Interpolation of the Irregular Curve Network of Ship Hull Form Using Subdivision Surfaces

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ABSTRACT

One of common problems in free-form surface modeling is how to smoothly interpolate a given curve network. In ship design, ship hullform is usually designed with a curve network, and hullform surfaces are generated by filling in (or interpolating) the curve network. Tensor-product surfaces such as NURBS and Bézier patches are typical representations to this interpolating problem. However, tensor-product surface can represent the surfaces of irregular topological type by only partitioning the model into a collection of individual rectangular patches or using degenerated patches. Then, adjacent patches should be explicitly stitched together and modified to meet complicated geometric continuity constraints. Subdivision surface offers an alternative. They are capable of modeling everywhere-smooth surfaces of arbitrary topological type using a simple recursive subdivision rule as a whole surface. In this study, we suggest a method of automatically generating hullform surface by interpolating the given irregular curve network using Catmull-Clark subdivision surfaces. Suggested method has an advantage over previous works in that it does not require any additional data except the given curve network. There are three steps in our method; generating initial meshes automatically, modifying the initial meshes for curve interpolation, and recursively subdividing the modified meshes for smooth hullform subdivision surfaces. In this paper, we concentrate on describing the first step, automatic generation of initial meshes. In addition, we also show several examples of resulting initial meshes from curve network data of actual ship hullform to justify the proposed method.

Keywords: Subdivision Surface, Curve Network, Interpolation, Ship Hullform Surface.

1. INTRODUCTION

Curve network is often used to design complicated free-form surfaces. Using the curve network is easier than directly manipulating control points of surfaces, and it also enables more intuitive modeling. Especially, in ship design, ship hullform is usually designed with a curve network that consists of several cross sectional curves and characteristic curves. Because of the complicated shape of ship hullform, a curve network around the fore- and after-body includes some irregular (i.e. three, five or higher sides) topological regions (Fig. 1). Hull-surface then can be generated by filling in (or interpolating) the curve network with appropriate surface patches. Tensor-product surfaces such as NURBS and Bézier patches are typical representations to this filling problem. However, tensor-product surface can only represent the surfaces of irregular topological type by partitioning the model into a collection of individual rectangular patches or using degenerated patches. Then, adjacent patches should be explicitly stitched together and modified to meet complicated geometric continuity constraints such as C^1, C^2, G^1, or G^2 conditions. In general, interpolating such irregular topologies using tensor-product surfaces is not a trivial problem [1, 2].

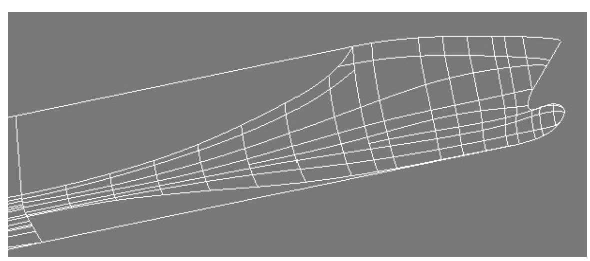


Fig. 1. Example of a irregular curve network for ship hullform design

Subdivision surface, first introduced by Doo-Sabin and Catmull-Clark, is an alternative to tensor-product surfaces. They can easily generate smooth surfaces of arbitrary topological type by recursively subdividing initial polyhedral meshes with C^1 or C^2 continuity [3, 4]. But, boundary control and curve interpolation of subdivision surfaces are important problems to be solved. Recently, several researches on interpolating specified curves using subdivision surfaces are presented. Levin [5, 6] proposed Combined Subdivision Scheme which modifies the subdivision rules around curves to be interpolated, while Nasri [7-12] modifies the initial polyhedral meshes for curve interpolation instead of changing subdivision rules. However, in addition to user-specified curves that should be interpolated, both of them require initial polyhedral meshes as input, which are usually not available in initial ship design stage in practice. We give a method how to build the initial polyhedral meshes utilizing Coons patches [16].

We suggest a method that can automatically generate hullform surfaces from given irregular curve network using Catmull-Clark subdivision surfaces without any additional inputs except given curve network. Suggested method consists of three steps. In the first step, initial polyhedral meshes are automatically generated by using piecewise bilinearly blended Coons patches and regular N-sided control meshes [18]. Then, in the second step, the initial meshes are modified for their limiting subdivision surfaces to interpolate given curve network. In the last step, smooth subdivision surfaces are obtained by recursively subdividing the modified initial meshes. In this paper, we concentrate on describing the first step, automatic generation of initial meshes (Section 3). Several examples of initial meshes generated from actual hullform curve network data are included to justify our proposed method in Section 4. Finally, concluding remarks and future researches are discussed in Section 5.

2. BASIC IDEA

While subdivision scheme can generate smooth limiting surfaces of arbitrary topological types by recursively applying simple subdivision rules, it is difficult to generate surfaces interpolating specified curves, which is actually an inverse problem. Related works on this problem and basic idea of our proposed algorithm are summarized in this section.

2.1 Related Works

Levin [5] proposed combined subdivision scheme which modifies subdivision rules for interpolating boundary curves. And Levin also extended his scheme to interpolate curve network that consist of inner curves and boundary curves [6]. Levin's scheme is simple to generate smooth surfaces with arbitrary topological type that interpolate networks of curves given in any parametric representation, but it cannot handle extraordinary points where more than three curves meet, which is frequent in curve network especially for ship hullform design. And initial polyhedral meshes whose limiting surfaces interpolate the given curve network are also required as input, which are not available in ship hullform design.

Nasri [7-15] suggested algorithm which generates face strips that converge to uniform quadratic (or cubic) B-spline curve by Doo-Sabin (or Catmull Clark) subdivision rules. These face strips are named as polygonal complexes. Using this algorithm, Nasri proposed two approaches that generate smooth subdivision surfaces interpolating given uniform quadratic B-spline curves: complex approach [11] and polygonal approach [10]. In complex approach as shown in Fig. 2(a), polygonal complexes of the given interpolated curves should firstly be provided by the user based on Nasri's algorithm. Next, the user has to complete the polyhedral meshes by filling in some vertices inside the regions enclosed by the polygonal complexes. So, the complex approach is not suitable for automatic surface generation. On the other hand, polygonal approach as shown in Fig. 2(b) is to start with an initial polyhedral meshes and tagged control polygons whose vertices and edges are chosen from those of initial meshes to define the curves to be interpolated. The basic idea is to construct polygonal complexes, one for each curve, by modifying some panels of the initial meshes or its one time uniform subdivision depending on whether the curve to be interpolated is a boundary one or an interior one. However, Nasri's method also needs initial polyhedral meshes as given data. Fig. 2 shows the procedure of generating subdivision surfaces interpolating given uniform quadratic B-spline curve using complex approach and polygonal approach, respectively.

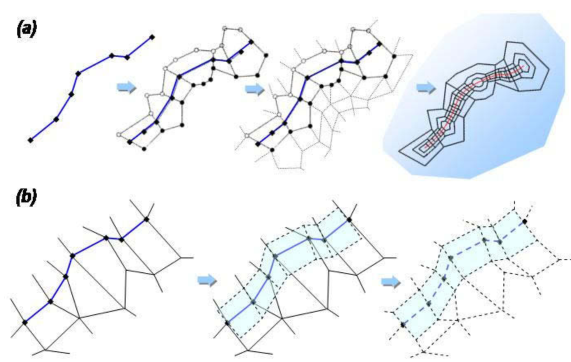


Fig. 2. Procedure of complex approach (a) and polygonal approach (b) in Doo-Sabin Subdivision setting

2.2 Basic idea of proposed algorithm

In this study, we suggest a method that can automatically generate hullform surfaces from given irregular curve network by using subdivision surfaces based on Nasri's polygonal approach. While the polygonal approach requires initial polyhedral meshes that include control polygons of curves to be interpolated, only curve network is available in ship hullform design. The basic idea to obtain initial meshes is to use bilinearly blended Coons patches that can be generated only with 4 boundary curves. If boundary curves can be represented in B-spline or Bézier form, their bilinearly blended Coons patches becomes B-spline or Bézier surfaces. Since the control meshes of the bilinearly blended Coons patches includes control polygons of their boundary curves (Fig. 3), these meshes can be used for initial polyhedral meshes for polygonal approaches. The procedure and several issues of constructing initial polyhedral meshes are described in the following section.

3. AUTOMATIC GENERATION OF INITIAL SUBDIVISION MESHES

3.1 Control meshes of bilinearly blended Coons patches

Bilinearly blended Coons patch is the simplest solution to generate surfaces that interpolate given four boundary curves [16]:

$$\mathbf{x}(u,v) = \begin{bmatrix}1-u & u\end{bmatrix}\begin{bmatrix}\mathbf{x}(0,v)\\ \mathbf{x}(1,v)\end{bmatrix} + \begin{bmatrix}\mathbf{x}(u,0) & \mathbf{x}(u,1)\end{bmatrix}\begin{bmatrix}1-v\\ v\end{bmatrix} - \begin{bmatrix}-1 & 1\end{bmatrix}\begin{bmatrix}\mathbf{x}(0,0) & \mathbf{x}(0,1)\\ \mathbf{x}(1,0) & \mathbf{x}(1,1)\end{bmatrix}\begin{bmatrix}1-v\\ v\end{bmatrix},$$

where $\mathbf{x}(u,0), \mathbf{x}(u,1), \mathbf{x}(0,v), \mathbf{x}(1,v)$ are given boundary curves.

If all boundary curves are given in B-spline or Bézier form, i.e. by their control polygons, and opposite boundary curves are defined over the same knot sequence and of the same degree, bilinearly blended Coons patches are also represented in B-spline or Bézier form. And its control meshes can be obtained in a simple way: interpreting the boundary polygons of given boundary curves as piecewise linear curves, the resulting Coons patch would then be the control meshes of a B-spline or Bézier surface that interpolates the boundary curves [17].

Fig. 3 shows the control meshes of bilinearly blended Coons patches that interpolates given four boundary curves. The control meshes keep the control polygons (bold lines) of given boundary curves as boundary polygons. So, if this bilinearly blended Coons patch scheme is applied to each region bounded by nets of curve, we can get the control meshes of piecewise bilinearly Coons patch that is a good candidate of initial polyhedral meshes for polygonal approaches (Fig. 4). This piecewise bilinearly blended Coons patches have a drawback that they, in general, are C^0 continuous. However, the control meshes of bilinearly blended Coons patch are only used for initial input polyhedral meshes of polygonal approach and will be modified to interpolate given all curve network. So, the resulting subdivision surfaces are not affected by this drawback.

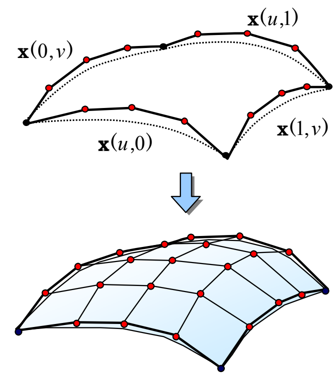


Fig. 3. Control mesh of bilinearly blended Coons patch

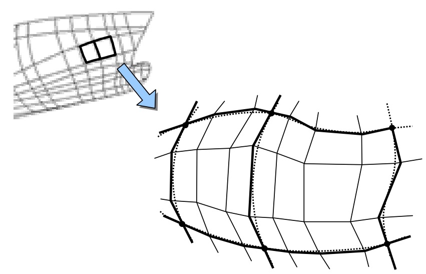


Fig. 4. Generation of initial meshes for polygonal approach using bilinearly blended Coons patches

3.2 Gap problem of the initial polyhedral meshes generated by Coons patches

To get the control meshes of bilinearly blended Coons patches, opposite curves must have the common knot sequence (or same number of curve segments). If their knot sequences are different from each other, knot insertion can be applied. For example, in Fig. 5, all boundary curves are cubic and have two different knot sequences: {0, 0, 0, 1, 1, 1} and {0, 0, 0, 0.5, 1, 1, 1}. Knot insertion operation at parameter 0.5 makes the knot sequences and the number of control polygons of opposite boundary curves same. Then, we can get the control meshes by using of bilinearly blended Coons scheme.

However, there is one problem to obtain the initial polyhedral meshes for polygonal approach from piecewise bilinearly blended Coons patches. Fig. 6 shows this situation. The control mesh of the left region can be generated by bilinearly blended Coons scheme with four boundary curves that have the common knot sequence, {0, 0, 0, 1, 1, 1}. But, the right region needs knot insertion operation for all boundary curves to have the common knot sequence, {0, 0, 0, 0.5, 1, 1, 1}.

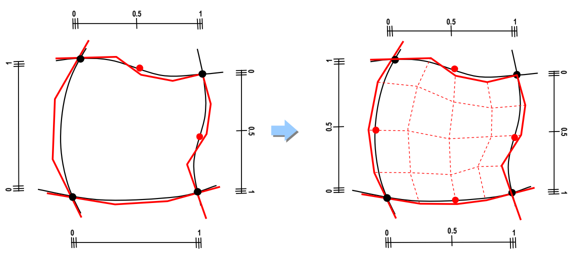


Fig. 5. Coons patch interpolating boundary curves with different knot sequence

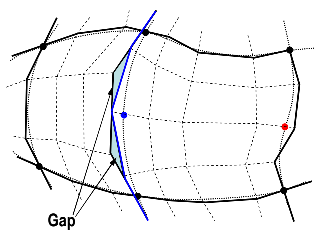


Fig. 6. Gap problem in generation of initial meshes by using of bilinearly blended Coons patches

Consequently, the gaps between the left and right control meshes may occur. Having this gap, the control mesh can not be used as initial input for the polygonal approach. One solution to this problem is mesh stitching. However, stitching may create ugly looking surface and remove 4-sided faces, along the boundary curves, that are necessary to generate Catmull-Clark polygonal complexes for curve interpolation in the mesh modification step. So, we solve this gap problem by approximating all given curve network with constant number of segments. This approximation guarantees that initial mesh can be generated without any gap.

3.3 Initial meshes for irregular topological regions

Initial meshes for rectangular regions can be obtained by bilinearly blended Coons scheme mentioned above. To complete the initial polyhedral meshes for polygonal approach, we need another algorithm to automatically generate initial meshes for non-rectangular regions (i.e., triangular region, pentagonal region, and so on). For these irregular N-sided regions ($N \neq 4$), we use regular N-sided control mesh topology (Fig. 7).

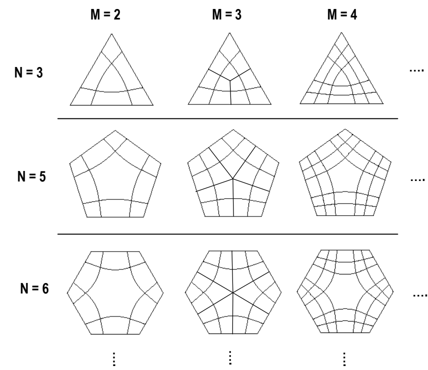


Fig. 7. Regular N-sided mesh topology
(N is the number of sides, M is the level of mesh)

Regular N-sided control mesh topology and construction algorithm are originally suggested to fill N-sided holes using Doo-Sabin subdivision surfaces [18]. We extended this algorithm for Catmull-Clark subdivision surfaces. Construction algorithm for regular N-sided control mesh is as follows:

Step 1: Generation of level M(=2) mesh (fig. 8)

- All boundary curves are approximated to uniform cubic B-spline curves with $2^{(M-2)}$ segments.
- Boundary points(black circle) of the level M mesh are set to control points of approximated boundary curves.
- N corner twist vectors are estimated by using of Adini's twist algorithm described in section 3.3.1.
- N inner-corner points(white circle) of the level M mesh are determined using estimated corner twist vectors.

Step 2: Generation of level M+1 mesh (fig. 9)

- All boundary curves are approximated to uniform cubic B-spline curves with $2^{(M-2)}$ segments.
- Generate initial level M+1 meshes by applying one step subdivision to the level M mesh.
- Boundary points of the level M+1 mesh are set to control points of approximated boundary curves.
- Inner-boundary points of the level M+1 are determined using estimated surface normal and cross boundary derivatives (section 3.3.2).
- All the others points od the level M+1 are unchanged.
- If $2^{(M-2)}$ is less than user-specified constant number of segments for curve approximation described in section 3.2, repeat Step 2.

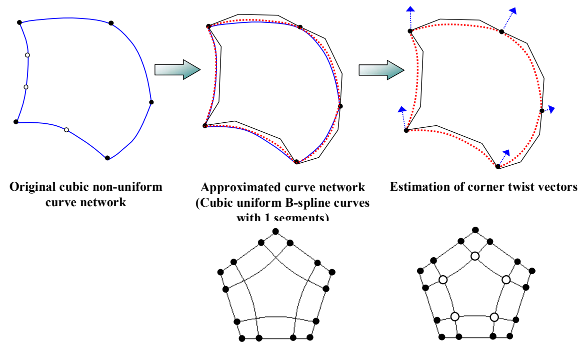


Fig. 8. Construction of regular 5-sided control mesh of level 2

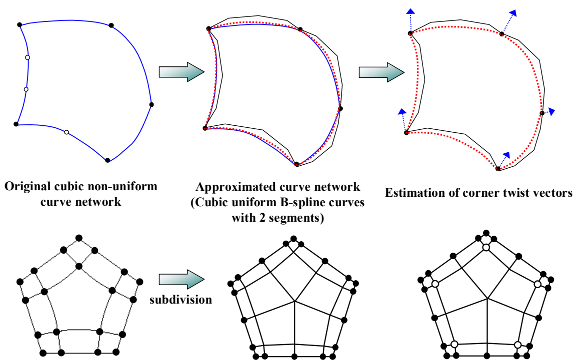
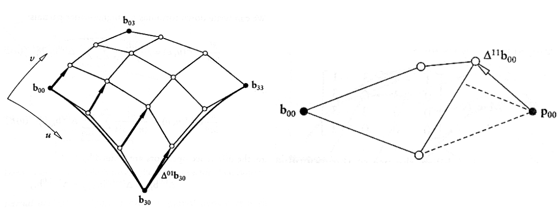


Fig. 9. Construction of regular 5-sided control mesh of level 3

3.3.1 Estimation of corner twist vectors

In control mesh of bicubic Bézier surface, geometric meaning of corner twist vector is a difference vector between an inner-corner point and parallelogram generated by corner point and two adjacent boundary points (Fig. 10).



$$Twist(u,v) = \frac{\partial^2}{\partial u \partial v} b(u,v)$$

$$\frac{\partial^2}{\partial u \partial v} b(u,v) = 3 \cdot 3 \sum_{j=0}^{2} \sum_{i=0}^{2} \Delta^{1,1} b_{i,j} B_i^2(u) B_j^2(v)$$

$$\frac{\partial^2}{\partial u \partial v} b(0,0) = 3 \cdot 3 \Delta^{1,1} b_{0,0} = 9 \cdot (\Delta^{0,1} b_{1,0} - \Delta^{0,1} b_{0,0})$$
$$= 9 \cdot ((\Delta^{0,0} b_{1,1} - \Delta^{0,0} b_{1,0}) - (\Delta^{0,0} b_{0,1} - \Delta^{0,0} b_{0,0}))$$

Fig. 10. Geometric meaning of corner twist vectors

Around a corner of regular N-sided control meshes, the adjacent rectangular polyhedron can be considered as Bézier control mesh. So, if corner twist vectors are estimated, inner-corner points of regular N-sided control mesh can be determined.

For corner twist vector estimation, we use well-known Adini's twist vector estimation method. Adini's method deals with only 4-sided regions, and requires 4 surface corner points and 4 tangent vectors of boundary curves (Fig 11).

$$\frac{\partial^2}{\partial u \partial v} x(u,v) = [-1 \ \ 1] \begin{bmatrix} \frac{\partial}{\partial v} x(0,v) \\ \frac{\partial}{\partial v} x(1,v) \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial u} x(u,0) & \frac{\partial}{\partial u} x(u,1) \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} - [-1 \ \ 1] \begin{bmatrix} x(0,0) & x(0,1) \\ x(1,0) & x(1,1) \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$\frac{\partial^2}{\partial u \partial v} x(0,0) = x_v(1,0) - x_v(0,0) + x_u(0,1) - x_u(0,0) - (x(0,0) - x(1,0) - x(0,1) + x(1,1))$$

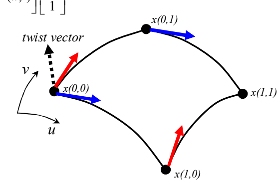


Fig. 11. Geometric meaning of Adini's twist vectors

To apply Adini's twist estimation method to non-rectangular regions, we determined the midpoint of opposite boundary curve as opposite corner point, and estimated 4 tangent vectors by simple geometric operations such as point-plane projection, linear interpolation, and so on (Fig. 12, 13). After estimation of corner twist vectors, we can determine inner-corner points of regular N-sided mesh by using of the geometric meaning of corner twist vectors.

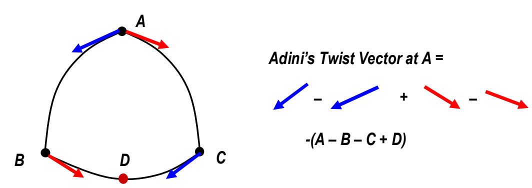


Fig. 12. Adini's twist vector estimation for 3-sided region

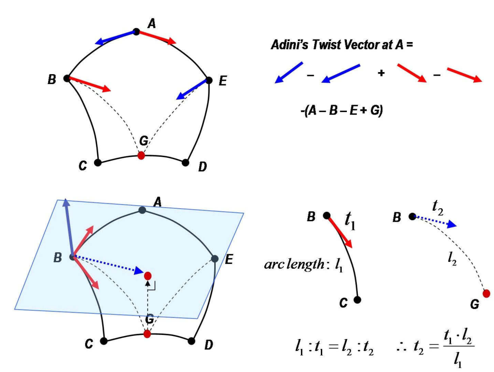


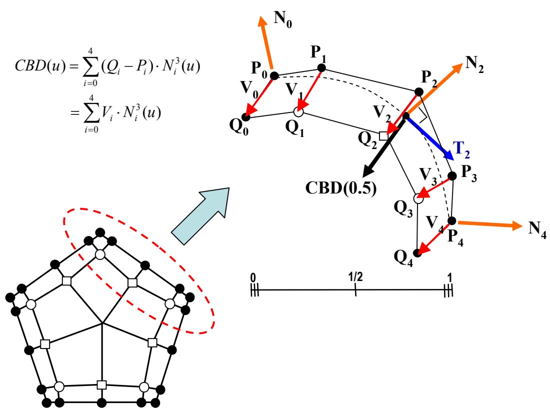
Fig. 13. Adini's twist vector estimation for 5-sided region

3.3.2 Determination of inner-boundary points

In N-sided control mesh of the level M(≥3), inner-boundary points (white rectangle in Fig. 14) are also determined for reasonable inner shape.

At each side of the N-sided region, first, surface normal vectors (N_2) along boundary curves are estimated by linearly interpolating both end surface normal vectors (N_0 and N_4) at the corner, and the cross boundary derivative vectors (CBD(0.5)) should be perpendicular to estimated surface normal vectors (N_2). Since regular N-sided control meshes can be considered as control meshes of bicubic B-spline surface around the boundary curves, the cross boundary derivative vectors can be derived from inner-boundary points. Then we can determine inner-boundary points by simple linear equations.

$$CBD(u) = \sum_{i=0}^{4} (Q_i - P_i)\cdot N_i^3(u) = \sum_{i=0}^{4} V_i \cdot N_i^3(u)$$



$$CBD(\tfrac{1}{2}) = \frac{1}{4}V_1 + \frac{1}{2}V_2 + \frac{1}{4}V_3 = \frac{1}{4}(Q_1 - P_1) + \frac{1}{2}(Q_2 - P_2) + \frac{1}{4}(Q_3 - P_3) \quad ①$$

$$CBD(\tfrac{1}{2}) = \frac{T_2 \times N_2}{\|T_2 \times N_2\|}\cdot \lambda, \quad ②$$

$$\text{where } \lambda = \tfrac{1}{2}\cdot\|V_0\| + \tfrac{1}{2}\cdot\|V_4\|$$

∴ ① = ②

Fig. 14. Determination of inner-boundary points using cross boundary derivative vectors

4. RESULT OF INITIAL MESHES

Algorithm proposed in this paper is basically based on Nasri's polygonal approaches except initial mesh generation using piecewise bilinearly Coons patches and regular N-sided control meshes. Nasri's approach has restriction that curves of network should be uniform cubic B-spline curves for Catmull-Clark setting, and this also is the same limitation in our algorithm. Since ship hullform is usually designed with non-uniform cubic B-spline curves, approximation of them to uniform cubic form is an indispensable process. For avoiding gap problems in initial meshes, there is another limitation of constant number of segments in curve approximation process.

Fig. 15 shows the curve network of actual ship hullform data, and the initial meshes generated by proposed algorithm. In general, the limiting subdivision surfaces from this initial meshes does not interpolate the given curves network. This is why mesh modification process is needed. Mesh modification is to embedding Catmull-Clark polygonal complexes that converge given curve network to the resulting initial meshes. This mesh modification algorithm for Catmull-Clark subdivision setting is almost developed. Since we are preparing this algorithm as another paper, this algorithm was excluded in this paper.

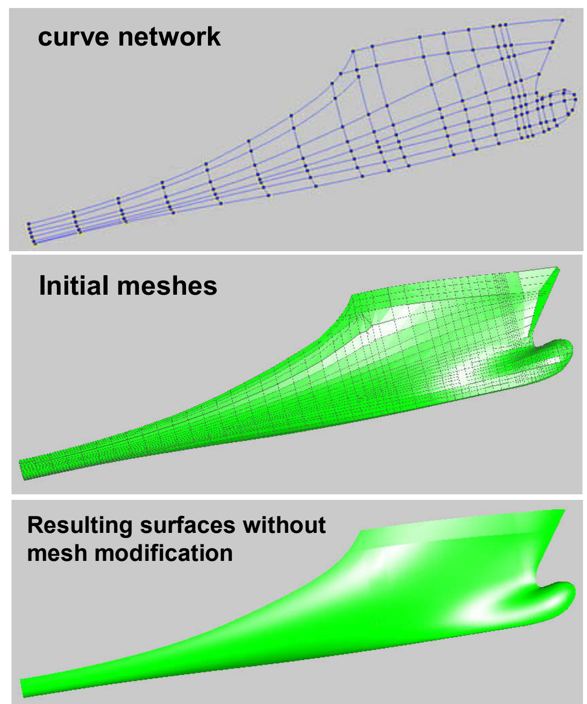


Fig. 15. Given curves network of ship hullform surfaces, resulting initial meshes and subdivision surfaces without mesh modification

6. CONCLUSIONS AND FUTURE WORK

In this paper, we suggested a method that can automatically generate hullform surfaces from given irregular curve network by using subdivision surfaces without any additional inputs except given curve network. The initial polyhedral meshes could be automatically generated from the control meshes of piecewise bilinearly blended Coons patches, and regular N-sided control meshes. Since curve network used for hullform surface is usually designed based on cubic non-uniform B-spline curves, curve approximation operation was needed. In order to generate ship hullform surfaces exactly interpolating the given curve network, researches on algorithm which makes it possible to directly interpolate non-uniform cubic B-spilne curves without curve approximation by uniform cubic subdivision surfaces can be considered.

7. ACKNOWLEDGEMENTS

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