Heterogeneous Object Design with Material Feature Blending

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ABSTRACT

A feature-based method is proposed to represent and design heterogeneous objects. Interrelations between the material governing features and material attributes are established and retained in the object model. Free-form functions are used to represent complex shapes of geometry and material features. Material properties are blended using a lofting process. An optimization problem is then constructed based on the object's functional requirements to calculate the optimum material variation. Variant models are easily generated by changing the geometric and material features using the constraints between them.

Keywords: heterogeneous object design, design optimization, material features.

1. INTRODUCTION

Unlike homogeneous objects, heterogeneous objects are made of different materials and are capable of satisfying multiple and conflicting functional property requirements. Besides geometry, the material variations of a heterogeneous object also need to be designed so that it can satisfy the functional requirements. It is possible to represent the continuous material variation with relation to the object's geometric features. In such a scheme, the geometric features can be changed to obtain variant models. Therefore, feature-based design methods [1] can be adapted to heterogeneous object design. However, obtaining the best material variation may not always be an easy task because there might be a large number of unknown design variables to solve for simultaneously. In these cases, suitable optimization techniques can be employed.

This paper proposes a heterogeneous object design methodology which involves establishment and integration of geometric and material features. Freeform features are used to model both geometry and material attributes. This method uses its own modeling technique and applies an optimization process to develop heterogeneous object model that best satisfies all the design requirements.

The rest of the paper is organized as follows: Section 2 reviews the literature. In Section 3, the developed methodology for feature based design of heterogeneous objects is described. Section 4 explains the methodologies for material variation design. Section 5 describes the optimization processes to establish the material variations. Implementation and examples are presented in section 6. Finally, conclusions are drawn in Section 7.

2. LITERATURE REVIEW

Several heterogeneous object modeling techniques have been proposed: the r_m-object approach by Kumar and Dutta [2], constructive methods of heterogeneous object representation by Shin and Dutta [3], grading-source based approach by Siu and Tan [4] and its application to fiber type reinforcement composites modeling [5]. In these research works, material variations were assumed to be given *a priori*.

The voxel based method by Ma *et al* [6] and the finite element based method by Jackson *et al* [7] have discretized the object into smaller units. Constant material compositions are assigned separately to each units (voxels). Voxel based methods are based on discrete units and their accuracy is determined by the number of voxels used in the model. Therefore the voxel based representation may not be as accurate as a continuous representation. Moreover a voxel model after calculated cannot be altered to obtain similar variant models.

A few researches on design optimization have been reported, such as design of heterogeneous flywheel [8] and injection mold cooling systems [9]. The methods used in these researches appear to be rather case-specific

and possible extensions and applications of these methods to generic design cases require further study.
Conventionally, in a heterogeneous object design problem, the designer assumes that the material varies according to a polynomial of a certain order whose coefficients need to be calculated [10]. This method may sometimes be erroneous because the actual variation may not be adequately represented by a polynomial.

Biswas et al [11] have shown that any material function can be converted to a canonical form of material variation based on Taylor series approximation. The canonical form is an approximate polynomial function of Euclidean distances.

In [12] by Qian, the author presented feature-based design and fabrication methodologies for heterogeneous objects. The author developed a direct face neighborhood alteration method [13] for combination operations for heterogeneous object features and a physics (diffusion) based method [14] to specify the material variation. In [14], material properties are specified only at the object control points, of the geometry. Knot vector insertion is used to present material variation when the material variation does not follow the geometry of the object.

In a prototype CAD system developed by Bhashyam *et al* [15], the material variations in the model are chosen from in-built library functions which are mostly expressed as polynomials. The library functions are not derived as a part of the design process, rather they are collected from papers on manufacturing listed in the literature.

The axiomatic design principle based approach by Chen and Feng [16] involves discretizing the object into "regions." In this method, constant material composition is stored explicitly in every region, and therefore memory requirement for complex objects might be prohibitively large.

In this paper, the geometry-material relationships inside an object are maintained by using feature based design methods. Freeform features [17, 18] are used to model arbitrary shaped objects. To represent the continuous spatial distribution of material compositions, B-spline functions are used which can represent virtually any shape of material variation. These functions are not known *a priori* but are derived through an optimization process presented in [19]. The details of the proposed methodology are presented in the following sections.

3. FEATURE-BASED DESIGN OF HETEROGENEOUS OBJECTS

In the feature based design approach [1], relationships among object features are constrained by means of various parameters. By means of *variational design* process, features of an existing model are changed to obtain a new model called a *variant* in which all the constraints of the parent model are maintained. In a similar fashion in the context of heterogeneous objects, the material attributes are developed with relation to the geometric features in this paper. These relationships are established as object-material constraints. The principles of feature based heterogeneous object modeling are discussed in the following subsection.

3.1 Feature-Based Heterogeneous Object Modeling

In feature-based design, the required object properties are specified, either explicitly or implicitly, at some of its form features. These features dictate both the object geometry shape and the constituent material. They are termed as *material governing features* (GF).

However, there can be different property requirements at more than one feature in a single object, which introduces a conflict of material selection. Usually, in such cases, the best solution is to use more than one constituent material. Since the material composition plays an important role behind the object's performance, the variation of material composition is defined as a *material feature*. In this paper, it is assumed that the number of material features of an object is same as the number of primary materials the object is composed of.

To represent a feature based heterogeneous object, a new model is introduced in our earlier work [19]. A point-set constituting a heterogeneous object made of n primary materials $M_1, M_2, ..., M_n$ is denoted as follows:

$$\begin{aligned}
&\mathbf{O} = (\mathbf{F}, \mathbf{R}, \mathbf{M}, \mathbf{C}) \\
&\mathbf{F} = \left\{ \{FF_a\}_{a=0,\ldots,A}, \{GF_b\}_{b=0,\ldots,B} \right\} \in E^3 \\
&\mathbf{R} = \{R_{a_1 a_2}\}_{a_1=0,\ldots,A;\, a_2=0,\ldots,A;\, a_1 \neq a_2} \\
&\mathbf{M} = \left\{ \{M_k, v^{(k)}(s,t)\}_{k=1,\ldots,n} \right\}, \{M_k\} \in M^n \\
&\mathbf{C} = \{C_b\}_{b=0,\ldots,B} \qquad (1)
\end{aligned}$$

where E^3 is the three-dimensional Euclidean space and M^n is the n-dimensional material space. The model hierarchy is shown in Fig. 1. The object $\mathbf{O}$ has a geometry feature set F which contains a subset of form features, $\{FF_a\}$, and a subset of material governing features $\{GF_b\}$.

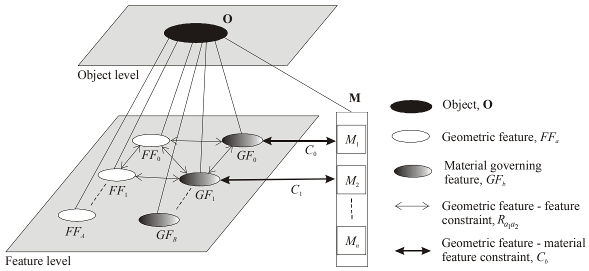

Fig. 1: Proposed feature model hierarchy [19].

The geometric constraint set, **R**, specify the relationships between the various form features of the object. The material composition vector, **M**, contains the n primary materials as its elements. Each material feature consists of the name of the primary material M_k and the mathematical form of its variation represented by B-spline functions $v^{(k)}(s,t)$. In other words, the actual volume fraction of a material at a point inside the object is as a function of parameters related to material governing features. Whenever a material governing feature is changed, the related material features also change accordingly which allows variant model creation.

The material variation can be 1-D or 2-D and they are represent by B-spline curves and B-spline surfaces as [19]:

$$v^{(k)}(t) = \sum_{j=0}^{\varepsilon} N_{j,\rho}(t)\, Q_j^{(k)} \tag{2}$$

$$v^{(k)}(s,t) = \sum_{j1=0}^{\varepsilon} \sum_{j2=0}^{\phi} N_{j_1,\rho}(s)\, N_{j_2,\theta}(t)\, Q_{j1,j2}^{(k)} \tag{3}$$

The overall properties at any point $P \in \mathbf{O}$ are directly proportional to the volume fractions of the constituent materials. If the materials $M_1, M_2, ..., M_n$ have associated properties $\pi_{M_1}, \pi_{M_2}, ..., \pi_{M_n}$, respectively, the overall property at a point *P* is given as:

$$\Pi^P = v^{(1)}.\pi_{M_1} + v^{(2)}.\pi_{M_2} + ... + v^{(n)}.\pi_{M_n} \tag{4}$$

where $v^{(k)}$ is the volume fraction of material M_k.

The object model in (1) contains a geometry-material relations set C. A relation specifies the datum or origin (i.e. a material feature GF_b) from where the material variation parameters are measured to calculate the variation function $v^{(k)}(r,s,t)$.

The proposed feature-based heterogeneous object modeling scheme supports creation of *variant* models. Because of the fact that all the feature-constraint sets, **R** and **C**, are retained in the variants. Variant modeling is very much suitable for designing objects with similar shapes. Examples are shown in Figs. 2 and 3.

In the rectangular block shown in Fig. 2(a), the material composition is varied between two material governing features, GF_1 and GF_2 and the variation direction t is from GF_1 and GF_2. Plotted next to each model are the B-spline curves that represent the material features. The curve shows how the composition of the red material (darker color), $v^{(red)}$, varies in direction t. The first and last control points of the curve are located on GF_1 and GF_2, respectively (thereby constrained). The middle control point is not constrained and is used to get a variant model in Fig. 2(b) keeping the geometric features unchanged. In Fig. 2(c), another variant is obtained by changing a few geometric features while keeping the material features same.

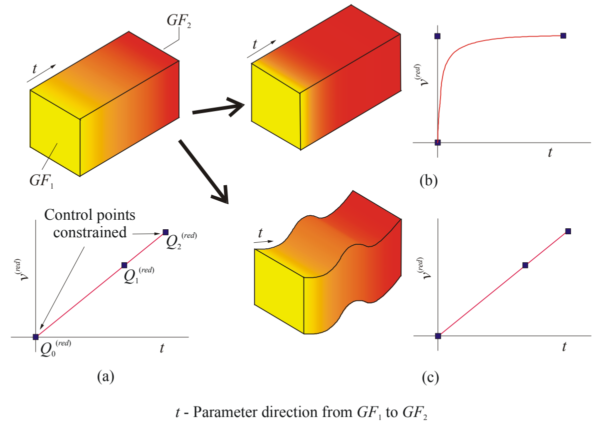

Fig. 2. 1D material feature (curves), corresponding variation in the solid model and variants, (a) initial model with 1D material features, (b) and (c) variant models.

In the block in Fig 3(a), material composition is varied between $GF_1 - GF_2$ and $GF_3 - GF_4$. The associated B-spline surfaces represent the red material features. Variant models from are shown in Figs. 3(b) and 3(c).

4. MATERIAL VARIATION MODEL: BLENDING

In this section, a mathematical model for determining the material composition variation with respect to the material governing features (*GF*s) is presented. A blending (lofting) model is proposed for representing the continuous variation of property requirements (and therefore the material composition) among a set of material governing features (*GF*s). The material features are constrained to these governing features and the direction of the variation is the same as the parametric lofting direction.

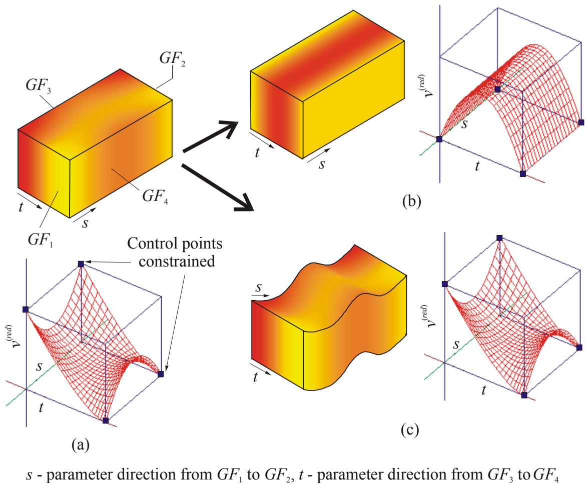

Fig. 3. 2D material features (surfaces), corresponding variations in the solid models and variants, (a) initial model with 2D material features, (b) and (c) variant models.

Traditionally, the blending or lofting operation [20] is used to generate an entity which is a blend among a set of lower dimensional entities called *generators* (curves and surfaces in 1-D and 2-D respectively). As shown in Figs. 4 and 5, the process can blend not only the geometric shape of the generators but also the property requirements at each of the generators. In the same way lofting can be used to get a smooth transition from one governing feature to another. It is assumed that each isoparametric entity in the blend direction will represent constant property requirements. Therefore, to find out and establish the material features, the loft entity must be constructed first.

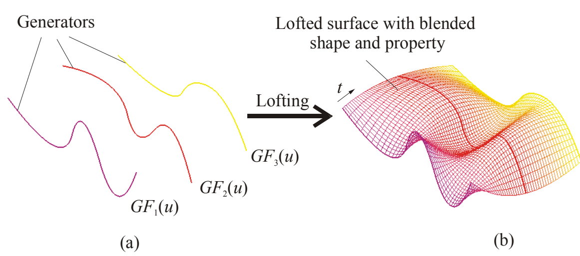

Fig. 4. (a) Generator curves with different property requirements (represented by different colors) and (b) lofted surface blends both geometric shape and property requirements.

In Fig. 6, lofting (blending) is used to represent the transition of both shape and property requirements between the governing features. Fig. 6 shows how two different material property requirements are blended together using a lofting process. Two curves, $GF_1(u)$ and $GF_2(u)$ are exposed to high and low temperatures, respectively and therefore exhibit different property requirements. A loft surface is constructed using $GF_1(u)$ and $GF_2(u)$ as generators. The surface is the geometric domain through which heat will flow from $GF_1(u)$ to $GF_2(u)$. It is a known fact that from a heated body, heat flows in the normal direction from every point of the body. Therefore, the isoparametric curves on the loft surface will represent iso-conditions (iso-temperatures). In Fig. 6, the thicknesses of the curves show the temperature intensity and therefore different property requirements.

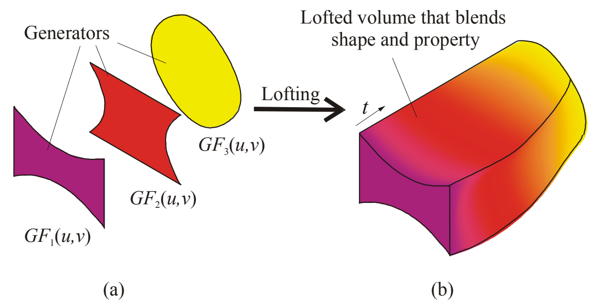

Fig. 5. (a) Generator surfaces with different property requirements (represented by different colors) and (b) lofted volume blends both geometric shape and property requirements.

The above discussion implicitly results into a proposition that the property requirement at a point can be expressed as a function of the parametric distance from a governing feature, *GF*. In prismatic (regular) shaped objects, these functions are available.

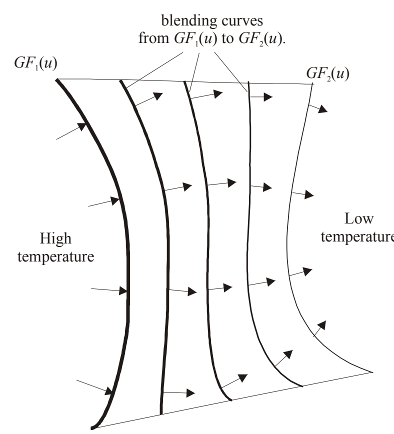

Fig. 6. Isoparametric curves between $GF_1(u)$ and $GF_2(u)$.

As example, consider a pressure vessel carrying fluid at high temperature T_{in} under high pressure P_{in} and the outside of the vessel is exposed to ambient pressure P_{out} and temperature T_{out}. The vessel of length L has its inner and outer radii equal to R_{in} and R_{out}, respectively. The vessel is given as a feature model with four form features, namely inside, outside, top and bottom, as shown in Fig. 7(a). The designer identifies the inside and outside surfaces of the vessel as the material-governing features, GF_1 and GF_2, as shown in Fig. 7(a). The material variation vector *t* is the parametric direction from the inside surface GF_1 to the outside surface GF_2, which means, the material varies in the radial direction.

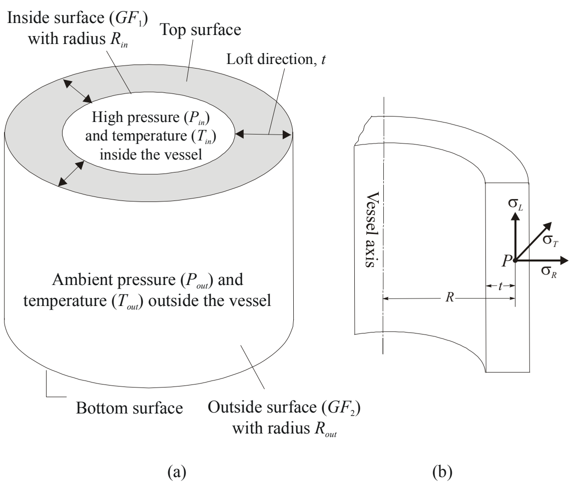

Fig. 7. (a) Feature based pressure vessel model and (b) various stress components at point P within the vessel.

The pressure and temperature gradients together develop thermo-mechanical stresses inside the vessel. At a point *P*, which is at a radial distance *R* from the vessel axis and at a parametric distance *t* from the inner surface GF_1, there are three stress components, σ_T, σ_R and σ_L. As shown in Fig. 7(b), these components are normal to each other and they are given as [19, 21]:

$$\sigma_T = \sigma_t + \sigma_{thermal}$$

$$\sigma_t = C_1 - \frac{C_2}{t^2}; \ \sigma_R = C_1 + \frac{C_2}{t^2}; \ \sigma_L = +\sigma_{thermal}$$

$$\sigma_{thermal} = \alpha(t)E(t)[T(t) - T_{out}]; \ t = \frac{R - R_{in}}{(R_{out} - R_{in})} \quad (5)$$

where C_1 and C_2 are constants. Note that the stress components are given as a function of parametric distance form the governing feature GF_1. The properties $\alpha(t)$ and $E(t)$ are the local thermal expansion coefficient and the local Young's Modulus at a point with parameter *t*, respectively. Overall material properties at a point can be calculated using Equation (7).

So that the vessel does not fail under these developed stresses, the total yield strength σ_Y of the point P must be greater than the resultant von-Mises stress σ_{VM}, which is given as [19, 21]:

$$\sigma_{VM} = \sqrt{\frac{(\sigma_R - \sigma_T)^2 + (\sigma_T - \sigma_L)^2 + (\sigma_L - \sigma_R)^2}{2}} \quad (6)$$

However, in cases of freeform objects, these functions are not readily available and usually cannot be derived from the geometry. In such cases, a suitable function can be assumed which adequately represents the property requirement at a point. For example, the Equation (5) can be used for cases where the pressure vessel walls are not exactly cylindrical in shape. By properly choosing the values of the constants C_1 and C_2, a function can be constructed that will more closely match the stresses.

4.1 Lofting entity construction

As explained in the previous subsection (also shown in Fig. 5), a lofted volume is obtained by performing a lofting operation on the material governing features. A procedure for generating lofted volume can be found in the earlier paper [19]

5. MATERIAL FEATURE OPTIMIZATION

After establishing the object model and the object-material constraints, the material features need to be established. The task of establishing the material features is considered to be an optimization problem because only the optimal material feature will ensure that all the requirements are met and the objective achieved. The design methodology is depicted as a flowchart in Fig. 8.

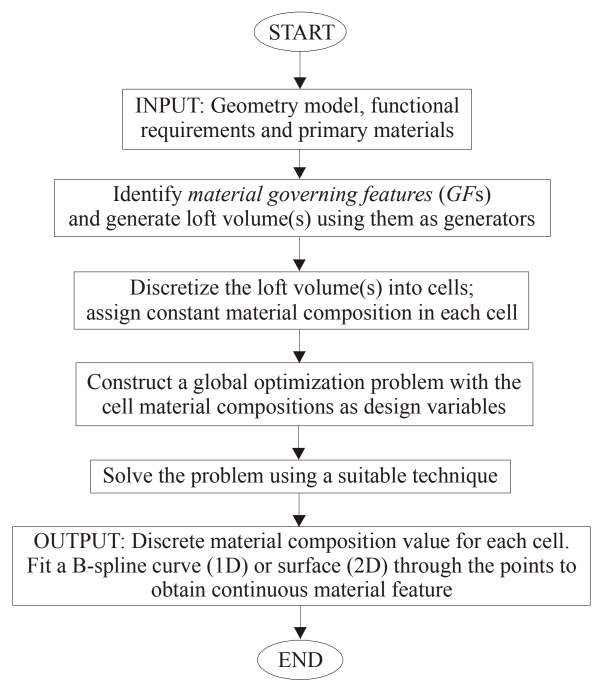

Fig. 8. Flowchart of overall design process

The actual design variables are the control points of the material variation function for each material. However, finding out all the control points simultaneously is too computationally expensive to be considered as a part of an interactive design system. Therefore, the alternative method of geometry discretizing has been employed. The lofted volume(s) are discretized into a set of disjoint cells along the lofting directions. All points in a cell are assumed to exhibit a constant property requirement. Fig. 9 shows an example of cell formation along the lofting direction *t* for the pressure vessel model mentioned in section 4. Details of cell generation can be found in our earlier work [19].

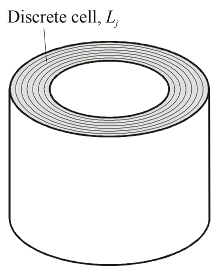

Fig. 9. Lofting and cell formation of pressure vessel model.

The material composition in a cell should be such that the resulting material property can meet the requirement at that cell. Therefore, the design variables are denoted as $\mathrm{DV}=[\mathrm{V}_j]$, where, V_j is the material fraction vector for the *j*-th cell L_j and is given as

$$\mathrm{V}_j = \left[v_j^{(1)}, v_j^{(2)}, \ldots, v_j^{(n)}\right]^T_{j=0,\ldots,\varepsilon} \quad (7)$$

and

$$\mathbf{V}_{j_1,j_2} = \left[v_{j_1,j_2}^{(1)}, v_{j_1,j_2}^{(2)}, \ldots, v_{j_1,j_2}^{(n)}\right]^T_{j_1=0,\ldots,\varepsilon;\, j_2=0,\ldots,\phi;} \quad (8)$$

for 1-D and 2-D, respectively. After the cells are formed, the design problem is formulated as an optimization problem as follows:

Min (Max): Objective function $f=(\mathbf{DV})$

Subject to:

(i) All material volume fractions must add to unity.

$$\sum_{k=0}^{n} v_j^{(k)} = 1 \qquad \forall j;$$

(ii) Inequality constraints:

$$G_p(\mathbf{DV}) \le 0 \qquad p=1,\cdots,g$$

(iii) Equality constraints:

$$H_q(\mathbf{DV}) = 0 \qquad q=1,\cdots,h$$

There are g numbers of inequality design constraints $G_1,\cdots,G_g$ and h numbers of equality design constraints $H_1,\cdots,H_h$. Examples of design constraints include upper limit of weight of object, minimum failure stress etc.

The optimization problem can be solved using a suitable solving algorithm or a commercial solver. In case of two materials M_1 and M_2, *incremental search* algorithm can be implemented to solve the problem [19]:

After the optimization problem is solved, the optimum values of $[\mathrm{V}_j]$ for each cell are known in the 1-D case. To represent the material variation as a continuous function, B-spline curves are fitted through each set of values of $v_j^{(k)}$. There will be one curve for each material M_k. A curve fitting algorithm, adapted from [20], is given in our earlier paper [19]. Similarly, for the 2-D case, the optimum values of $[\mathbf{V}_{j_1,j_2}]$ will be obtained after solving the optimization problem. To represent the material function as a continuous function, a B-spline surface is fitted through each set of values of $v_{j_1,j_2}^{(k)}$. There will be one surface for each primary material. Algorithm for surface fitting can be found in [20].

5.1 Optimizing Variant Models

The resulting object model obtained at the end of the process is called an *initial model*. Unlike the initial model which has the optimized features for a specific set of design constraints, the variants will not necessarily be the most suitable models for their respective sets of design conditions. Therefore, it might be required to construct and solve a new optimization problem to establish the material features of a variant.

However, since the variant model already has material attributes, the re-optimization will take less time than it did for the initial model. This is because the optimization process for the initial model starts form scratch where the input model had no material attribute at all. In case of re-optimization of the variant model, the existing material feature will represent an upper bound (in case of minimization) or a lower bound (in case of maximization). Therefore, the variant optimization process will be faster. This property of feature based modeling and design is very useful in case a large number of similar shaped models need to be created at an interactive rate.

6. IMPLEMENTATION AND EXAMPLES

The proposed design methodology is implemented on a PC using Microsoft Visual C++. OpenGL library functions have been used for displaying the model along with their material variations. Some example models were designed in Rhinoceros 2.0 [22] to obtain control point coordinates.

Example part I is the simplified heterogeneous pressure vessel mentioned in section 4. An optimization problem is modeled where the objective is to minimize heat flow H from inside to outside of the vessel and the constraints are to withstand the stresses in each cell. Two materials M_1 and M_2 are chosen as primary constituents. M_1 has low heat conductivity and low mechanical strength. Material M_2 has high mechanical strength but has high heat conductivity.

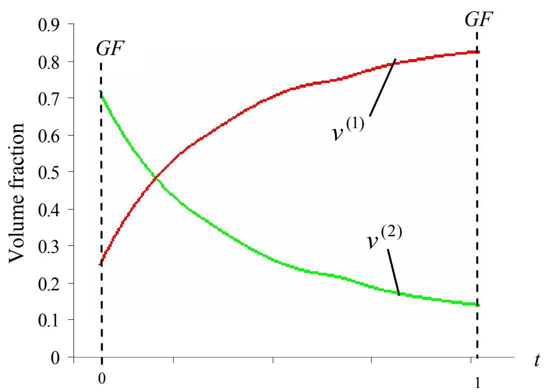

Fig. 10. Smooth B-spline curves representing material variation for Example part I.

The incremental search technique is used for solving the problem. Smooth B-spline curves are fitted through the points which represent the variation function $v^{(1)}$ for material M_1 as shown in Fig. 10. The first and last control points of the B-spline curves are constrained to be on material governing features GF_1 and GF_2, respectively. Fig. 11 shows the optimum heterogeneous model.

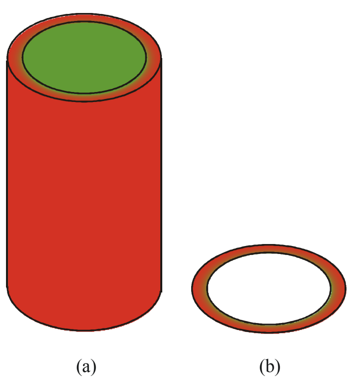

Figure 11. Example part I: optimized heterogeneous pressure vessel, (a) initial model and (b) cross section of initial model.

After the optimal heterogeneous model is designed, variant models are created by modifying the features as shown in Figs. 12(a) and 12(b). In Fig. 12(a) the variant model is cylindrical shaped where the wall thickness and the height have been changed keeping the material feature unaltered. In Fig. 12(b), a free-form model is obtained by repositioning some of the geometric feature control points from Fig. 11, while maintaining the material variation profile. As the geometry control points are repositioned to change the shape, the material feature control points automatically reposition themselves to maintain the feature relationships.

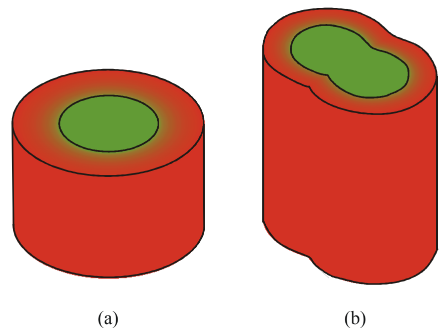

Figure 12. Variant models of Example part I, (a) cylindrical and (b) free-form.

Example part II is a mold with a freeform shaped cavity, as shown in Fig. 13(a). The molten metal is poured at a high temperature T_H and high pressure P_H. The outside surface of the cube is exposed to coolant which is at low temperature T_L and pressure P_L. The design requirement for the part is that the mold must dissipate the heat quickly to allow for rapid cooling of the molded part while withstanding the thermo-mechanical stresses developed due to the pressure and temperature gradients. Two candidate materials, M_1 and M_2 are chosen. M_1 has a high mechanical strength but low heat conducting properties whereas heat conductivity of M_2 is higher. An optimum material variation needs to be calculated to achieve the design requirements.

The material governing features are identified as the cavity surface (GF_1) and the outside cube surface (GF_2) and the lofting direction t is from GF_1 outwards to GF_2 as shown in Fig. 13(a). All the stress components are the same as Equations (5) and (6). A sectional view of the lofting and cell generation is shown in Fig. 13(b). An optimization problem is modeled with the design variables as in Equation (7).

After solving the problem, smooth B-spline curves are fitted to represent the material variations which are shown in Fig. 14. The output design with material variations is shown in Fig. 15.

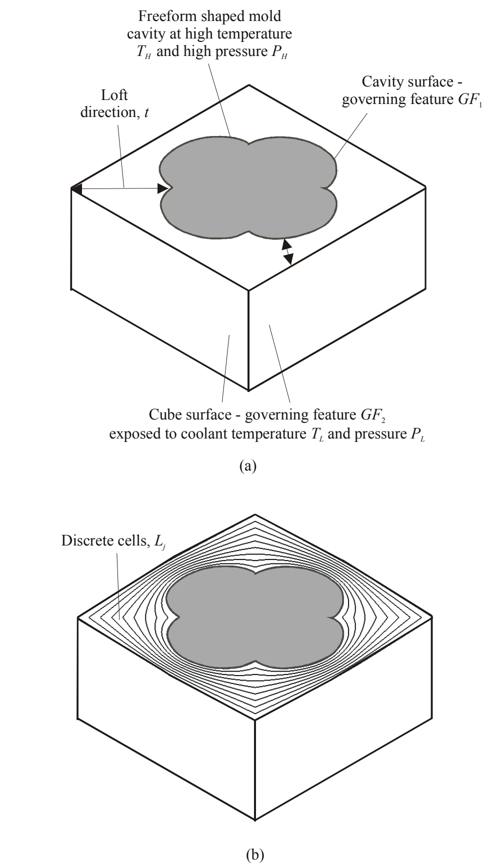

Fig. 13. (a) Example part II: mold with a freeform cavity and (b) blending and cell formation.

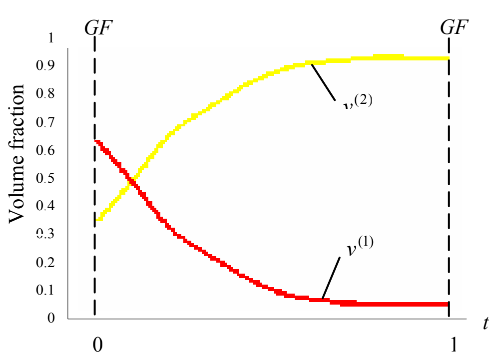

Fig. 14. Smooth B-spline curves representing material variation for Example part II.

The solid model of example part III is shown in Fig. 16(a). Property requirement at governing features GF_1 and GF_2 is different from the property requirement at governing features GF_3 and GF_4. Therefore, this is a case of 2-D material feature design. Two loft volumes are generated in the parametric directions s and t, respectively, and the cells are shown in Fig. 16 (b).

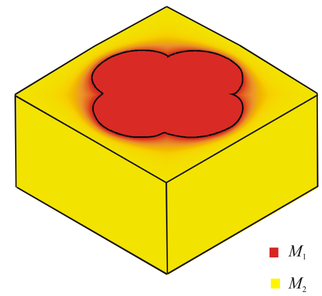

Fig. 15. Example part II: heterogeneous mold.

Two different materials M_1 and M_2 are selected as primary materials. M_1 satisfies one set of property requirement but doesn't satisfy the other whereas the reverse is the case for M_2. The design variables are same as in Equation (8). Solution of the optimization process gives the optimum values of the design variables. Two fitted B-spline surfaces that represent material features are shown in Fig. 17. The solid model with continuous material variation is shown in Fig. 18.

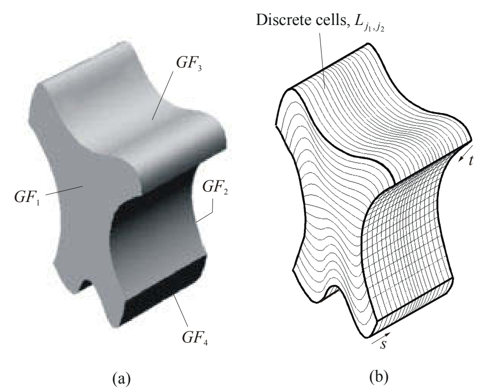

Fig. 16. (a) Example part III: Freeform solid object and (b) lofting and cell formation.

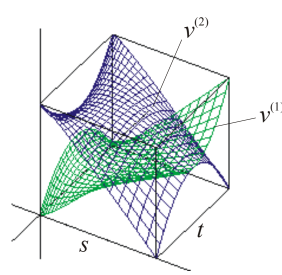

Fig. 17. Smooth B-spline surfaces representing two dimensional material features for Example part III.

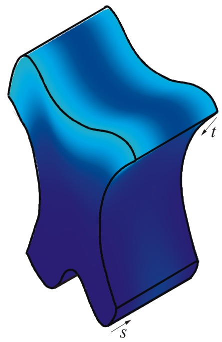

Fig. 18. Example part III: heterogeneous freeform solid object .

7. CONCLUSIONS

In this paper, feature-based design methodologies have been developed to the design of freeform heterogeneous objects. Freeform (sculptured) object features has been used to model and represent heterogeneous objects and material features. Given the initial object geometry, property requirements and candidate materials, a suitable optimization problem is formulated and solved to construct the material features. Under the assumption that the property requirement is given as a function of parametric distance from a material governing feature, this methodology will generate valid feature based objects where all the features relations are retained. Variant models are created easily by changing material or geometric features.

8. REFERENCES

[1] Shah, J. J. and Mantyla, M. *Parametric and Feature-based CAD/CAM*. John Wiley and Sons, New York, NY, 1995.

[2] Kumar, V. and Dutta, D., An approach to modeling and representation of heterogeneous objects, *ASME Journal of Mechanical Design,* Vol. 120, No. 4, 1998, pp 659–667.

[3] Shin, K.-H. and Dutta, D., Constructive representation of heterogeneous objects, *Journal of Computing and Information Science in Engineering*, Vol. 3, No. 1, 2001, pp 205–217.

[4] Siu, Y. K. and Tan, S. T., 'Source based' heterogeneous solid modeling, *Computer-Aided Design*, Vol. 34, No. 1, 2002, pp 41–55.

[5] Siu, Y. K. and Tan, S. T., Modeling the material grading and structures of heterogeneous objects for layered manufacturing. *Computer-Aided Design*, Vol. 34, No. 10, 2002, pp 705–716.

[6] Ma, D., Lin, F. and Chua, C. K., Rapid prototyping applications in medicine. Part 1: NURBS-based volume modeling, *International Journal of Advanced Manufacturing Technology*, Vol. 18, No. 2, 2001, pp 103–117.

[7] Jackson, T. R., Patrikalakis, N. M., Sachs, E. M., and Cima, M. J., Modeling and designing components with locally controlled composition, in *Proceedings of Solid Freeform Fabrication Symposium*, August, Austin, Texas, 1998.

[8] Huang, J. and Fadel, G. M., Heterogeneous flywheel modeling and optimization, *Materials & Design*, Vol. 21, No. 2, 2000, pp 111–125.

[9] Huang, J. and Fadel, G. M., Bi-objective optimization design of heterogeneous injection mold cooling systems, *ASME Journal of Mechanical Design*, Vol. 123, No. 2, 2001, pp 226–239.

[10] Markworth, A. J. and Saunders, J. H., A model of structure optimization for a functionally graded material, *Mat. Letters*, Vol. 22, 1995, pp 103–107.

[11] Biswas, A., Shapiro, V., and Tsukanov, I., Heterogeneous material modeling with distance fields, *Computer Aided Geometric Design*, Vol. 21. No. 3, 2004, pp 215–242.

[12] Qian. X., *Feature methodologies for heterogeneous object realization*, Ph.D. Dissertation, University of Michigan, Ann Arbor, MI, 2001.

[13] Qian, X. and Dutta, D., Heterogeneous object modeling through direct face neighborhood alteration, *Computers and Graphics*, Vol. 27, No. 6, 2003, pp 943–961.

[14] Qian, X. and Dutta, D., Physics based modeling for heterogeneous objects, *ASME Transactions Journal of Mechanical Design*, Vol. 125, No. 3, 2003, pp 416–427.

[15] Bhashyam, S., Shin, K. H. and Dutta D., An integrated CAD system for design of heterogeneous objects, *Rapid Prototyping Journal*, Vol. 6, No. 2, 2000, pp 119–135.

[16] Chen, K. and Feng, X., Computer-aided design method for the components made of heterogeneous materials, *Computer-Aided Design*, Vol. 35, No. 5, 2003, pp 453–466.

[17] Au, C. K. and Yuen, M. M. F., A semantic feature language for sculptured object modeling, *Computer-Aided Design,* Vol. 32, No. 1, 2000, pp 63–74.

[18] Berg, E. V. D., Bronsvoort, W. F. and Vergeest, J. S. M., Freeform feature modeling: concepts and prospects, *Computers in Industry*, Vol. 49, No. 2, 2002, pp 217–233.

[19] Samanta, K. and Koc, B., Feature-based design and material blending for freeform heterogeneous object modeling, *Computer-Aided Design*, (in Press)

[20] Piegl, L. and Tiller, W., *The NURBS Book*, Springer, New York, NY, 1997.

[21] Shigley, J. E. and Mitchell, L. D., *Mechanical Engineering Design*, McGraw Hill, NewYork, NY, 1977.

[22] Robert McNeel & Associates, www.rhino3d.com, Rhinoceros 2.0.