Feature-based Decomposition of Trimmed Surfaces

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ABSTRACT

A trimmed surface is usually represented by a parametric surface and a set of trimming curves. Because of the complexity in manipulating trimmed surfaces, many CAD processes and algorithms cannot be applied to trimmed surfaces directly. It is thus desirable to represent a trimmed surface by a group of regular surfaces. In this paper, an algorithm for decomposing a trimmed surface is presented. First, bisectors of the Voronoï diagram developed in the parametric space are used to define an isolated region for every trimming curve. Feature points on the trimming curves are extracted by considering curvatures of the curves. Correspondence between feature points and vertices on the bisectors are established by considering the similarity between the trimming curves and the bisectors. Regions of parametric patches are then identified. Finally, a group of regular surfaces are constructed by interpolating a set of sampled surface points on each of the identified regions.

Keywords: trimmed surface, decomposition, feature points, surface fitting.

1. INTRODUCTION

Trimmed surface plays an important role in CAD/CAM technology. However, there is no standard representation for trimmed surface among different types of CAD systems. The differences between systems result in difficulties for exchanging trimmed surface data. Moreover, there are geometric algorithms that cannot be directly applied to trimmed surface. For example, trimmed surfaces are usually the result of intersecting different surfaces. Deformation of a trimmed surface may require re-evaluating the trimming curves which is a time consuming process. One solution to this problem is to decompose a trimmed surface into other types of basic standard elements that can be easily recognized by most CAD systems. In this paper, NURBS surface is adopted as the basic element. A trimmed surface is decomposed into a set of non-intersecting NURBS surfaces. The union of these NURBS surfaces resembles the original trimmed surface. The main issue is to develop an algorithm for the decomposition of a trimmed surface.

2. PREVIOUS WORK

In general, there are two approaches for decomposing trimmed surfaces. Triangulation, the tessellation of a trimmed surface into a set of triangular facets, is the most popular approach. The other approach is to decompose a trimmed surface into regular Bezier or NURBS surfaces.

In the context of triangulation, Piegl and Richard [8] proposed to triangulate a trimmed NURBS surface in the parametric space. The triangles in parametric space are projected to the physical space. Deviation between the triangles and the trimmed surface is evaluated. The process is repeated until the deviation is less than a prescribed tolerance. This gives a piecewise planar approximation of the trimmed surface. Cho et al. [2] adopted an unstructured Delaunay mesh approach for tessellating trimmed rational B-Spline surface. The algorithm constructs 2D triangulation domains that preserve sufficiently the shape of the corresponding triangles in the 3D space. Abi-Ezzi and Subramaniam [1] used the graphical data compilation concept to dynamically tessellate a trimmed NURBS. Liu et al. [7] proposed an algorithm for splitting irregularly trimmed shapes into regular convex regions which are then triangulated. Piegl and Tiller [9] introduced a method for tessellating trimmed NURBS surface into triangular facets based on its geometric characteristics. Cho et al. [3] proposed to use an auxiliary planar domain for the triangulation of parametric trimmed surface.

Triangulation gives a mesh of triangular facets ready for rendering and visualization. However, the triangular facets only gives an approximation to the trimmed surface, further editing of the surface will require manipulating the triangle vertices which may not be desirable. The other approach for tessellating trimmed surface is to decompose a trimmed surface into a set of Bezier and NURBS surfaces. Vries-Baayens and Seebregts [12] proposed a technique for decomposing non-rational Bezier surface based on the combined 'triangulation-quadrangulation' of a trimmed surface in the parametric space.

Hamann and Tsai [5] decomposed a trimmed NURBS surface by a set of planar, ruled surfaces in the parametric space. A Voronoï diagram in the parametric space of the trimmed surface defines regions around each trimming curve. Scan-lines are used to identify non-horizontal segments of the trimming curves. Pairs of these segments are linearly interpolated in the horizontal parametric direction thereby partitioning the parametric space. Surfaces corresponding to the regions are then created, and hence decompose the trimmed surface into regular surfaces.

3. BRIEF DESCRIPTION

Although Hamann and Tsai's method effectively decomposes a trimmed surface into regular surfaces, the surfaces created are horizontal strips in the parametric space of the surface. This may lead to an excessive number of surfaces being created as the partitioning of the surface does not take into consideration the shapes and features of the surface and the trimming curves. The method introduced in this paper extended Hamann and Tsai's approach to decompose a trimmed surface by considering shape features of the trimming curves and the surface boundaries. The result of using the method is a group of surface patches approximating the original trimmed surface with no irregular shapes on the boundary. There are four stages in the algorithm, namely, Voronoï diagram development, feature point determination, correspondence establishment, and surface approximation.

In the first stage, Voronoï diagram of the trimming curves and the boundary curves is developed in the parametric space of the surface. The Voronoï diagram is constructed by locating a set of uniformly sampled points on the bisectors of the Voronoï diagram. Bisectors are then constructed by fitting cubic B-Splines through these data points. Feature points on all the curves are identified. A feature point is a sharp corner or a curve point whose radius of curvature to arc length ratio is less than a predefined value. The feature points identified in the 3D space are projected to the parametric space for further processes.

Similar feature points and bisector vertices are identified. The similarities between the feature points and the bisector vertices are measured by comparing the distance between the vertices, and the angles at the bisector vertices. Correspondence are established by matching feature points to the nearest and sharpest bisector vertices.

A pair of consecutive feature points and their corresponding bisector vertices thus defines the parametric region of a regular surface. In general, the boundary of each regular parametric patch formed consists of four parts: one bisector segment, one trimming curve segment and two correspondence links between the bisector and the trimming curve. A set of surface points corresponding to each of the parametric regions is sampled. B-spline surfaces interpolating these data points are then constructed. Details of the technique will be discussed in the following sections.

4. REPRESENTATION OF A TRIMMED SURFACE

In the following discussion, a trimmed surface is defined with a B-spline surface and a set of trimming curves. A B-spline surface is expressed as

$$\mathbf{S}(u,v) = (x(u,v), y(u,v), z(u,v)) = \sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)\mathbf{P}_{i,j} \qquad u \in [u_0,u_1],\ v \in [v_0,v_1] \tag{1}$$

where $\mathbf{P}_{i,j} = (x_{i,j}, y_{i,j}, z_{i,j})$ is a $(n+1)\times(m+1)$ matrix of three-dimensional control points, $N_{i,p}(u)$ is the i-th B-Spline basis function of order p in the u-direction, u_0 and u_1 are the minimum and maximum knots in the u-direction respectively. Similarly, $N_{j,q}(v)$ is the j-th B-Spline basis function of order q in the v-direction, v_0 and v_1 are the minimum and maximum knots in the v-direction respectively. A trimmed shape on the surface is represented as a closed loop of curves on the parametric space. A trimmed shape is called "trim" and the curves composing the trim is called trimming curves. In this paper, trimming curves are represented as B-Spline curves. Denote the i-th trim as T_i, the j-th trimming curves on the i-th trim is expressed as

$$\mathbf{C}_{i,j}(t) = (u(t), v(t)) = \sum_{k=0}^{n} N_{k,p}(t)\mathbf{P}_k \qquad t \in [t_0,t_1] \tag{2}$$

where $\mathbf{P}_k = (u_k, v_k)$ is a control points of the curve, $N_{k,p}(t)$ is the k-th B-Spline basis function of order p, t_0 and t_1 are the minimum and maximum knots of the curve respectively. In most cases, each curve segment of the trim is C^1 continuous and every trim must be C^0 continuous. The curve corresponding to the outer boundary is denoted as T_0. In general, the boundary trim T_0 may not coincide with the boundary of the parametric space. Fig. 1. shows the parametric space of a trimmed surface. The corresponding surface is shown in Fig. 2. The curves on the boundary trim are arranged in an anticlockwise direction while curves of the other trims are arranged in a clockwise direction. The dots on the curves in Fig. 1. represent the end points of the curve segments. The coordinates of the lower left corner in the parametric space is (u_0, v_0) and that of the upper right corner is (u_1, v_1).

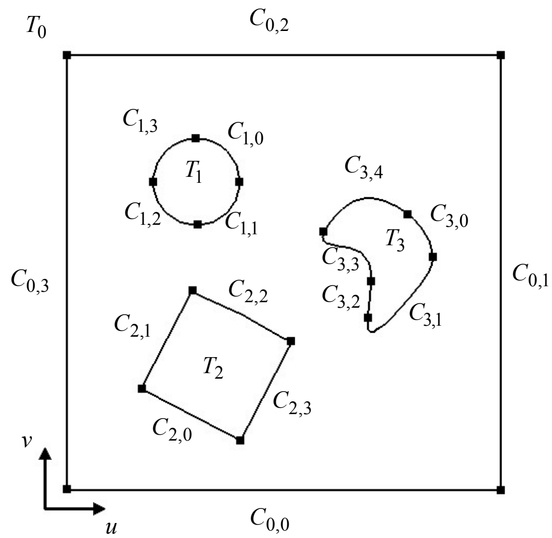


Fig. 1. The parametric space of a trimmed surface

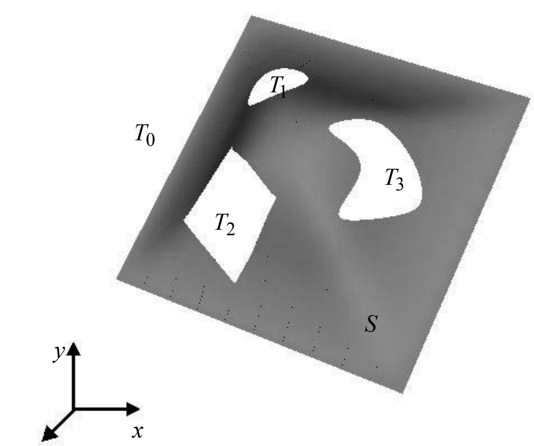


Fig. 2. A trimmed surface

5. VORONOÏ DIAGRAM

Based on Hamann and Tsai's approach [5], a Voronoï diagram [11] is developed for the trims. A closed loop of bisectors defines a region containing each trim. A bisector is a series of straight lines with equal distance from the trims and the boundaries. A regular triangulation is constructed in the parametric space of the surface. For each vertex of every triangle, labels are assigned indicating the trim that is closest to the vertex. For example, if T_1 is the closest trim to a triangle vertex, then the label of that vertex is 1. According to the combination of labels in a triangle, three different cases are classified as listed below.

1. All three vertices labels of the triangle are the same - there is no bisector vertex on the triangle edges or inside the triangle.
2. There are two different labels among the three triangle vertices - two bisector vertices exist on the triangle edges whose vertices labels are different.
3. All three vertices labels are different from each other - there are three bisectors cutting the edges and they meet inside the triangle.

In cases 2 and 3, an iterative process is invoked to locate a bisector vertex that is equidistant between the two closest trims. For case 3, a bisector vertex equidistance from its three closest trims is also determined. This type of bisector vertex is referred to as bisector centroid. The bisector vertices are then connected to form loops as shown in Fig. 3. In the figure, there are four trims (including the boundary), and four bisector centroids are located. As a result, each trim is enclosed in a loop of bisectors.

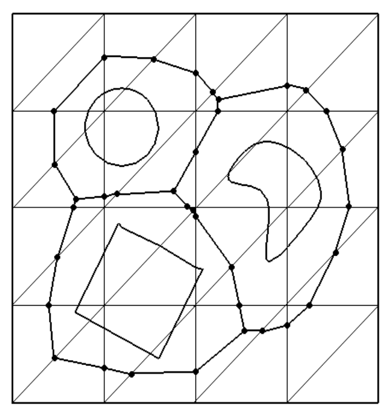


Fig. 3. Voronoï diagram and trims in the parametric space

In order to obtain a set of evenly distributed points on each of the identified regions of the Voronoï diagram, the bisectors are approximated with cubic B-spline curves. Fig. 4. illustrates two examples. Evenly distributed bisector vertices on the Voronoï diagram (right) and their corresponding points on the surfaces (left) are presented in each of the examples. The region between a trim and its bisector in the parametric space is called a parametric tile. Each tile is to be partitioned into regions corresponding to a regular patch as discussed in the following sections.

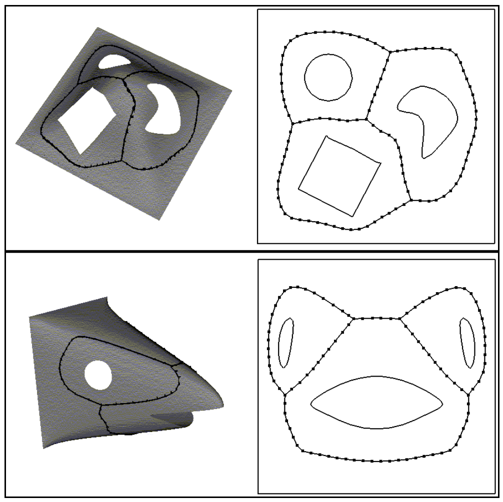


Fig. 4. Evenly distributed bisector vertices

6. FEATURE POINTS

Feature points refer to sharp turns on the trims. Since a feature point in the parametric space may not be a feature point on the surface, and vice versa, feature points are determined on the trims of the surface in the Euclidean space. The approach in [6] is adopted for identifying whether a data point on the trim is a sharp turn or not. Basically, the ratio between the total arc length of the trim and the radius of curvature at the data point is used for measuring the sharpness of a turn. For any point lying on a unit circle, this ratio is $1/2\pi$. This is used as the threshold for identifying feature points. A point with a sharpness ratio less than the threshold is considered a feature point (Fig. 5.). Given a curve $\mathbf{C}_{i,j}(t) = (x(t), y(t), z(t))$ of T_i, the radius of curvature at $\mathbf{C}_{i,j}(t)$ is

$$\rho_{i,j}(t) = \frac{\left|\dot{\mathbf{C}}_{i,j}(t)\right|^3}{\left|\dot{\mathbf{C}}_{i,j}(t) \times \ddot{\mathbf{C}}_{i,j}(t)\right|} \tag{3}$$

where $\dot{\mathbf{C}}_{i,j}(t)$ and $\ddot{\mathbf{C}}_{i,j}(t)$ denote respectively, the first and second derivatives of $\mathbf{C}_{i,j}(t)$. Assuming the total arc length of the trim T_i is $L(T_i)$, the sharpness ratio of the data point is

$$\varepsilon_{i,j}(t) = \frac{\rho_{i,j}(t)}{L(T_i)} \tag{4}$$

If $\varepsilon_{i,j}(t) < 1/2\pi$, the data point $\mathbf{C}_{i,j}(t)$ is a feature point of the trim. This type of feature point is referred to as continuous feature point.

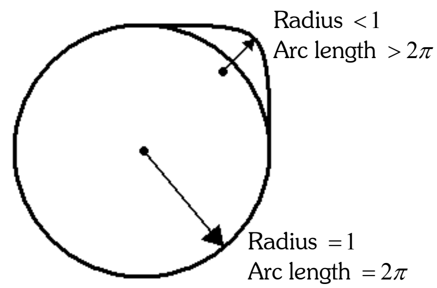


Fig. 5. Identifying sharp turns

Since a trim is composed of one or more curve segments, sharp turns may exist at the junctions of the curves. These sharp turns are called discrete feature points. Consider the junction of the curves $\mathbf{C}_{i,j}(t)$ and $\mathbf{C}_{i,j+1}(t),\ t \in [t_0, t_1]$ on trim T_i. Since the curves are C^0 continuous,

$$\begin{aligned}\mathbf{C}_{i,j}(t_1) &= (x(t_1), y(t_1), z(t_1)) \\ &= \mathbf{C}_{i,j+1}(t_0) = (x(t_0), y(t_0), z(t_0))\end{aligned} \tag{5}$$

To identify if a junction is a discrete feature point, the angle between the slopes at the junction is measured. The angle between the slope vectors is given by

$$\theta_{i,j,j+1} = \pi - \cos^{-1}\left(\frac{\dot{\mathbf{C}}_{i,j}(t_1) \cdot \dot{\mathbf{C}}_{i,j+1}(t_0)}{\left|\dot{\mathbf{C}}_{i,j}(t_1)\right|\left|\dot{\mathbf{C}}_{i,j+1}(t_0)\right|}\right) \tag{6}$$

where $\dot{\mathbf{C}}_{i,j}(t_1)$ and $\dot{\mathbf{C}}_{i,j+1}(t_0)$ are the tangent vectors of $\mathbf{C}_{i,j}(t_1)$ and $\mathbf{C}_{i,j+1}(t_0)$ respectively. In general, the angle θ at a sharp turn is equal or less than $\pi/2$ (Fig. 6.). The parametric coordinates of the feature points are computed using the point projection and point inversion methods [10].

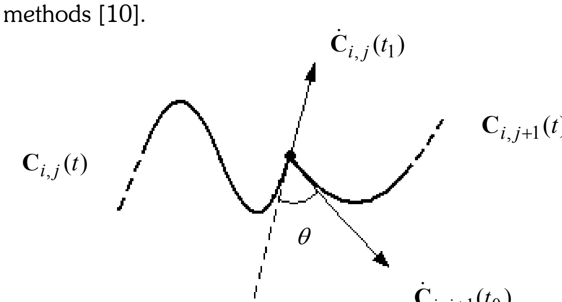
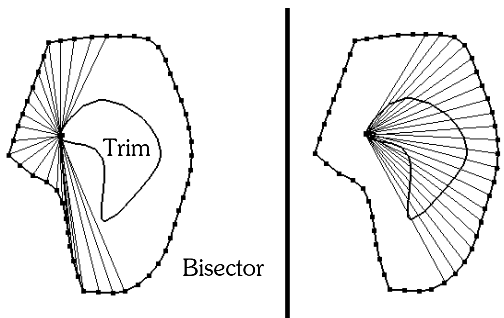


Fig. 6. Discrete feature point

7. VERTICES CORRESPONDENCE

Partitioning of the surface is attained by partitioning the Voronoï diagram. This in turn is attained by partitioning the parametric tiles constructed in the previous process. Since a tile is a loop of bisectors enclosing a trim, a segment of the trim and a corresponding bisector segment of the tile define the parametric region of a regular surface. The correspondence between segments of the trim and that of the bisectors is established by considering the correspondence between the feature points on the trims and the bisector vertices. This is achieved by considering the similarity of the feature points and the corresponding bisector vertices. The approach in [6] is adopted. A trim and its bisector loop are each enclosed in their respective minimum enclosing boxes which are normalized to unit squares. A ranking process is performed to locate the sharpest bisector vertex that is closest to a feature point. If there exists any bisector centroids on the bisector loop, correspondence is established between the bisector centroids and their nearest data points on the trim. Lines or links connecting corresponding vertices, together with the corresponding trim and bisector segments thus partition the parametric space into regions for the subsequent surface construction.



(a) (b)

Fig. 7. Correspondence links of parametric tile: (a) Valid correspondence, (b) Invalid correspondence

A basic criterion in establishing correspondence is that the line connecting a feature point and a corresponding bisector vertex cannot intersect any trim. For example, in Fig. 7., a straight line is connected between a feature point on the trim and its possible corresponding bisector vertices. The connecting lines in Fig. 7(b). intersect the trim at points other than the feature point. The corresponding bisector vertices are thus invalid points for the correspondence, whereas those bisector vertices in Fig. 7(a). are valid possible correspondence vertices.

Assume the diagonal coordinates of the enclosing rectangle for the trim are $(u^{t}_{min}, v^{t}_{min})$ and $(u^{t}_{max}, v^{t}_{max})$, and those for the bisectors are $(u^{b}_{min}, v^{b}_{min})$ and $(u^{b}_{max}, v^{b}_{max})$. Then the centers of the trim and the bisector loop are

$$(u^{t}_{cen}, v^{t}_{cen}) = \left(\frac{u^{t}_{max}+u^{t}_{min}}{2}, \frac{v^{t}_{max}+v^{t}_{min}}{2}\right) \text{ and}$$

$$(u^{b}_{cen}, v^{b}_{cen}) = \left(\frac{u^{b}_{max}+u^{b}_{min}}{2}, \frac{v^{b}_{max}+v^{b}_{min}}{2}\right) \quad (7)$$

respectively. The normalized coordinates of the feature point (u^{t}, v^{t}) and bisector vertex (u^{b}, v^{b}) are

$$(u'^{t}, v'^{t}) = \left(\frac{u^{t}-u^{t}_{min}}{u^{t}_{max}-u^{t}_{min}}, \frac{v^{t}-v^{t}_{min}}{v^{t}_{max}-v^{t}_{min}}\right) \text{ and}$$

$$(u'^{b}, v'^{b}) = \left(\frac{u^{b}-u^{b}_{min}}{u^{b}_{max}-u^{b}_{min}}, \frac{v^{b}-v^{b}_{min}}{v^{b}_{max}-v^{b}_{min}}\right) \quad (8)$$

respectively. For each parametric tile, all feature points on the trim and all valid bisector vertices are normalized using Eqn. (8). Correspondence between the feature points and the bisectors can then be established based on the normalized shapes. However, the way the shapes are positioned may affect the result. If the shapes are aligned at their centers, undesirable result may be obtained (Fig. 8.). In order to take into consideration the relative positions of the shapes, the distance between the shapes is used for positioning the normalized shapes. Denote the displacement between the shapes' centers as,

$$(\Delta u, \Delta v) = (u^{t}_{cen} - u^{b}_{cen}, v^{t}_{cen} - v^{b}_{cen}) \quad (9)$$

In the normalized shape domain, the corresponding displacement is given by

$$(\Delta u', \Delta v') = \left(\frac{\Delta u}{u^{b}_{max}-u^{b}_{min}}, \frac{\Delta v}{v^{b}_{max}-v^{b}_{min}}\right) \quad (10)$$

All bisector vertices are translated by the displacement $(\Delta u', \Delta v')$ for the subsequence ranking process. Given a feature point (u'^{t}, v'^{t}) on a trim, all valid bisector vertices (u'^{b}_{i}, v'^{b}_{i}) are ranked according to their normalized distance from the feature point, and the sharpness of the bisector at the vertex on the surface. A score is assigned to the bisector vertex and is expressed as

$$s_i = \frac{\left|(u'^{t}, v'^{t}) - (u'^{b}_{i}, v'^{b}_{i})\right|}{\max\left(\left|(u'^{t}, v'^{t}) - (u'^{b}_{j}, v'^{b}_{j})\right|\right)} + \frac{\alpha_i}{\max(\alpha_j)}$$

$$j = 0, 1, \ldots, n \quad (11)$$

where α_i is the interior angle of the i-th bisector vertex on the surface and n is the total number of valid bisector vertices. The lower is the score of the bisector vertex, the higher is the vertex's ranking. Recalling from the previous definition, a bisector centroid is a bisector vertex which is shared by three bisectors. Since a bisector centroid is where two or more regular patches meet, a bisector centroid is given a higher priority in the ranking process. A certain amount of score is deducted from Eqn. (11) if the bisector vertex being ranked is a bisector centroid. This increases the tendency of a feature point to be associated with a bisector centroid. In this research, 5% of the maximum value of s_i is deducted from s_i when a bisector centroid is processed.

If a bisector centroid is not found to correspond to any feature point, a data point on the trim closest to the bisector centroid will be selected for the correspondence. Throughout the process, tests are performed to avoid cross correspondence, i.e. a link intersecting with other links. In case there is no feature point and bisector centroid in a parametric tile, any two bisector vertices, which are half the bisector perimeter apart, will be linked to the nearest data points of the trim. A bisector vertex that has been connected to a feature point is considered as a bisector centroid in the process. In this way, this vertex will finally be connected to two feature points. The parametric tiles sharing this bisector centroid will share the same sets of bisector loop segments. This helps to avoid having a patch vertex lying on an edge of another patch.

Fig. 9. shows four examples of the partitioning in the parametric space. Correspondences are established for all the feature points and bisector centroids. In Fig. 9(d)., there is no feature point or bisector centroid on the trim and the bisector loop. In this case, two vertices from each of their bisectors link to the nearest data points on the trim.

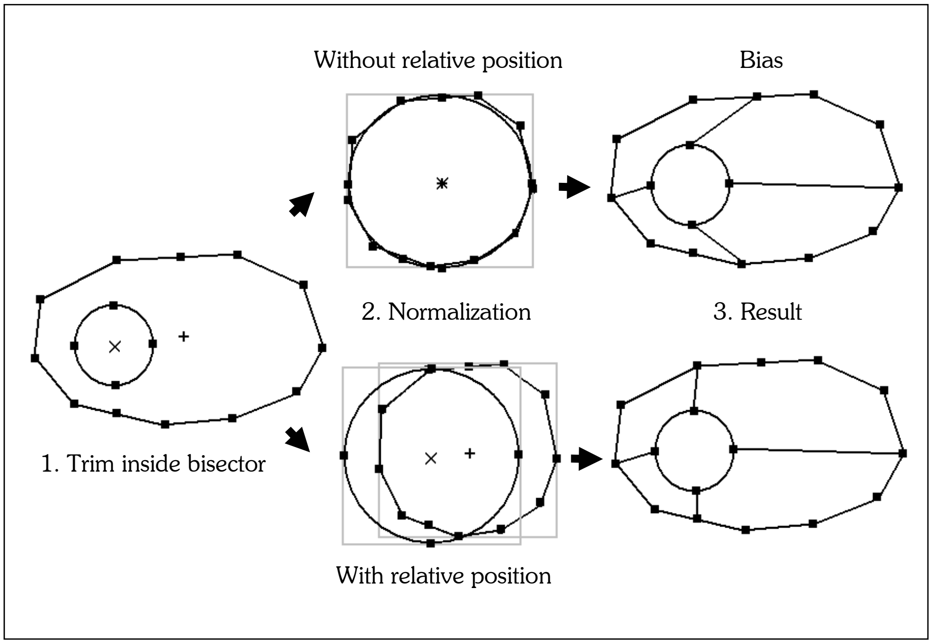


Fig. 8. The effect of relative position in normalization

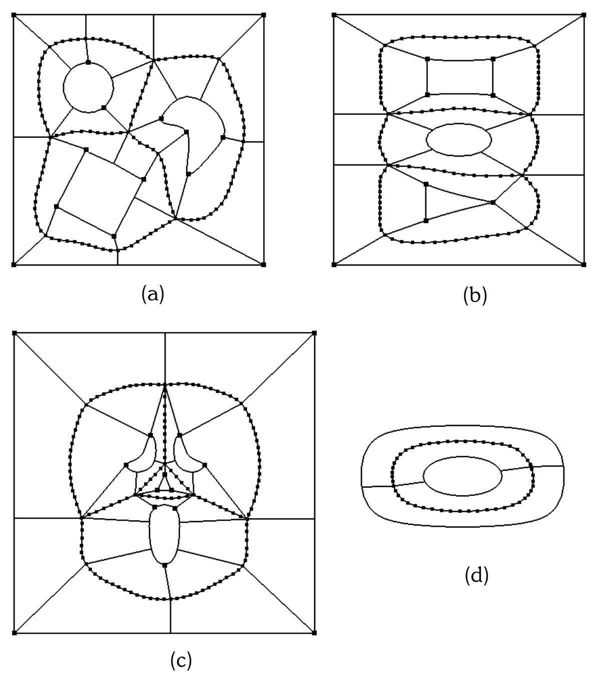


Fig. 9. Examples of correspondence

8. SURFACE FITTING

In the previous process, feature points on the trims and their corresponding bisector vertices are extracted. A pair of consecutive feature points defines a segment of the trimming curve. The bisector vertices corresponding to these feature points define a corresponding bisector segment. A trim segment and a bisector segment, together with the lines (or links) connecting the feature points and their corresponding bisector vertices defines a parametric region of a regular surface. However, there are cases when one or more bisector vertices correspond to a feature point. In this case, triangular patches with a degenerated edge are created. Parametric grids are constructed in the patch regions. Surface points are generated at the grid points for the subsequent surface fitting. A local parametric coordinate system is defined for each parametric region. The u-direction of the patch is defined along the bisector and the trim segments. The v-direction is defined along the links connecting feature points and bisector vertices (Fig. 10.). Assume n as the number of bisector vertices on the bisector segment, n evenly distributed points are generated on the trim segment. Denote p as the order of the patches in the u-direction. If $n < p$, then $p - n$ vertices are inserted into the segment. A point on the trim segment and a point on the bisector segment are linearly interpolated to obtain points in the v-direction of the grid. Points on the surface corresponding to the grid points are computed using Eqn. (1). Using chord-length parameterization, a surface defined by each parametric region is then constructed. Some results are shown in Fig. 11. The grids on the parametric space are shown on the left. Three different views of the partitioned surface are shown on the right. Fig. 12. shows the surface of a toy car with trims for the windshields, the wheels and the headlights.

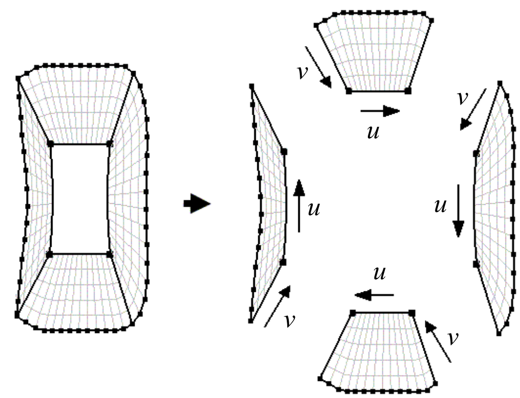


Fig. 10. Local parametric coordinate systems

Adjacent patches sharing the same common bisector share the same set of bisector vertices as sampled points on the boundaries (in the u-directions) of the patch. Along the local v-directions, adjacent patches share the same set of data points sampled on their common correspondence links. The patches are therefore C^0 continuous.

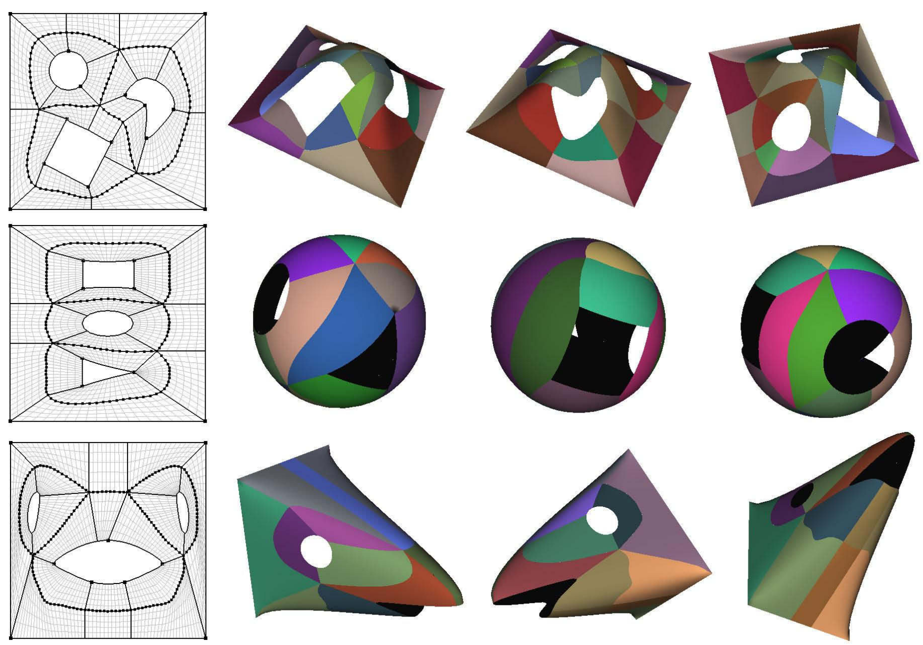


Fig. 11. Results of decomposing trimmed surfaces

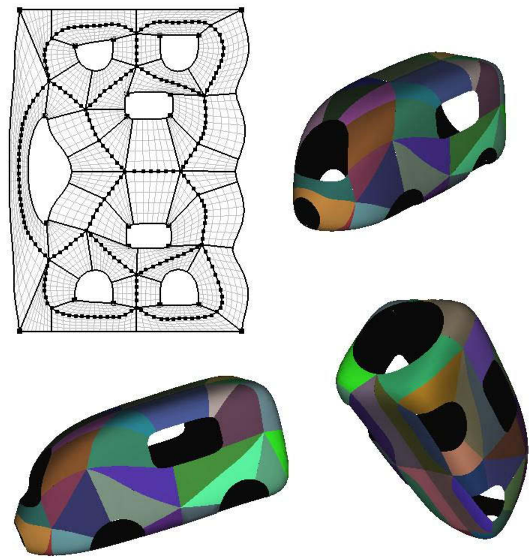


Fig. 12. Decomposing a trimmed surface of a toy car

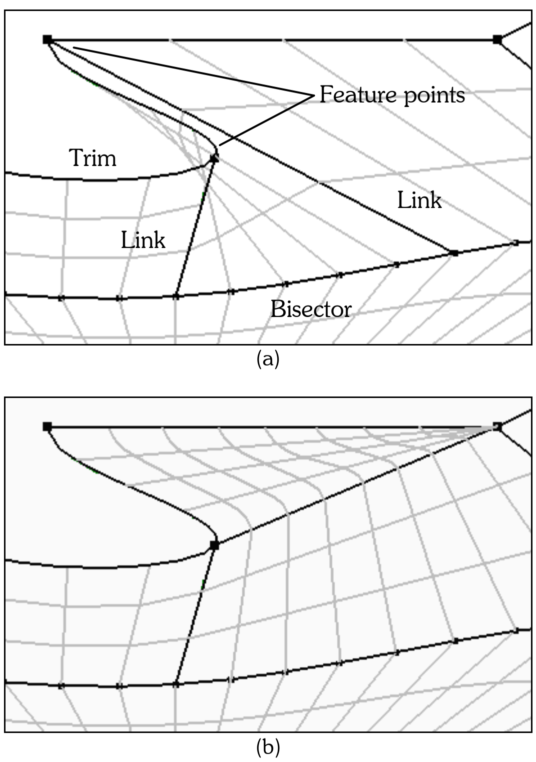


Fig. 13. S-Shaped feature: (a) Invalid patch, (b) Valid patch

C^1 continuity between the patches depends on the number of sample points used. Increasing the number of sample points decreases the distance between the points, and hence, improves the continuity between the patches. In case C^1 continuity has to be ensured, the derivatives of the surface at the sample points along the common boundaries can be used as constraints in the interpolation process.

There is a special case when the above surface fitting technique results in an undesirable surface. As shown in Fig. 13(a)., the S-shaped feature on the trim causes the extension of one of the links to intersect the other link. This causes the resulting surface to be twisted. In this respect, a degenerated patch is constructed by using neighboring points on the trim as shown in Fig. 13(b). This effectively removes the S-shaped feature so that the normal correspondence method can be applied.

9. A COMPARISON

Both the feature-based technique and Hamann's approach effectively decompose a trimmed surface into a set of regular B-spline surfaces. However, using Hamann's technique, the number of patches developed depends on the number of local extremas on the bisectors and trimming curves. For the feature-based approach, the number of patches depends on the number of the feature points on the trims and the number of bisector centroids. The maximum number of patches is of the order $O(f + nm)$, where f is the total number of feature points, n is the number of trims, and m is the total number of bisector centroids. Unlike Hamman's approach, the feature-based technique considers the boundary of a surface as a trim as well. One more bisector is developed for the boundary trim in the feature-based algorithm. As a result, one more parametric tile has to be processed. In general, with the same surface, fewer patches are usually obtained using Hamann's method. However, there are cases as shown in Fig. 14., when the number of patches obtained with the feature-based approach is less than that obtained with Hamann's method. This usually occurs when the number of extremas is larger than the number of feature points. Fig. 14(a). and Fig. 14(b). shows the result of decomposing a surface using respectively Hamann's and the feature-based method.

In the feature-based algorithm, the shape of the patches created depends on the shape of the trims and the bisectors. In most cases, the patches are four-sided. Degenerated patches may be avoided by not allowing two or more bisector vertices to correspond to the same feature point, or vice versa. The shapes of the patches created using Hamann's method depend on the distribution of the extremas. Patches are degenerated wherever there is a single pair of adjacent maxima and minima. Moreover, narrow patches may be obtained as shown in Fig. 14(a).

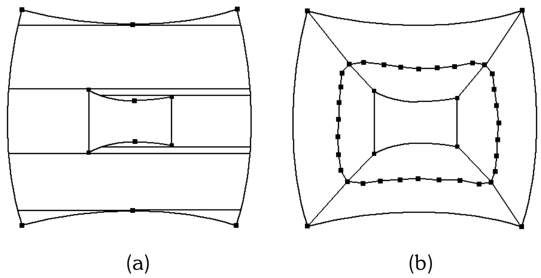


Fig. 14. An example using Hamann's method and the feature-based technique: (a) Result using Hamann's method, (b) Result using the feature-based technique

10. CONCLUSION

A method for decomposing a trimmed surface into a set of regular B-spline surfaces is presented. Voronoï diagram is developed in the parametric space of a trimmed surface. Bisectors in the Voronoï diagram isolate the trims from each other in the parametric space. Feature points are detected by locating sharp turns on the trimming curves. Correspondence between the feature points and vertices on the bisectors of the Voronoï diagram are established. By connecting feature points and the corresponding bisector vertices, the parametric space is partitioned into regions. A set of surface points corresponding to points in each of the parametric regions is generated. A B-spline surface interpolating the surface points corresponding to each parametric region is determined. Comparing with Hamann's approach, the feature-based method generates more patches. However the patches created with the feature-based method are more regular in shape and size.

11. ACKNOWLEDGEMENT

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12. REFERENCES

[1] Abi-Ezzi, S. S. and Subramaniam, S., Fast dynamic tessellation of trimmed NURBS surfaces, *Eurographics '94*, Vol. 13, No. 3, 1994, pp 107-126.

[2] Cho, W., Maekawa, T., Patrikalakis, N. M. and Peraire, J., Robust tessellation of trimmed rational B-spline surface patches, *Proceedings of Computer Graphics International CGI '98*, 1998, pp 543-555.

[3] Cho, W., Patrikalakis, N. M. and Peraire, J., Approximate development of trimmed patches for surface tessellation, *Computer-Aided Design*, Vol. 30, No. 14, 1998, pp 1077-1088.

[4] Farin, G., *Curves and Surfaces for Computer Aided Geometric Design*, 3rd Ed., Academic Press, San Diego, 1993.

[5] Hamann, B. and Tsai, P. Y., A tessellation algorithm for the representation of trimmed NURBS surfaces with arbitrary trimming curves, *Computer-Aided Design*, Vol. 28, No. 6, 1996, pp 461-472.

[6] Hui, K. C. and Li, Y. A feature-based shape blending technique for industrial design, *Computer-Aided Design*, Vol. 30, No. 10, 1998, pp 823-834.

[7] Liu, D., Dong, J. and Tong R., A new approach for tessellating trimmed parametric surfaces, *The Fifth International Conference for Young Computer Scientists Aug. 1999*, Nanjing, P. R. China.

[8] Piegl, L. A. and Richard, A. M., Tessellating trimmed NURBS surfaces, *Computer-Aided Design*, Vol. 27, No. 1, 1995, pp16-26.

[9] Piegl, L. A. and Tiller, W., Geometry-based triangulation of trimmed NURBS surfaces, *Computer-Aided Design*, Vol. 30, No. 1, 1998, pp 11-18.

[10] Piegl, L. A. and Tiller, W., *The NURBS Book*, 2nd Ed., Springer-Verlag, New York, 1997.

[11] Preparata, F. P. and Shamos, M. L., *Computational Geometry*, 3rd Ed., Springer-Verlag, New York, 1990.

[12] Vries-Baayens A. E. and Seebregts C. H., Exact conversion of a trimmed nonrational Bezier surface into composite or basic nonrational Bezier surfaces, *Topics in Surface Modeling*, SIAM, Philadelphia, PA, 1992, pp 115-143.