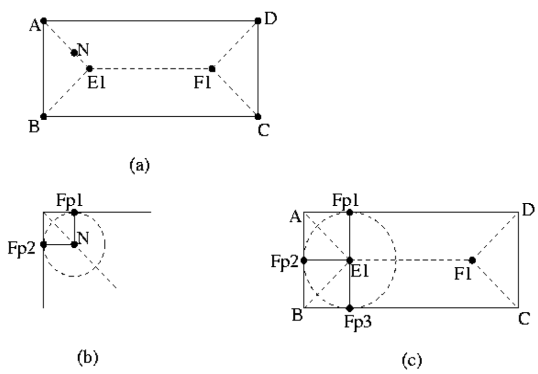


Fig. 2. Classification of points on MAT.

3. OVERVIEW OF THE ALGORITHM

The algorithm takes as input a boundary representation (B-rep) model of the part and the 2D MAT of each face in the B-rep. The 2D MAT is obtained by the algorithm described in [10]. Mid-curves for each face are first constructed from the 2D MAT. This step involves removal of some MAT segments and addition of some new segments to account for the corners (convex and concave) in each face. Based on the connectivity and the associated radius function, mid-curve segments across the faces in the part are connected to form chains or loops. The face-pairs that define the mid-surface are then identified from these chains (and loops) of mid-curve segments. The chains/loops that result in valid face-pairs are flagged. In some special cases, the loops of mid-curve segments directly yield bounding edge loop of a mid-surface patch. The identification of face-pairs terminates when all the faces in the object either have their mid-curves flagged or are one of the faces in the face pair associated with a mid-curve that has been flagged. The mid-surface patches are constructed from each face pair identified, by an offset based procedure. The information available with the mid-curve loops/chains is then used to connect/trim the mid-surface patches to obtain the mid-surface.

4. ALGORITHM DETAILS

The algorithm for constructing the mid-surface consists of the following steps.

- Constructing mid-curves from MAT
- Connecting mid-curves to form mid-curve graphs
- Identifying valid face pairs
- Constructing mid-surface patches

4.1 Constructing Mid-curves from MAT

In this step, mid-curve for each face in the part is constructed from the 2D MAT of that face. The construction of the mid-curve involves deletion of MAT segments that correspond to corners (convex and concave) and addition of segments that account for the portion of the boundary in the vicinity of the corners, that is not accounted for by the modified MAT. As mentioned earlier, it is the MAT segments corresponding to convex and concave corners that prevent the MAT from reflecting the local topology exactly. The remaining MAT segments (with some modifications as mentioned above) are part of the mid-curve. By definition of MAT, these segments are equidistant to their corresponding boundary segments that are not adjacent to each other. Hence they form part of the mid-curve.

4.1.1 Replacing MAT Segments due to Concave Corner

For each concave vertex in the face, the extreme points of MAT segments corresponding to the concave vertex are identified. There will be two such extreme points that have as their corresponding boundary entities - the concave vertex and one of the two edges incident at the concave vertex respectively. Continuity of the MAT ensures that these two points exist. The non-linear segment between these two points are removed. The removed segment is replaced by segments that are parallel to the respective edge incident at the concave vertex, through the extreme points identified. Intersection of these two segments is a new branch point, termed the secondary branch point (SBP). The new segments are trimmed at the SBP. Fig. 3. illustrates this process.

4.1.2 Replacing MAT Segments at Convex Corners

MAT segments emanating from a convex corner are removed. If these segments (starting from a convex corner) share a branch point then these are replaced by a new segment between the branch point and the mid-point of the boundary segment formed by the two convex corners.

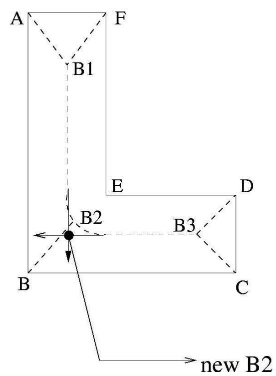


Fig. 3. Replacing MAT at a concave corner.

The new point on the boundary segment is called 'mid-curve-end (mc-end)' point. In Fig. 4., S1 is the mc-end point and S1-E1 is the equidistant segment replacing the MAT segments A-E1 and B-E1. The radius function for the points on the new segment (S1-E1) is defined by the disk centered at a point on S1-E1 and that has foot points on the other edge incident on the two convex vertices respectively. The disk is therefore termed to be locally maximal as it need not be maximal with respect to the entire object.

The addition of the new segment is required to account for the portions of the domain that is left uncovered due to the removal of the MAT segments at the convex corners. As can be seen in Fig. 4., the domain covered by the segment E1-S1 (that has been added) is the rectangle A-B-Fp1-Fp2. This is equal to the sum of the areas covered by the MAT segment A-E1 (area A-Fp1-E1-S1) and the MAT segment B-E1 (area B-Fp2-E1-S1). Hence the segment E1-S1 can replace the MAT segments A-E1 and B-E1.

It is now shown that the above construction of the mid-curve from the MAT satisfies the properties of a skeleton (Theorems 1 and 2).

Theorem 1 The result of the above procedure has no interior.

Proof : The MAT does not have any interior in the same dimensional space as that of the object [11]. Deleting a MAT segment does not create any interior. The segments between branch points that are retained belong to MAT segments and hence they also create no interior. Assume that there is an interior formed by segments added to form the mid-curve. Let 'q' be a point on such a segment (refer Fig. 4.) and 'f' be its footpoint. Let the corresponding radius of the locally maximal ball be r. If there exists an interior, then some point in the vicinity of 'q' such as 'p' should belong to the interior. Clearly there cannot be a locally maximal ball at 'p' that has the same radius. Therefore 'p' cannot be on the mid-curve. Similar argument holds if 'q' were to be in the interior. Hence the theorem.

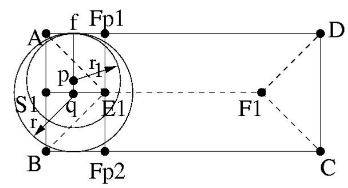


Fig. 4. Illustration of mc-end point.

Theorem 2 The above procedure preserves homotopy.

Proof : Proof is by contradiction. Assume the above procedure 'creates' holes. MAT segments that have holes as their corresponding boundary edge will have at least one branch point. The above procedure only modifies an existing branch point (corresponding to concave corner) but does not create any extra branch points. Therefore no extra holes are created. Similar argument holds for 'deleting' holes. Hence the procedure preserves homotopy.

Fig. 5. illustrates the steps in the construction of mid-curve from the MAT for a simple domain. The 2D MAT for the L-section is input (Fig. 5(a).). B1, B2 and B3 are the primary branch points. Replacing the non-linear segments due to the concave corner results in the secondary branch point B2' (Fig. 5(b).). Processing the MAT segments at convex corners that share a branch point results in the segments B1-S1 and B3-S2 along with the removal of corresponding MAT segments (Fig. 5(c).). Finally, the MAT segment between convex corner B and branch point B2 is removed to obtain the mid-curve (Fig. 5(d).).

The mid-curve of a face is stored in the form of a graph. The nodes/vertices of this graph are the end points of each mid-curve segment and the edges of this graph are the mid-curve segments. The nodes carry an attribute indicating the type of vertex to be either a mc-end point or SBP. The edges have three attributes. The first attribute is the face on which the mid-curve is defined. This is also referred to as the *parent face*. The second is the radius of the mid-curve segment (which is the radius of the locally maximal disk). The third attribute is the face pair which consists of the two faces (other than the parent face) respectively incident at the two edges defining the mid-curve segment. The structure representing the mid-curve is referred to as attribute mid-curve graph (AMG).

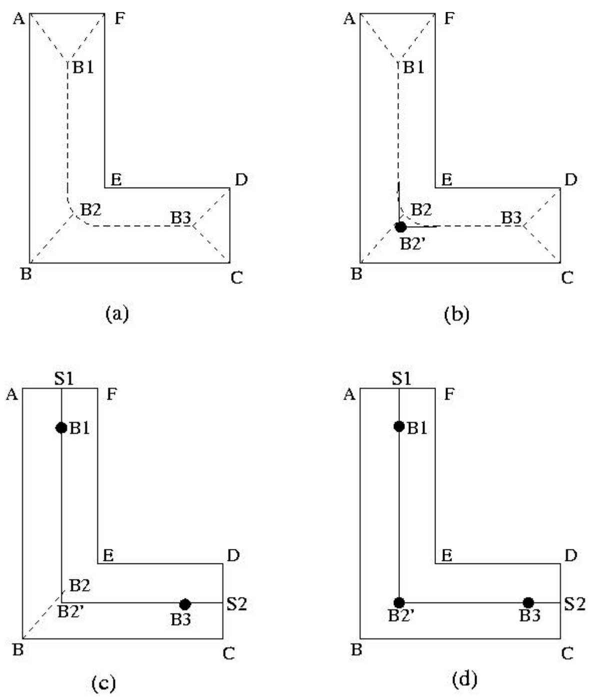


Fig. 5. Constructing mid-curve of a face from the 2D MAT.

4.2 Connecting Mid-curves to Form Mid-curve Graphs

In this step mid-curve segments obtained for each face are connected to form chains or loops. The mc-end points associated with the mid-curve of each face are used to connect mid-curves across the faces of the object.

Definition 2 *(correspondence)*: A vertex (having mc-end point as its attribute) of a mid-curve of a face is said to have correspondence if that vertex is a mc-end point in the mid-curve of an adjacent face (that shares an edge).

mc-end points in the mid-curve of a face are used to connect mid-curves.

Definition 3 *(Union of AMG's through mc-end points):* Union of mid-curve graph through mc-end points is defined as follows. The nodes or the vertices of the graph union are the set union of vertices of all the mid-curves of the faces having common mc-end points. The edges of the union are the union of the edges of the mid-curves of all such faces having common mc-end points.

The union of AMGs across faces is also an AMG. The resulting mid curve graph is classified further as a loop-graph or a chain as follows. Union of AMGs obtained above is said to form a cross loop graph (CLG), if there exists correspondence for every mc-end point on a face. Otherwise, it is labelled as a Chain (CH).

Fig. 6(b). and Fig. 6(c). show a CLG and CH respectively obtained for the object in Fig. 6(a).

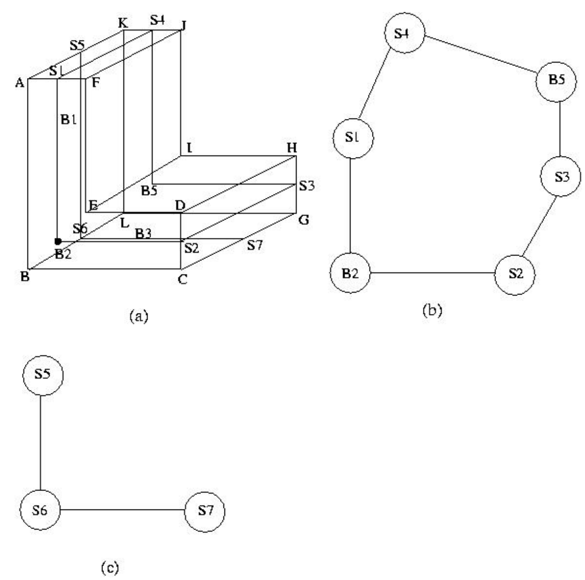


Fig. 6. (a) Mid-curve segments for L-section (b) Cross loop graph (c) Chain.

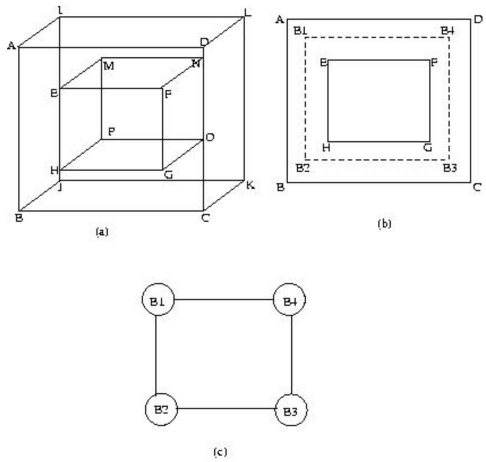


Fig. 7. (a) Object with a hole (b) Mid-curve of the multiply connected face (c) Self loop.

If there is no mc-end point present on the face, then the AMG on that face is tested for 'cycles'. An AMG having a cycle but no mc-end point in its vertex attribute is called a self loop (SL).

Fig. 7(a). shows an object with a through hole and Fig. 7(b). shows the mid-curve on a face of that object. Fig. 7(c) illustrates the self-loop.

4.3 Identifying Valid Face Pairs

In this step, the graphs of mid-curves obtained in the previous step are used to identify the face pairs forming the mid-surface patches. The CLG is used to identify the edges of the mid-surface of an object while SL, CH are used to identify face pairs for which the mid-surface patch is to be generated.

Mid-curve identifies non-adjacent edges in a face that are closest to each other. Consequently, the mid-curve also establishes a distance norm between the faces (other than the face for which mid-curve is defined) incident at these two edges. For example in Fig. 8., the ball Ba1 connects the edges AB and EF in face F7 and this can be treated as a distance metric between faces F1 and F3 incident on these two edges. The 'maximal ball' constraint of MAT and the locally maximal ball constraint of mid-curve ensures that these two edges (and therefore the two faces) are closest to each other. The above properties of mid-curve along with the presence of mc-end points facilitates the identification of pairs of faces that could form the mid-surface. Connecting the mid-curves across faces then allows identifying valid face pairs that would define mid-surface patches.

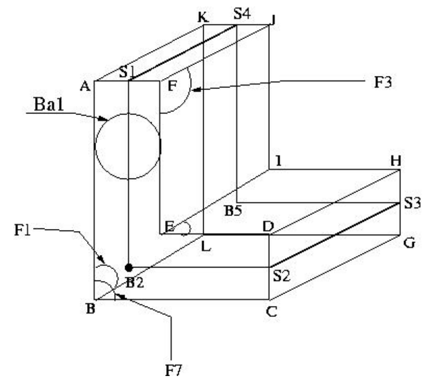


Fig. 8. Proximity relation between faces through mid-curves.

If there are more than one cross-loop-graph or chains then parallel cross-loop-graph and parallel chains from amongst these are identified. Two cross-loop-graphs are said to be parallel if every edge in the cross-loop-graph is associated with the same face pair (Fig. 9(a).). Similarly, two chains are said to be parallel if the mc-end point of the terminal edge in each chain share a common face (Fig. 9(b).). Once pairs of parallel cross-loop-graphs or chains have been identified, face pairs forming the mid-surface patches are obtained as follows. For every pair of parallel cross-loop-graph or chain, parent faces of the edges in the respective cross-loop-graph/chain that are parallel form a valid face-pair. If there are more than two edges parallel to each other, a procedure similar to the scan-line algorithm [8] is used to pick the right pair.

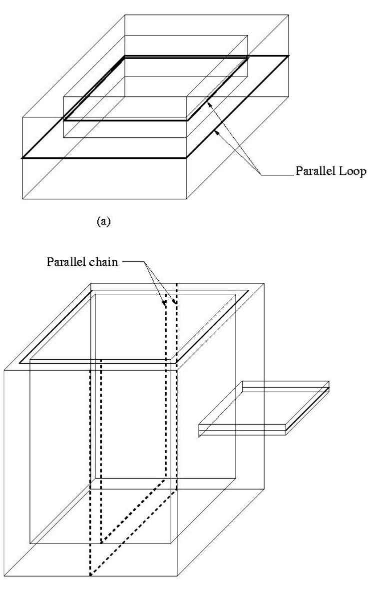


Fig. 9. (a) Parallel Loop (b) Parallel Chain.

If there is only one chain then if the edges in the chain correspond to the same face pair then that face pair is a valid face pair.

If there are self loops identified in the mid-curve of the faces, the face pair associated with each mid-curve segment forms a valid face pair.

It may be noted that some valid face pairs may be identified twice (usually in the presence of self loops). Since the face pairs have unique identifiers (in terms of the labels of each object face in the pair) the repetition can be trapped. The edges (mid-curve segments) in the loop-graphs (cross and self) and chains used to identify the face pairs are flagged.

4.4 Constructing the Mid-surface Patches

Once the face pairs are identified, the mid-surface patches are generated. The mid-surface patches can be generated using a geometric interpolation [4] or by generating an offset-surface between the given face pair. Each patch definition can then be obtained using the corresponding surfaces in the object.

In special cases (described below) mid-curve segments form the edges of the mid-surface. The converse is not true because the mid-surface patches do not always terminate on the faces of the object. The edges (mid-curve segments) in cross-loop-graphs (that are not parallel to any other cross-loop-graph) form the edges of a mid-surface patch. Mid-curve segments in the self loops also form the edges of the mid-surface patches. This can be used to trim and connect the mid-surface patches. For example, consider the object shown in Fig. 9(b). The self-loop, parallel chain and the chain in that object determine the respective face pairs. Since the parallel chains terminate on a face corresponding to that with the self loop, the edges forming the self loop are used to trim and connect the mid-surface patches obtained from the face pairs identified from the parallel chain. In a similar manner the connection between the mid-surface patch obtained from the single chain to the mid-surface patch identified from the parallel chain/self loop can then be established from the fact that the chain terminates on one of the faces in the face pair obtained from the self loop. Therefore, the mid-curve segments not only help in automatically identifying face pairs but also enable establishing the connectivity of the mid-surface patches.

4.5 Termination of the Algorithm

The procedure to identify valid face pairs terminates when one of the following is true for all the faces.

1. The mid-curve associated with the face is flagged.
2. The face belongs to the face pair associated with a mid-curve that has been flagged.

These two conditions ensure that all the valid face pairs have been identified. The construction procedure terminates when the mid-surface patch for all the valid face pairs identified have been constructed and trimmed with the other patches.

5. RESULTS AND DISCUSSION

The algorithm described has been implemented and this section presents the results obtained for some typical objects. The input to the algorithm is the B-rep of the object. Curved edges are discretised into straight lines. At present the implementation is not linked with a procedure to generate offset surfaces. Once the face pairs are identified, the implementation will determine the connectivity between the patches denoted by face pairs. Results shown here therefore do not have objects where the edges of the mid-surface patches are in the interior (and therefore not mid-curve segments).

Mid-surface obtained for some typical objects are shown in Fig. 10. to Fig. 16. Mid-surface is generated for objects having multiply connected segments as well (Fig. 10. to Fig. 13.).

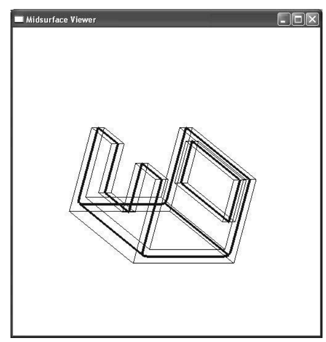


Fig. 10. Object with a slot and its mid-surface.

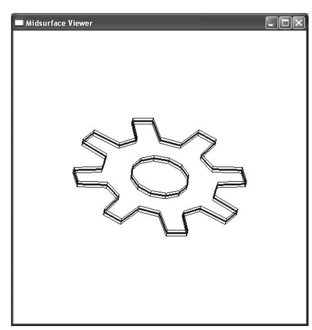


Fig. 11. Test object and its mid-surface.

Tab. 1. shows the time taken for generating mid-surface (including the generation of MAT for each face) for some of the figures. These times do not however include the time for generating offset surfaces. The implementation is on a PIII 450 MHz machine with 256MB RAM.

5.1 Discussion

Fig. 14. shows the results for a 2.5D test object obtained by extruding the same profile across different lengths (frames (a) and (c) respectively). The corresponding mid-surfaces are shown in frames (b) and (d) in the same figure. Note that there is a significant change in the geometry of the mid-surface because of this difference in the span of extrusion.

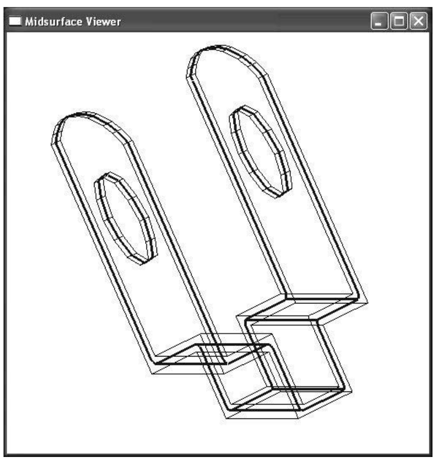


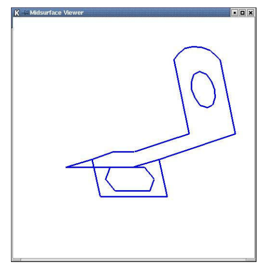
Fig. 12. Test object and its mid-surface.

Therefore the mid-surface of a 2.5D object **can not always** be obtained by just extruding/rotating the mid-curve of the profile curves from which the object is generated [7]. While the mid-surface for the object in Fig. 14(a). can be obtained by extruding the mid-curve of the profile curve, this is not the case for the object in Fig. 14(c). This is only possible if the span of extrusion is above a threshold value.

Unlike the method in [7], the method described here does not create any loops and hence loop elimination procedures are not required. This method can also be applied to parts that are obtained by rotating the profile curve of the object (Fig. 15., Fig. 16.). This algorithm also requires no separate procedure to get the mid-surface for RAMP part as in [4].

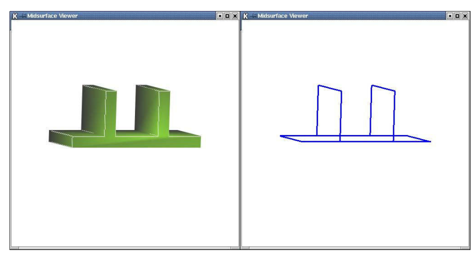
The proposed algorithm uses the mid-curve to generate each face pairs for which the mid-surface patch is to be generated rather than using a complex ray-tracing procedure which avoids the need to use separate pairing strategy. Moreover, no user intervention is required. It also generates the mid-surface of variety of part models and not restricted to shell and extruded geometries. The result will be as accurate as the 2D MAT used.

(a) Test object.

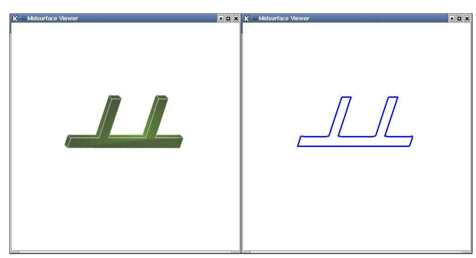


(b) Mid-surface.

Fig. 13. Test object and its mid-surface.

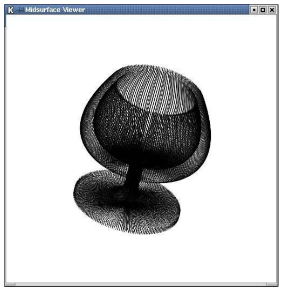


(a) (b)

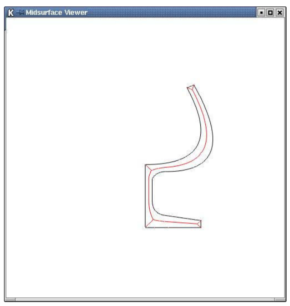


(c) (d)

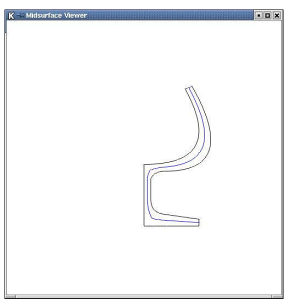
Fig. 14. (a) Test object (b) Mid-surface (c) Thinner object (d) Mid-surface.



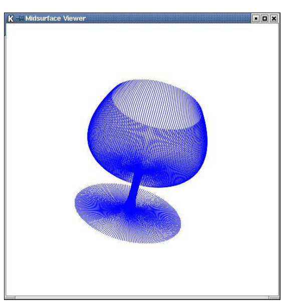
(a)



(b)

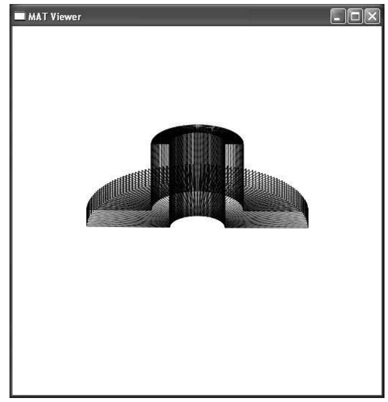


(c)

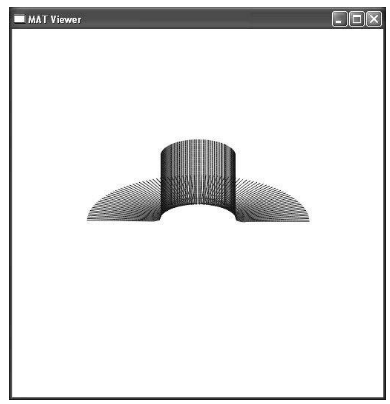


(d)

Fig. 15. (a) Test object (b) MAT of the profile (c) Mid-curve of the profile (d) Mid-surface.



(a)



(b)

Fig. 16. (a) Test object (b) Mid-surface.

Figure No.	*Time (s)*
11	37
12	70
13	46

Tab. 1. Time taken for generation of mid-surface (including MAT of each face) for typical objects.

6. CONCLUSION

In this paper, a new method for determining the mid-surface of a 3D object using a 2D MAT has been described. Computational complexities are alleviated because of a dimension reduction in solving the problem. At present, the curved edges are discretised into line segments. Moreover, the present algorithm for generating mid-curve from MAT works on one level only. For example, for a rounded corner, in its piecewise linear approximation, the MAT will look like a tree of several levels. Future work includes solving for curved edges without discretisation, handling more than one level in the MAT and providing formal proofs for some of the assertions regarding the loop-graphs and chains, and the face pairs forming the mid-surface.

7. REFERENCES

[1] Atul Sudhalkar, Levent Gursoz, Friedrich Prinz, Box-skeletons of discrete solids, *Computer-Aided Design*, Vol. 28, No. 6/7, 1996, pp 507-517.

[2] Azriel Rosenfeld, Axial representations of shape, *Computer vision, graphics and image processing*, No. 33, 1986, pp 156-173.

[3] Blum, H., A transformation for extracting new descriptors of shape, models for the perception of speech and visual form, pp 362-381.

[4] Harry Blum and Roger N. Nagel, Shape description using weighted symmetric axis features, *Pattern Recognition*, Vol. 10, 1978, pp 167-180.

[5] Fischer, A., Smolin, A. and Elber, G., Mid-surfaces of profile-based freeforms of mold design, *Journal of Manufacturing Science and Engineering,* Transactions of the ASME*,* Vol. 121, May 1999, pp. 202-207.

[6] Fischer, A. and Wang, K.K., A method for extracting and thickening a Mid-surface of a 3D thin object represented by NURBS, *Journal of Manufacturing Science and Engineering,* Transactions of the ASME, Vol. 119, Nov. 1997, pp. 706-712.

[7] James D. Foley, Andries van Dam, Steven K. Feiner and John F. Hughes, Computer graphics, Addison-Wesley, 2nd Edition, 1995.

[8] Mohsen Rezayat, Mid-surface abstraction from 3D solid models: general theory and applications, *Computer-Aided Design*, Vol. 8, No. 11, 1996, pp. 905-915.

[9] Quadros, W.R., Gurumoorthy, B., Ramaswami, K. and Prinz, F.B., Skeletons for representation and reasoning in engineering applications, *Engineering with Computers*, No. 17, 2001, pp 186-198.

[10] Ramanathan M. and Gurumoorthy B., Constructing medial axis transform of planar domains with curved boundaries, *Computer-Aided Design*, Vol. 35, No. 7, 2003, pp 619-632.

[11] Seng-Beng Ho and Charles R. Dyer, Shape smoothing using medial axis properties, *IEEE Trans. on Pattern Analysis and Machine Intelligence*, Vol. PAMI 8, No. 4, July 1986, pp 512-519.